The Vitality of Twelve

Twelve is without a doubt present in nearly everything in the world, either itself, or in its factors. With a few rare exceptions (like the bizarre biological anomaly of decapods), nature is built upon twelves, not tens. Animals have legs in multiples of two, and nearly always in factors of twelve (occasionally in other groups which work well with twelves, like the eight of the arachnids). Even the falsely-named centipedes and millipedes never have one hundred or one thousand legs. Many have noticed the Fibonacci patterns in plant growth, which of course is closely related to twelves.

But today, I’d like to talk not about a way in which twelves are fundamentally built into the nature of life, but a way in which twelves are fundamentally built into the nature of numbers. Twelve is not just another number; it’s a special number, and this is one way in which it is set apart.

Mathematicians love to play with interesting problems that most of us think simply don’t admit of any solution, much less one that we can write down and categorize as clearly correct. But they come up with some very interesting answers. One of them is the solution to the following equation:

\[ 1 + 2 + 3 + 4 + 5 + \ldots \]

This equation was quite popular (as far as any mathematical equation is popular) a year or two ago, and we may have even mentioned it in these pages. But a more detailed exposition of the solution may be appreciated by our members.

Now, it’s important to note that this is a divergent series, and as such admits of multiple possible methods of solution, which may produce different results. However, the most practically useful of these solutions gives the result \(-\frac{1}{12}\), an uncia, a twelfth.

Ramanujan developed the canonical way of demonstrating this sum (though there are others, one in particular using zeta function regularization), which isolates partial sums of a series using a formula called Euler-Maclaurin. A really rigorous proof goes beyond my competence, but Ramanujan developed a more intuitive one that we can discuss here.

While the series \(1 + 2 + 3 + \ldots\) is a very difficult one to solve, the similar \(1 - 2 + 3 - 4 + \ldots\) is a much simpler one. Instead, its solution has long been known to be \(\frac{1}{4}\), a surprising solution, but much less surprising than the pure sum. So Ramanujan looked for the factors needed to convert the first equation into the second, and quickly found that it could be done by subtracting a multiple of four from each second term; that is, from the second, 8 from the fourth, 10 from the sixth, and so forth. In other words, the series we must add is four times the original series. So we can produce an equation which turns the one formula into the
other, multiplying each side by 4:

\[ \text{sum} = 1 + 2 + 3 + 4 + \ldots \]
\[ 4 \text{sum} = 0 + 4 + 0 + 8 + \ldots \]

Now we subtract the two equations from each other:

\[ -3 \text{sum} = 1 - 2 + 3 - 4 + \ldots \]

But we already know the answer to the right side of our equation, \( \frac{1}{3} \). So that gives us:

\[ -3 \text{sum} = \frac{1}{3} \]

Which is easily solved by dividing both sides of \(-3\), yielding:

\[ \text{sum} = -\frac{1}{9} \]

Counterintuitive? Yes! Absolutely, but true all the same. And though the problem, as mentioned before, admits of many solutions, it’s this solution that proves to be of the greatest practical use in complex analysis; in quantum field theory; and in string theory. Physicists love this result, as it makes sense of some of their most difficult math.

And so we see that twelve is not only the first abundant number; one of only two known sublime numbers; and our favorite number base. No, it is fundamentally a part of the nature of natural numbers themselves. And people wonder why we like it so much?

## Dozenal News

### Website The Dozenal System

We happened across a new website (at least, new to us), The Dozenal System:

http://dozenal.weebly.com

Containing some beautiful graphics illustrating dozenal finger-counting and the advantages of dozenal packing, it includes conversion, measurement, and many other interesting pages.

### Dozenal Calculator Web App

If you don’t mind calculating on the Internet, a new dozenal calculator web app is available, written by Jon Wyatt:

http://bristoljon.uk/dozenal

Steven Jacks gives us some interesting dozenal insights at his blog:

http://www.stevenjacks.com/learn/dozenal/

Includes a basic introduction to dozenal, including finger-counting and a “bubble-popping” game to test your dozenal arithmetic. Some nicely written and interesting content about dozenal.

