A BASE TWELVE PUZZLE

---See page 1#
THE DOZENAL SOCIETY OF AMERICA
(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of
research and education of the public in the use of base twelve in numeration, mathe-
matics, weights and measures, and other branches of pure and applied science.

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SOLUTION TO "REMEMBERING PI"

(Winter 1991 -- p. 1#)

In the last issue of the Bulletin we asked readers to provide us with a mnemonic for remembering the duodecimal value of Pi, which was given as:

3;184 809 493 #91 866 457 3*6 212...

a total of 21 digits.

Our first solution comes from Charles Ashbacher, Hiawatha, IA whose suggestion contains 15 digits. Please note that since it is impossible to have a word of length zero, Charles simply placed the word "zero" in that position.

Gee. A computer that displays zero answering time, beguiling all individuals, beginning a stampede toward owning many.

Our second solution contains 14 digits and comes from new member Monte J. Zerger, Adams State College, Alamosa, CO. Monte uses the period between the two sentences to signify the zero in the expansion.

Ten, I maintain, isn't suitable. Examining real carefully the mathematics, discloses a superior number, twelve.

Monte also writes:

I found it pleasing that the first three digits in the base twelve representation of Pi are the first three digits in the base ten representation of 1/Pi.

Pi = (3;18)12

1/Pi = (.318)10

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.
which would be involved in introducing a systems of units completely incompatible with the present ones. (What the distant future might bring is less predictable. Circumstances could be different.) However, that does not prevent one looking into the question of the relative advantages of different scales, and, indeed, there are even new situations in which the use of a different base is possible and advantageous. For example, the rules for winning quite a lot of games depend on the use of the scale of 2.

The relative advantages of different bases depend very much on the sort of calculations one wants to do. Suppose that one wants to find the factors of a number. In the usual denary scale, one knows that a number is divisible by 2 if the last figure is even, and it is divisible by 5 if the last digit is 0 or 5. One can go further, and say that it is divisible by 4 if the number formed by the last 2 figures is exactly divisible by 4, for example, 1325244 is exactly divisible by 4. To find whether a number is divisible by 3 is more trouble; the rule is that a number is divisible by 3 if the total of the figures is divisible by 3, and it is divisible by 9 if the total is divisible by 9. Thus 1 + 3 + 2 + 5 + 2 + 4 + 4 = 21, which is divisible by 3 but not by 9, so 1325244 divides exactly by 3 but not by 9. One can construct rules for divisibility by other numbers, such as by 7, but most of such rules tend to be rather complicated. If, instead of using the scale of ten, we used that of twelve, matters would be easier. A number would then be divisible by 2 when the last figure is divisible by 2, and similarly for 3, 4, and 6. However, tests for divisibility such as these are more matters for amusement than of practical use these days. They may well have been useful in the past, when one could have to add fractions such as 1/2 + 3/5 + 5/6 + 1/4, and so had to reduce them to a common denominator. But how often does one now have to deal with fractions? Fractions can be useful in certain technical contexts, such as some algebraic calculations, but are rarely needed in everyday life, apart from halves and quarter, which can anyway be written as .5 and .25, etc.

(Continued)

The Society regrets to announce the death of member number 1*3, James A Forster, on October 28, 1990.

Mr. Forster had been a member of the DSA for over two dozen years, and had made use of the base twelve circular slide rule during a long teaching career.

We extend our deepest sympathies to his family.

Before looking into the question further, one needs to think what sorts of calculations need to be done at all often at present. The situation has changed dramatically in the last 50 years. In 1940 calculating machines would do the four elementary operations of addition, subtraction, multiplication and division, but rather slowly, and they were expensive. Anything beyond that was bothersome. Tables gave the values of square roots, logarithms, antilogs, sines, cosines, etc., conveniently to about 4 figures, but any greater accuracy involved the slow process of looking them up in substantial books of tables. Nowadays a scientific pocket calculator has a very modest price, and can perform all these calculations and many more (quite often to an accuracy of something like a dozen figures), and do them virtually instantaneously. We can classify the typical sorts of calculations which are now regularly performed somewhat as follows:

(1) Financial calculations (e.g., bank accounts). These involve doing very simple operations, such as adding, subtracting, classifying, recording, sorting large masses of data, best done by a large computer network.

(2) Technical and scientific calculations. These possibly involve doing rather complicated mathematical operations on a medium sized body of data. Usually, a moderate sized computer will cope adequately, although there are occasions on which extremely long calculations are performed, which are only possible when large versatile computers are available.

