COUNTING IN DOZENS

1  2  3  4  5  6  7  8  9  10
one  two  three  four  five  six  seven  eight  nine  ten

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written 12, and is called do, for dozen. The quantity one gross is written 100, and is called gro. 1000 is called wo, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozens, or gross. This number would be called 2 gro 6 do 5, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

54  136
31  694
96  322
192  1000

Five ft. nine in.   5'9'
Three ft. two in.  3'2'
Two ft. eight in.  2'8'
Eleven ft. seven in. 11'7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozons, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication tables.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2E, which is 28 + 7, or 35. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

Dоценal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12² (or 144) times the third figure, plus 12³ (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by 12, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or 1.

<table>
<thead>
<tr>
<th>Numerical Progression</th>
<th>Multiplication Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 One</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
</tr>
<tr>
<td>10 Do</td>
<td>Edo</td>
</tr>
<tr>
<td>100 Gro</td>
<td>Egro</td>
</tr>
<tr>
<td>1,000 Mio</td>
<td>Emo</td>
</tr>
<tr>
<td>10,000 Do-mo</td>
<td>123456789101112131415</td>
</tr>
<tr>
<td>100,000 Gro-mo</td>
<td>Egro-mo</td>
</tr>
<tr>
<td>1,000,000 Bi-mo</td>
<td>Ebi-mo</td>
</tr>
<tr>
<td>1,000,000,000 Trin-mo</td>
<td>and so on.</td>
</tr>
</tbody>
</table>

THE DUODECIMAL SOCIETY OF AMERICA

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TOWARDS A BASE TWELVE COMPUTER

B. A. M. Moon, Director,
Computer Centre,
University of Canterbury,
Christchurch, 1, New Zealand.

For the internal representation of numbers in a computer, the advantages of using a base which is a power of two are well known. In particular, only two-state storage elements are needed and the logical circuitry required for performing arithmetic is relatively simple.

However, a power-of-two base does have some disadvantages, such as the inability to represent exactly any fraction whose denominator is not a power of two. Reflections such as this have led to other possibilities being investigated, and several recent references have been made to ternary (base three) organization. Devices certainly exist which lend themselves to ternary operation, for example a magnetic element magnetized in either direction or not at all.

The Russians have built a ternary computer and a recent paper in "Cybernetics", a leading Soviet computer journal, is entitled "One Class of Three-valued Algebras and Its Application for Synthesis of Ternary Logical Circuits of Ternary Components".

Knuth also discusses this topic, pointing out especially the advantages of balanced ternary notation, using the numerals −1, 0, +1 rather than 0, 1, 2, and remarking that "perhaps its symmetric properties and simple arithmetic will prove to be quite important some day (when the 'flip-flop' is replaced by a 'flip-flap-flap')".

In an alternative approach suggested by Bucholz and reported by Walker an attempt is made to determine the radicx which offers minimal cost. While his assumptions about the relations of cost to radix used are probably not realistic, his method does show that even if base-three elements cost half as much again for each digit represented, the over-all cost is less since each digit contains more information.

Attached hereto (Table 1) we give a modified form of Bucholz' approach, which meets one criticism of it, and demonstrates the relative cost advantage of a power-of-three system, by comparison with other bases up to 2^6. Of course, a power-of-three-based computer in its turn suffers the disadvantage that no fractions which do not have power-of-three denominators may be represented exactly; that is, even 1 can only be represented approximately. (It is .1 recurring in ternary notation, both balanced and conventional).

In view of the importance of the halving process, this would be better avoided. In short, we find that factors such as this...
Table 1