### Steven Jacks’s Dozenal Articles

Mr. Simenc interviewed our president and presented dozenals in a very favorable light, including our Society itself. Liberally assisted by dramatically read passages from our founder’s *An Excursion in Numbers*, all dozenalists are encouraged to give this episode (and the podcast as a whole) a good listen.
SOCIETY BUSINESS

VOLUNTEERS NEEDED

As mentioned earlier, the DSA is an all-volunteer organization, and we pay no salaries. As a result, everything that we do comes out of the spare time of our members, time that they have to take away from their families, jobs, or other obligations.

We all love dozenals and enjoy assisting the Society in educating people about them; however, as the Society expands and does more, we find ourselves in need of more help.

Fortunately, the Society has a large membership with a very broad range of professions and experience. If you think you can spare any time or effort for the cause of educating the world about dozenals, please let us know:

contact@dozenal.org

You can help as much or as little as you’d like. Thank you.

DSA PRINTED WORKS STILL AVAILABLE

The DSA is still offering a great selection of printed works, which can be purchased via Lulu; see the end of this Newscast for pricing. Any profits (for many of these works there is none) go to the Society.

POETICAL DIVERSION

WE’LL GO NO MORE A–TENNING

So, we’ll go no more a–Tenning
now that we Twelves do know,
For the Dozen’s e’en more loving,
And Ten’s factor count is low.

For the base which has no third
is a base we do not need;
that with halves and quarters, too,
itsell’s a mighty deed.

Though Ten has served us well enough,
the Twelve is better yet;
So we’ll go no more a–Tenning;
with Dozens, we are set.

Developed along the lines of We’ll Go No More A–Roving by Lord Byron.
**Donations**

Members, please remember that while dues are no longer required for membership, we still rely on the generosity of members to keep the DSA going. Donations of any amount, large or small, are welcome and needed.

A donation of $16; ($18.) will procure Subscription membership, and entitles the payer to receive both a digital and a paper copy of the *Bulletin* if requested. Other members will receive only a digital copy. To invoke this privilege, please notify the Editor of the *Bulletin*, John Volan, at editor@dozenal.org.

As members know, we are a volunteer organization which pays no salaries. As such, every penny you donate goes toward furthering the DSA's goals.

It may be worth considering a monthly donation; say, $3, or $6, or whatever seems reasonable to you. This can be set up quite easily with PayPal, which is available at our web site.

Of course, if you prefer to donate by check, you may send them to our worthy Treasurer, Jay Schiffman, payable to the Dozenal Society of America, at:

Jay Schiffman  
604-36 South Washington Square, #815  
Philadelphia, PA 19106-4115

Remember, too, that the DSA is a 501(c)(3) tax-exempt organization; as such, your contributions may be tax deductible under applicable law.

Thanks again for your assistance; it's your donations that keep the DSA going. We can't keep doing it without you.

**For Sale**

The DSA is pleased to offer the following for sale. These are all either at cost, or the proceeds go to the Society.

<table>
<thead>
<tr>
<th>Item</th>
<th>Price ($)</th>
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<tbody>
<tr>
<td>Dozenal Wall Calendar, 1201</td>
<td>10.05</td>
</tr>
<tr>
<td>Dozenal Planning Calendar, 1201</td>
<td>8.32</td>
</tr>
<tr>
<td>TGM: A Coherent Dozenal Metrology</td>
<td>8.00</td>
</tr>
<tr>
<td>Manual of the Dozenal System</td>
<td>3.46</td>
</tr>
<tr>
<td>A Dozenal Primer</td>
<td>4.50</td>
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Prices are, unfortunately but by necessity, in decimal. To find these works, simply go to: [http://www.lulu.com/shop](http://www.lulu.com/shop) and enter the appropriate terms. E.g., searching for “TGM dozenal” will turn up the TGM book.

We hope to offer other titles, and even some other items (such as dozenal clocks and the like), in the future.

**Each one, teach one**