(3) Simple technical and mathematical calculations, arising from time to time in an investigation, which can be adequately performed on a pocket calculator or personal computer. With the increasingly modest prices and ready availability of calculators and simple computers, these will rarely be done by hand.

(4) Small, off-the-cuff calculations which can best be done by hand, or which occur when no calculator is readily available. In practice, these will almost always amount to some additions, subtraction and multiplications, as when one buys goods in a shop, or measures a room for decorating, or wants to know how much a prospective purchase will cost, or something similar.

(Continued)

**PRIME ENDINGS**

Mathematicians know that every prime number greater than 3 is of the form 6k ± 1, where k is a natural number. In base twelve this means that all primes end in either 5, 7, # or 1.

Hence we know that

1) Numbers greater than 3 which end in the other eight digits (0, 2, 3, 4, 6, 8, 9, *) are factorable, and

2) When searching for prime numbers, we need only consider numbers which end in one of the four digits: 5, 7, # or 1.
Thus, in practice, the question of what would be a good scale resolves itself into 2 rather distinct questions: what would be a good scale for a computer, and what would be good for a human calculator?

Computers are not bound by tradition. Some of the first computers worked in the scale of ten, as we do. But it was soon realized that that was inefficient. It requires quite a deal of effort for a human to translate a number from one scale to another, so that changing the scale in an ordinary calculation is unproductive. But, on the other hand, a computer can change the scale very quickly, so that it can be designed to operate in any scale which gives the best performance. In practice, this almost always is taken to be the scale of 2, or binary scale. A positive binary number is simply a series of 1’s and 0’s. Thus 1 represents 1, 10 represents 2, 100 represents 4, 1000 binary = 2 x 2 x 2 = 8, and so on, giving a sequence of powers of 2. Any other positive number can be expressed as the sum of powers of 2, as 5 = 4 + 1, written 101, ten = 8 + 2, written 1010, 15 = 8 + 4 + 2 + 1, or 1111, and so on. This is convenient, since there is an enormous possible choice of “presence” and “absence” alternatives which can be used to denote 1 and 0. A black mark on a piece of paper can mean “1”, and the absence of such a mark “0”. A magnetic tape can be magnetized in one direction to signify “1”, the opposite direction “0”.

That means that a binary number can be recorded or transmitted in a number of different ways. The elements representing “1” and “0” are called “bits” (short for “binary digits”). The addition and multiplication tables are extremely simple:

\[
\begin{align*}
0 + 0 &= 0; \\
0 + 1 &= 1 + 0 = 1; \\
1 + 1 &= 10; \\
0 \times 0 &= 0 \times 1 = 1 \times 0 = 0; \\
1 \times 1 &= 1.
\end{align*}
\]

But the great snag is that the binary system, in this form, only represents positive numbers. The usual device to allow it to represent negative numbers is to let the left most digit take the values q1 (= -1) and 0, instead of 1 and 0. Thus, if we are dealing with 5 digit numbers, 00000 = 0, 00001 = 1, 00100 = 4, 00101 = 5 exactly as before, but 10000 = -16 (=q24), so that q10110 means -16 + 4 + 2 = -10 (= q10). Any number between -16 and +15 can then be represented.

A possible rival to the scale of 2 is the scale of 3, or ternary scale. This has several advantages:

1. There are only 3 digits, 1, 0, q1 = -1
2. Any whole number is represented in one and only one way, as 2 = 1q1 (= 3 - 1), 7 = 1q11 (= 3 x 3 - 3 + 1), -2 = q2 = q11, etc.
3. To reverse a number, one just reverses each digit = 7 = 1q11, -7 = q11q1.
4. The addition and multiplication tables are very simple:

\[\text{(Continued)}\]

\[\text{What Would Be A Good Base}\]

Addition of 0 does not change a number; also
\[
\begin{align*}
q1 + q1 &= q11 \\
q1 + 1 &= 1 + q1 = 0 \\
1 + 1 &= 1q1
\end{align*}
\]

Out of the 9 possible additions of pairs of digits, only 2 (q1 + q1 and 1 + 1) produce a carry.

Multiplication by 0 always gives 0, and
\[
\begin{align*}
q1 \times q1 &= 1 \times 1 = 1 \\
q1 \times 1 &= 1 \times q1 = q1
\end{align*}
\]

No one of these multiplications produces a carry.

\[\text{(Continued)}\]

\[\text{In A Perfect World...}\]

we would have had “Snow White and the Twelve Dwarfs”, not seven. Can you restore the correct numbers in the examples that follow?