<table>
<thead>
<tr>
<th>Base</th>
<th>Component elements</th>
<th>Relative Cost (sum of component values)</th>
<th>Information per digit (log r)</th>
<th>$\frac{s}{\log r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>842</td>
<td>7.23</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>638</td>
<td>8.33</td>
</tr>
<tr>
<td>4</td>
<td>2, 2 or 4</td>
<td>4</td>
<td>864</td>
<td>7.23</td>
</tr>
<tr>
<td>5</td>
<td>2, 3 or 5</td>
<td>5</td>
<td>793</td>
<td>7.87</td>
</tr>
<tr>
<td>6</td>
<td>2, 3</td>
<td>5</td>
<td>872</td>
<td>6.22</td>
</tr>
<tr>
<td>7</td>
<td>2, 2, 2</td>
<td>6</td>
<td>949</td>
<td>7.80</td>
</tr>
<tr>
<td>8</td>
<td>2, 2, 2</td>
<td>6</td>
<td>1006</td>
<td>7.23</td>
</tr>
<tr>
<td>9</td>
<td>3, 3</td>
<td>6</td>
<td>1214</td>
<td>6.93</td>
</tr>
<tr>
<td>10</td>
<td>2, 2, 3</td>
<td>7</td>
<td>1215</td>
<td>7.62</td>
</tr>
<tr>
<td>11</td>
<td>2, 2, 2, 2</td>
<td>8</td>
<td>1048</td>
<td>7.91</td>
</tr>
<tr>
<td>12</td>
<td>2, 2, 2, 2</td>
<td>8</td>
<td>1082</td>
<td>7.60</td>
</tr>
<tr>
<td>13</td>
<td>2, 2, 2, 2</td>
<td>8</td>
<td>1102</td>
<td>7.47</td>
</tr>
<tr>
<td>14</td>
<td>2, 2, 2, 2</td>
<td>8</td>
<td>1148</td>
<td>7.23</td>
</tr>
</tbody>
</table>

A modification of a result of Bucholz

The amount of information in one digit using a radix (or number-base) of r is proportional to $\log r$. (The base of logarithms is immaterial since only proportionality is involved).

Bucholz assumes the cost of providing that information is proportional to the number of physically distinct states required to represent it, i.e., to the base itself, and so the cost per unit of information, C, is given by:

$$C = \frac{K}{\log r}$$

where $K$ is a constant of proportionality.

This quantity is a minimum if $r = e$, the exponential constant, i.e., 2.7182818... approximately, which is between 2 and 3. Of course for practical purposes we consider whole number values only.

Where this procedure is subject to criticism is in its assumption about cost. Thus if it were sought to use base eight each digit in practice, would be represented by three base-two elements at a cost of 3, not 8, times the cost of one binary element, and probably most of the logical circuitry would in essence use base two, not base eight.

If we assume therefore that combinations of elements having less states are used whenever this gives a saving but that otherwise Bucholz’ cost assumption is valid, we obtain the results in Table 1. This table is expressed using base twelve throughout, the comparative cost figures in the right-most column being calculated by slide rule. It demonstrates for bases up to 2², a minimal cost for base 3 or 9, followed by 6, then twelve.

lead us to those very arguments which demonstrate the superiority of the dozen system. In addition if our modification of Bucholz' result be accepted there may be an opportunity to gain some cost advantage over a pure power-of-two system as well.

What we are suggesting is not a pure base-twelve computer, but one using a mixture of base two and base three storage elements and logical circuitry in a 2:1 ratio, in groups of three having in effect the "values" of "penny, threepence, and sixpence" in the British currency system of the 60's before disfiguration, so that representation in base twelve external to the computer, for example, in the usual twelve-ruled punched card, would be a very simple operation. In a similar way, information from a purely binary machine is often represented externally in octal format.

Clearly by using a mixture of binary and ternary components we are introducing some complication and additional expense not present when one kind only is used and this offsets some of the savings over a pure binary machine, but pending an investigation, it is not clear how significant this would be.

Factors affecting the choice of base are not limited to those we have discussed here. IBM altered the floating point arithmetic hardware in its major machines from binary to hexadecimal. One argument in favor of this change is that it simplifies the normalization process.

Others, notably "Brown and Richman, have argued that the choice of a prime base rather than any of its higher powers is superior both with respect to the range of values that may be covered and the effect of chopping (truncation) of low order digits in the results obtained.

It is not claimed therefore that a suggestion for a mixed binary/ternary machine will lead to the last word in efficient machine design. It is considered however that it does well merit a thorough design study.

(Continued at bottom of page 4)
The Duodecimal Bulletin

THE NUFT OR "NUBBIN"

A TRANSITIONAL DIMENSION FOR JET AGE?

By Henry G. Churchman

Since there are said to be 44 inches in one "Dometron" or "mètre duodécimal", therefore a "Quarter-dometron" (in science described as the equal of 461,376 wavelengths of orange-red kr. 86 light) might be said to equal eleven inches or one "Nuft". A smaller, not perfect ear of maize is a nubbun; and a "nuft" could be referred to as the "Nubbin" in foot measurements. The word is already in the dictionary and not inappropriate.