1; Twelve Commandments, not
2; Twelve seasons, not
3; The House of the Twelve Gables, not
4 & 5; Friday the Twelfth: Part Twelve, not
6; The Twelve Hills of Rome, not
7; One Thousand seven hundred and twenty eight Arabian Nights, not
8; The Wonderful Twelve Hoss Shay, not
9; The Twelve Wonders of the Ancient World, not
*; Twelve Seas, not
#; The Twelve, Lost Tribes not
11; Twelve Gentlemen of Verona, not
12; Twelve Years Before the Mast, not
13; Twelve Comedies & Twelve Interludes by Cervantes, not
14; The Mighty Twelve Russian composers, not
15; The Civil War Battle of the Twelve Forks, not
16; Twelve Characters in Search of an Author, not
17; The Twelve Sistres by Chekov, not
18; Valley of the Twelve Peaks in Canada, not
19; I Led Twelve Lives, not
*; Twelve River National Forest in CA, not
18; Goldilocks & the Twelve Bears, not
20; The Twelve Musketeers, not

\[\text{Answers - Next Issue}\]
What Would Be A Good Base

However, the use of such a scale of 3 requires a supply of "trits", i.e. 3 states in which an object can exist. Thus a magnetic tape could be magnetized in one direction or the other, or left unmagnetized. An electric current could flow in one direction along a wire, or the other, or there could be no current. A dot could be colored green, red, or left white. But it is rather more difficult to think of suitable trits than to think of suitable bits, and the idea has so far not found great favor.

But one could, of course, use a pair of bits to represent a trit. 10 could mean 1, 01 could mean 1, 1 could mean 0. This would make the representation a little longer than using the binary scale; to represent a number in this way, with pairs of bits representing each trit, would require about 26 (=3²×4) percent more space. (With the price of computer memories falling dramatically, this may no longer be so important.)

But it has a compensating advantage, in that there is a fourth combination, 00, which can be taken to mean "not known", or "uncertain". Think of the statement, "the population of England is 53 million. (Never mind, if that is not strictly correct; this is an illustration.) We would naturally take this to be stated to the nearest million, i.e., that the population lies between 52,500,000 and 53,000,000. The zeroes here do not really mean zero. They are just put in to fill the spaces, and represent figures whose values we are not worrying about. Similarly, if we say that the population is 50 million, we would again take that to mean to the nearest million, unless the contrary was stated. If we write that as 50 000 000, the first zero is a real exact zero, but the others just represent figures whose value is unknown or ignored.

There is nothing in the ordinary way of writing 50 000 000 to show which 0's are real zeroes and which are indeterminate or unknown figures. That is, 50 000 000 might just as well be the value to the nearest hundred thousand, instead of to the nearest million. There is nothing in the way the number is written to indicate which is meant. Thus there is a real need for a figure which can mean "unknown", rather than each figure expressing a precise value.

The second question is rather different. If we could go back in history, and choose which base of notation we liked best for calculations, which would we choose? In discussing this, we will suppose in any case that we are using two-way numbers, since they make arithmetic quite noticeably easier than using the traditional one-way numbers. There is also a personal element in this. Professor A. C. Aitken, FRGS, of Edinburgh, used to like calculating in the scale of sixty. But that gives addition and multiplication tables which are much too big for ordinary mortals to learn conveniently. Possibly sixteen is about the largest base which one could reasonably consider. And any small base, say, less than 6, will tend to make even reasonably large numbers so long that they would strain the memory. Remember that, for example, 1989 or 209 representing 111111000101 in the binary scale.

There seems little question that, for elegance, twelve is easily the best scale. We have already mentioned that in that scale we can immediately determine whether a number is divisible by 2, 3, 4 or 6 by looking only at the last digit. We can have analogs of "decimal fractions" in the scale of twelve, but we can not call them decimals. They are usually called "radix fractions". Thus, .6 in the scale of twelve would mean six twelfths, or 1/2; .4 would mean 4 twelfths, or 1/3, and .16 = (twelve + 6)/(twelve x twelve) = one eighth. One advantage of the scale of twelve is that so many fractions, like these, which occur fairly naturally, can be expressed as simple short radix fractions, whereas, in our ordinary decimal scale, 1/2 = .5, 1/4 = .25, 1/5 = .2, but 1/3 and 1/7 and 1/9 and many other fractions become recurring decimals. Also, in the multiplication table, various combinations such as 2 x 6 = 10, 3 x 4 = 10, give simple answers, whereas the only one such in decimals is 2 x 5 = 10 (ten). Further comments, showing the elegance of the dozenal scale, are to be found in Colson News, Vol. 3, issue 2.

However, what we are looking for is not elegance, but practical usefulness. One may very well hope that the two will go hand in hand, but it does not necessarily follow. We have seen that in practice by far the greatest number of calculations not done on a calculator or computer are additions (or subtractions, which amount to much the same with two-way numbers). The next most common operation is multiplication. Other operations, such as finding factors of numbers, or turning fractions into radix fractions, may be useful on rare occasions, but they form a very small part of everyday life. The operations of division and square roots are unquestionably useful in many contexts, but by far the easiest way of performing them is to use a pocket calculator.

What is wanted, then, is some measure of the difficulty of calculation, by which we can compare the merits of the different bases. And, because addition and multipli-

IT IS A DOZENAL UNIVERSE -- Official!!

Arthur Whillock

Recent studies in nuclear physics have shown that all matter in the Universe is composed of twelve fundamental particles only, arranged into three families, with four in each, which are bonded together with twelve forces. An article describing this significant fact is being prepared for the DSGB Journal. DSA members should refer to the Scientific American for December 1989: "The Lost Generation," p. 15. British readers can more conveniently see New Scientist for 28th October 1989: "Cosmology Limits the Number of Neutrinos," p. 30.

Do you have an idea to share with our members? Why not submit an article to the Bulletin?
tion are easily the two most common operations performed in practice, we will be particularly interested in how comparatively difficult they are. Now when we say that the multiplication $2 \times 5 = 10$ (base ten) is simple, the feature which would justify that is that there is only 1 nonzero digit, 1, in the product, whereas in a product like $3 \times 5 = 15$, there are 2 nonzero digits.

One possible measure of difficulty which springs to mind is the average number of nonzero digits in the entries in the addition and multiplication tables. For example, in the addition table for the scale of 2 there are 4 entries, with sums 0, 1, 1, 10 (=2), giving 3 nonzero digits, or an average of $3/4$ (decimally) .75 per entry. In the scale of 3 there are 9 (=1q1) entries in the table, and 8 nonzero digits giving an average number $8/9 = .89$ (=1.1q1) nonzero digits per entry. From that point of view, the binary scale is simpler than the ternary. As regards multiplication, the binary table gives 1 nonzero digit in 4 entries, or an average of .25, while the ternary scale has 4 out of 9, or $4/9 = .44$. Again, binary does better than ternary. But 2 and 3 are too small to be practical scales. Suitable candidates for a good base of notation could be 6, 8, 10, 12, and 16. One would expect that 12, being the most elegant, would turn out to be the simplest scale. The actual values are shown in Table 4.1.1q4.1.

<table>
<thead>
<tr>
<th>Base</th>
<th>In addition table</th>
<th>In multiplication table</th>
</tr>
</thead>
<tbody>
<tr>
<td>06 = 1q4</td>
<td>1.01</td>
<td>0.83 = 1.243</td>
</tr>
<tr>
<td>08 = 1q2</td>
<td>1.07 = 1.1q3</td>
<td>1.02</td>
</tr>
<tr>
<td>10</td>
<td>1.10</td>
<td>1.18 = 1.2q2</td>
</tr>
<tr>
<td>12</td>
<td>1.13</td>
<td>1.20</td>
</tr>
<tr>
<td>16 = 2q4</td>
<td>1.16 = 1.2q4</td>
<td>1.40</td>
</tr>
</tbody>
</table>

These results are somewhat unexpected. As regards addition, 10 is a bit simpler than 12 or 16, though not quite as simple as 8. As regards multiplication, where one would have expected 12 to score most heavily, it actually comes out as slightly worse than 10. So from this particular point of view, 10 is not really such a bad choice, though 8 would have been better.

However, in this calculation we have neglected another important factor, namely the length of the numbers to be added. The larger the base, the shorter the number when written down, and hence the smaller amount of work to be done in adding or multiplying it. This is more important in multiplication, since in multiplying 2 numbers, say, $x$ and $y$, the number of multiplications to be done is the product of the number of digits in $x$ and the number of digits in $y$. For long numbers, containing many digits, this means that in base B the number of operations to be performed in addition is $\log_{10} B$ times the number required with base 10. We will call this the "length factor". For multiplication, the length factor is the square of the length factor for addition. Thus, with base 12, we find that, in this sense, an addition only requires about 93 per cent as much work as in base 10, and a multiplication only 86 per cent as much work. The results are set out in Table 4.1.1q4.2.

<table>
<thead>
<tr>
<th>Base</th>
<th>For addition</th>
<th>For multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>06 = 1q4</td>
<td>1.29 = 1.3q1</td>
<td>1.65 = 2.3q4</td>
</tr>
<tr>
<td>08 = 1q2</td>
<td>1.11</td>
<td>1.23</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>0.93 = 1.1q3</td>
<td>0.86 = 1.1q4</td>
</tr>
<tr>
<td>16 = 2q4</td>
<td>0.83 = 1.2q3</td>
<td>0.69 = 1.3q4</td>
</tr>
</tbody>
</table>

These length factors are more than large enough to offset the difficulty factors in the previous table. So, on multiplying the two together, it seems that the larger the base (in our table) the smaller the difficulty, and 12 is, after all, better than 10.

It would seem that the best measure of the difficulty of calculation, if we are to measure it in terms of the expected number of nonzero digits would be the product of the factors in the two Tables, 4.1.1q4.1 and 4.1.1q4.2, as Table 4.1.1q4.3.

<table>
<thead>
<tr>
<th>Base</th>
<th>For addition</th>
<th>For multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>06 = 1q4</td>
<td>1.30</td>
<td>1.37 = 1.4q3</td>
</tr>
<tr>
<td>08 = 1q2</td>
<td>1.19 = 1.2q1</td>
<td>1.25</td>
</tr>
<tr>
<td>10</td>
<td>1.10</td>
<td>1.18 = 1.2q2</td>
</tr>
<tr>
<td>12</td>
<td>1.05</td>
<td>1.04</td>
</tr>
<tr>
<td>16 = 2q4</td>
<td>0.96 = 1.0q4</td>
<td>0.96 = 1.0q4</td>
</tr>
</tbody>
</table>

The general conclusion suggested by this table is that the larger the base the easier the calculation, though there is no very dramatic difference between different bases among those in this table.

However, this is still not the end of the story. We have already remarked that there is also a psychological factor, which may vary from person to person. Prof. Aitken could calculate in the scale of 60. But that means that (even if he used two-way numbers) he would have had to learn effectively 1860 (=2q4q40) entries in the addition table, and 435 entries in the multiplication table. Not many people would have the ability to do that. The larger the base, the larger the number of entries in the addition and multiplication tables to be learnt, and recalled when needed. The effort needed to do this will vary from person to person, so it is not easy to put forward a suitable measure of the difficulty that presents. In brief, though disappointingly, it doesn't seem possible to give a really objective measure of comparison of the difficulties of calculating in the different bases.

Remember -- your gift to the DSA is tax deductible.
By 1949, Alan Turing had lost interest in doing certain kinds of basic computer work...

"The 'fussy little detail' of binary to decimal conversion, for instance, he now found not worth bothering about. He himself found it simple to work directly in the base-32 arithmetic in which the machine could be regarded as working, and expected other people to do the same.

"To use base-32 arithmetic it was necessary to find 32 symbols for the 32 different 'digits'. Here he took over the system already used by the engineers, in which they labelled the five-bit combinations according to the Baudot teleprinter code. Thus the 'twenty-two' digit, corresponding to the sequence 10110 of binary digits, would be written as 'P', the letter that the sequence 10110 encoded for an ordinary teleprinter. To work in this system meant memorising the Baudot code and the multiplication table as expressed in it -- something he, but few others, found easy.

"The ostensible reason for sticking to this hideously primitive form of coding, which entailed so much work for the user, was that the cathode ray tube storage made it possible -- indeed necessary -- to check the contents of the store by 'peeping', as Alan called it, at a monitor tube. He insisted that what one saw as spots on the tube had to correspond digit by digit to the program that had been written out. To maintain this principle of correspondence it was actually necessary to write out the base-32 numbers backwards, with the least significant digit first. This was for technical electronic engineering reasons, the same as those which obliged cathode ray tubes always to scan from left to right. Another awkwardness arose on account of the five-bit combinations which did not correspond to a letter of the alphabet on the Baudot code... Geoff Tootill had already introduced extra symbols for these, the zero of the base-32 notation being represented by a stroke '/'. The result was that pages of programs were covered with strokes."

---

**DON'T KEEP THIS MAGAZINE**

Do you discard your copies of the Bulletin after you have read them? Or do they gather dust on a shelf in your attic? Why not pass them along to your local library, or to a school library. Perhaps some nearby math teacher would appreciate a copy. You can also just leave them in a dentist's office or other waiting area.

"Each one teach one."
-- Ralph Beard, Founder of the DSA
153_{dek} (Number One)
Brian Dean
Ohio

There is a text from the desert fathers called ‘On Prayer: One Hundred and Fifty-Three Texts’ by St. Evagrius the Solitary.

Now 153_{dek} = 109_{metro}, and 109, is an interesting number. It is a triangular number, i.e., it is a number such that it is a member of \( t(n) = 1 + 2 + 3 + \ldots + n \), and in fact 109 = \( t(15) \).

It is also a hexagonal number which means that it is a member of \( h(n) = 1 + 5 + 9 + 13 + \ldots + n \) (109 = \( h(29) \)).

It is also the sum of 84 (which is a square number), 24 (which is triangular \( 24 = t(7) \)) and 21 (which in base * is 25). This means that 25 is a spherical/circular number -- one that is a square number whose square root gets reproduced as the last digit (in base *, \( 25 = 5 \times 5 \), also 36 = \( 6 \times 6 \)) although in the dozenal system 21 would not be circular/spherical.

***

153_{dek} (Number Two)
S. Ferguson
Dozenal Society of Great Britain

While working on patterns involved with the “153” problem (where \( 1^3 + 3^3 + 5^3 = 153_{dek} \)) I happened to note that for \( 7^3 \) in base dek there is a pattern:

\[ 7^3 = 343, \text{ and } (3+4)^3 = 343. \]

Was this an isolated case? or could there be similar examples in base twelve or some other base?

Up till now I have the following:

\[ (1+2)^3 = 123 \text{ in base } 4 \]
\[ (3+4)^3 = 343 \text{ in base } * \]
\[ (3+1)^4 = 314 \text{ in base } 9 \]

and
\[ (1+4)^5 = 144 \text{ in base } 1# \]

(Continued)

Since the unit digit of the number written in a given base is the same as the power of the bracketed expression, we are in fact looking at numbers which fit the congruence:

\[ x^3 \equiv n \pmod{\text{base}} \]

I suspected (a) that the above four might well be the only solutions with three digits and (b) that there aren't any solutions with four or more digits, but further investigation led me to discover:

\[ (9+0+8)^4 = 9084 \text{ in base } 19 \]

and

\[ (4+3+1+5)^4 = 43154 \text{ in base } 9 \]

***

153_{dek} (Number Three)
Gene Zirkel
New York

In response to Shaun Ferguson’s contribution “153_{dek} - Number two,” note that:

\[ 2^1 = 24_{xx} \]
\[ 2^2 = 26_{2x} \]
\[ 2^3 = 28_{(4)x} \]

e tc.

In fact,

\[ 2^n = 2(2^{n-1} - n) + 2n \text{ is an identity as is } \]

\[ k^{n+1} = k(k^{n-1} - n) + kn \]

where the base (\( k^{n-1} - n \)) > the 2 digits k and kn.

This latter yields examples such as:

\[ 3^5 = 33_{x}, 3^6 = 36_{(1)x} \text{ and } 4^4 = 44_{(0)x} \]

End
We tend to think of things being codified and universal. However many measurements were regional and things were written differently in different localities. Thus we find Roman Numerals not quite as standard as we were taught in elementary school. For example:

IV is not the only four - IIII was also used. Two hundred appears as II hundred as well as CC. But sometimes CC stands for two long hundreds! V**x is found as denoting 5 times 20 and VI is used for six thousand.

The reader who is not aware of these variations would have difficulty in attempting to decipher the meaning of some passages in medieval texts.

To some extent, it appears that the popular culture preferred the long hundred while the narrow hundred prevailed in sacred use.

Professor Ulf-Müller is due to return to the States in November. We hope to be able to announce a date when he will speak to us again, and we trust that you will have the opportunity to hear him.

Don't be surprised if you read that:

$100 \text{ fish} = 6 \text{ score fish}$,
$3 \text{ (units) of fish} = 100$, and
$1 \text{ (unit)} = 40$.

Just pity the poor historian who is trying to comprehend a text in which the symbol 100 has two different interpretations in the same document.

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THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
3. Manual of the Dozen System by George S. Tetry. ($1.00)
4. New Numbers by F. Emerson Andrews. ($10.00)
5. Douze: Notre Dix Futur by Jean Essig. In French. ($10.00)
6. Dozenal Slide Rule, designed by Tom Linton. ($3.00)
7. Back issues of the Duodecimal Bulletin, as available, 1944 to present. ($4.00 each)
8. TGM: A Coherent Dozenal Metrology by T. Pendlebury ($1.00)
9. Modular Counting by P.D. Thomas ($1.00)
10. The Modular System by P.D. Thomas ($1.00)
DOZENAL JOTTINGS

... from members and friends... News of Dozens and Dozenalists...

JOHN D. HANSEN, JR. (30#, Vista, CA) writes: "I find myself more optimistic about dozenal's chances of eventual adoption. Until recently I felt we had a better chance of getting mankind to grow twelve fingers than to adopt base twelve... But when I think of all the improbable things that have happened on the world scene in the past three years, I begin to think that anything is possible." John also reports that his robotics professor has read our "Manual of the Dozen System" and is interested in other material from the Society...

Member JAY L. SCHIFFMAN (Jersey City State College, NJ) recently presented a paper "Congruences and Divisibility Tests" at the 49th Annual Meeting of the Metropolitan New York Section of the Mathematical Association of America...

An announcement about PETER D. THOMAS'S work in modular counting and arithmetic appeared in the March-April 1991 issue of the American Metric Journal. (Peter received the DSA Ralph Beard Memorial Award posthumously in 1990, having passed away early in that year.)...

Member PAUL SCHUMACHER (Cherry Hill, NJ) recently had an article published in Topical Time: "The Presidential Election of 1856"...

Bulletin Editor DR. PATRICIA ZIRKEL (St. John's University, NY) recently presented two papers on medieval topics: one at the Twelfth Medieval Forum, Plymouth State College (USNH), Plymouth, NH, and a second at the 26th International Congress on Medieval Studies, Western Michigan University, Kalamazoo, MI...

Thanks to HENRY C. CHURCHMAN (Council Bluffs, IA) for his recent gift to the DSA, which was forwarded to us by his son JOHN P. CHURCHMAN...

Thanks also to KAY McKIERNAN for her recent gift to the DSA in memory of her husband DR. ELLIS R. VON ESCHEN. Kay wrote to express appreciation for the tribute to Ellis which appeared in the last issue of the Bulletin. She said: "Ellis was very fond of the group and enjoyed the times he spent with us." Dr. Von Eschen had twice presented papers at DSA Annual Meetings...

New member MONTE J. ZERGER (Adams State College, Alamosa, CO) writes:

I notice that the symbols for dek and el are apparently from two of the twelve push buttons on the telephone.

(1) To obtain Directory Assistance for numbers outside their area code, callers must dial 1 + area code + 555-1212. Curiously, the number 5551212 ends with a repetition of 12, and is divisible by 12.

(Continued)

(2) How appropriate then that BELL ends in a repetition of the 12th letter of the alphabet. And what about HELLO's successive L's?

(3) Counting from either side of the alphabet to the 12th letter we find L and O. These letters exactly balance in TELEPHONE as well.

ABCDEF GH IJ L M N O PQRSTUVWXYZ
TE LH O NE

BRUCE MOON, DSA Fellow, writes from Diamond Harbour, NZ:

Thank you for my copy of Bulletin 67; which I enjoyed reading. I am glad to know that the Society is going from strength to strength. I noticed too the paragraph apologising for your rather amusing error in confusing New Zealand and Australia -- apologies accepted of course.

I have come to the conclusion that you folk down under are really rather droll, for you go on to say that JAMISON and VERA HANDY visited us this past summer. Now that really is quite comical because their visit was in the depths of winter. While her son John went skiing, Jamie, Vera and John's friend Corinne drove with me across the South Island from Diamond Harbour (east coast) to Greymouth (west coast). During the trip across the Southern Alps we had magnificent views(...)

Dating of Bulletin 67; as "Winter 1991" is also rather quaint since we haven't had winter yet this year!...

Welcome to New Members:

310; JOANNE ALICE YOUNG Crosslanes, WV
311; NADER RAFATY MALAKY University of Tabriz Malekan, IRAN
312; ROBERT S. HARRIS Knoxville, TN

Bob writes: "If it's good enough for eggs, it's good enough for me!" (See "Eggactly a Dozen," James Malone; Whole Number 42; (1981), p. #). He is also interested in sexagesimal counting and would like to hear from other aficionados...

313; MONTE JAMES ZERGER Adams State College Alamosa, CO

(Continued)
A BASE TWELVE CROSSNUMBER PUZZLE

Charles Ashbacher

(Each entry is a non-negative digit.)

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & & \\
2 & & \\
3 & & \\
\end{array}
\]

ACROSS:
1. A perfect square and a perfect cube.
2. The three most widely used bases are divided by two.
3. Palindromic in two common bases.

DOWN:
1. A permutation of the digits form an arithmetic sequence.
2. The dozenal sum of these digits occurs as part of another solution in this puzzle.
3. Easily seen to be divisible by 3 in base ten and base twelve.

SOLUTION IN NEXT ISSUE!

"In numerology, the number 12 has always represented completeness, as in the 12 months of the year, the 12 signs of the zodiac, the 12 hours of the day, the 12 gods of Olympus, the 12 labors of Hercules, the 12 tribes of Israel, the 12 Apostles of Jesus, the 12 days of Christmas and so on. Since 13 exceeds 12 by only one, the number lies just beyond completeness and, hence, is restless to the point of being evil."

from the Smithsonian, 2/87
An integer is palindromic if it is the same when read forward or backward. That is, a palindrome is identical with its reverse. Every palindrome with an even number of digits is divisible by 11. Thus 11 is the only prime palindrome with an even number of digits.

Of the 110 three-digit palindromes, 18 are prime. They are:

```
 111  181  565  727  797
131  1#1  575  737  1#1
141  535  585  747  2#1
171  545  5#5  767  6#1
```

Included above are four pairs and two trios of consecutive palindromes. Note that 565 and 575 are consecutive primes, while 727, 737, 747 and 767 are alternate primes.

**HEADDINES AND EXCERPTS**

from a recent issue of the American Metric Journal  
(Volume XVIII, Jan/Feb 1990)

‘No case has been made for metric’ 1990

Britain Still Dragging Its Feet Over Metric System  
(Feet? - Ed.)

Conversion would hurt the U.S. economy (from the Government Accounting Office Report to the Congress)

There is no need to convert to the metric system (from the GAO Report)

European countries do **NOT** all use the same metric. They even mix in the inch/pound system and always have.  
(Emphasis added - Ed.)

**COSTS IN THE BILLIONS**  
Conversion would be enormously expensive

---

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COUNTING IN DOZENS

1 2 3 4 5 6 7 8 9 # # 10
one two three four five six seven eight nine ten

Our common number system is decimal—based on 10. The dozen system
uses twelve as the base, which is written 10, and is called do, for dozen. The
quantity one gross is written 100, and is called gro. 1000 is called me,
representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent suc-
cessive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to
tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more
important in dosenal counting. For example, 265 represents 5 units, 6 dozen,
and 2 dozen-dozen, or gross. This number would be called 2 gro 6 do 5, and by a
coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;3.

Place value is the whole key to dosenal arithmetic. Observe the following
additions, remembering that we add up to a dozen before carrying one.

| 94 | 135 | Five ft. nine in. | 5'9' |
| 31 | 694 | Three ft. two in. | 3'2' |
| 36 | 362 | Two ft. eight in. | 2'8' |
| 10 | 1000| Eleven ft. seven in.| 11'7' |

You will not have to learn the dosenal multiplication tables since you al-
ready know the 12-times table. Mentally convert the quantities into dozens,
and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so
set down 63. Using this "which is" step, you will be able to multiply and divide
dosomal numbers without referring to the dosenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are
35 years old, dozentially you are only 28, which 12\[368\] 3 years old, dozenally you are only 28, which
in two dozen and eleven. For larger numbers, 12\[50\] + 5
keep dividing by 12, and the successive re-
mainders are the desired dosenal numbers.

Dozenal numbers may be converted to decimal numbers by setting down
the units figure, adding to it 12 times the second figure, plus 12\[2\] (or 144)
times the third figure, plus 12\[4\] (or 1728) times the fourth figure, and so on as far as
needed. Or, to use a method corresponding to the illustration, keep dividing
by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications,
instead of divisions, by 12 or *.

For more detailed information see Manual of the Dozen System ($1.00).

Application for Membership

We extend an invitation to membership in our society.

Application for Admission to the

Dosenal Society of America

Name ___________________________ ___________________________

Mailing Address (for DSA items) ___________________________

(See below for alternate address)

Telephone: Home ___________________________ Business

Date & Place of Birth ___________________________ Degrees

College ___________________________ Business

Business or Profession ___________________________

Annual Dues ___________________________ $12.00 (US)

Student (Enter date below) ___________________________ $3.00 (US)

Life ___________________________ $144.00 (US)

School ___________________________

Address ___________________________

Year & Math Class ___________________________

Instructor ___________________________ Dept. ___________________________

Other Society Memberships ___________________________

 Alternate Address (Indicate whether home, office, school, other)

Signed ___________________________ Date ___________________________

My interest in duodecimales arose from ___________________________

Use space below to indicate special duodecimal interests, comments, and other
suggestions, or attach a separate sheet:

Mail to: Dosenal Society of America

c/o Math Department

Nassau Community College

Garden City, LI, NY 11530