If we were to think in terms of precision then we know that 12 nuft exceed the length of 11 feet by about one 3000th part, and, therefore, 12,000 nuft are the equal of 11,000 feet, plus 1/3000 of 12,000, or quite exactly 4 feet more than 11,000 feet ----which we may ignore since only the most delicate altimeters are capable of disclosing this difference in present feet or in the nufut scale. Today's foot is a flyer's favorite unit. Let the inch presently retain its definition of 25.4 millimeters.

As one nuft is, by definition, the exact equal of one-fourth dometron or one-fourth mètre duodécimal, it follows that 12,000 nuft, or nubbuns, are the equal of 3,000 dometrons.

Hence, if you are flying at 11,000 feet, you are 12,000 nuft or 3,000 dometrons above sea level. Equally, at 33,000 feet we are 36,000 nuft above sea level, or 9,000 dometrons.

At 44,000 feet one's flying height in nuft may be said to be 48,000, which is the equal of 12,000 dometrons. At 55,000 feet one may be said to be 60,000 Nubbins above sea level or 15,000 dometrons. Here let us ignore some 20 feet difference over the 55,000 feet.

During a period of transition from feet to dometrons (mètres duodécimaux in French language) it would seem that the quarter-dometron, or one nuft or nubbun, might prove helpful.

- o - o - o -

A BASE TWELVE COMPUTER (Continued from page three)

References:

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The Duodecimal Bulletin

CHROMATIC MUSICAL SCALES AND NOTATIONS

By Erich Kathe

Part Three (Cont'd)

Chromatic Notations

Being placed on the staff caused some changes of the neumes. Figure 30 illustrates this by showing in column I a few of the neumes and in column II and column III some derivatives from them.

Figure 30

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virga</td>
<td>/</td>
<td>1</td>
<td>.</td>
</tr>
<tr>
<td>Puncrum</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Pes</td>
<td>✓✓✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
<tr>
<td>Clavis, Flexa</td>
<td>✓✓✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
<tr>
<td>Torculus</td>
<td>✓✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
<tr>
<td>Forcicrus</td>
<td>✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
<tr>
<td>Climasppr</td>
<td>✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
<tr>
<td>Scandius</td>
<td>✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
</tbody>
</table>

The notation which used Guido's four-line staff with signs as indicated in column II of Fig. 30 has been known as the Gothic Choral notation. Even more important became the notation which used the four-line staff with the signs as shown in column III and became known as the Roman Choral Notation, which proved to be so satisfactory for notating the plain songs that it is still in usage today.

Another interesting method of notating songs is shown in Fig. 31, and it had been used until the fifteenth century. It utilized Guido's four-line staff and placed the text on the staff. Thus the location of a syllable on the staff indicated its pitch.

Figure 31

Guido's staff used four lines. However, the number of lines to the staff was not settled until the sixteenth century, when four were used for plain chant (as today for the Roman Choral...
Notation) and five for secular music. In modern music the staff consists of five lines, and when more are wanted, short additional lines (called ledger lines) are written above or below. Should these ledger lines become excessive, the same are repeated with the expression "8" above or below the clef. The former indicates that the octave (or hon- nance) above is intended as shown on Fig. 32a; the latter indicates the octave (or hon- nance) below, as shown on Fig. 32b.

Figure 32

Although we speak of our staves as having five lines each, actually they are part of the great staff of eleven lines (the line of middle C in the center). The diagram on Fig. 33 shows the great eleven-line staff, with the different clefs applied to the selection of five lines which they serve to identify. Furthermore, the letters are given which indicate the absolute pitches.

Figure 33

Lately the tendency exists to substitute the C-clef for a lowered G-clef as shown on Fig. 32b. However, while doing so, the lowering number 8 is often left out, since in those cases it is usually understood in what range the melody is meant.

An interesting variation of the conventional staff notation was the so-called "Patent Notes" which were used in the southern U.S.A. during the last century. As shown in Fig. 34, they employed all signs of the conventional notes (including signatures which will be discussed later) with the addition of characteristically shaped noteheads. These peculiar shapes indicated the names of the notes. (Ref.: W. S. B. Mathews and E. Liebling, "Dictionary of Music", Chicago, 1896).

Figure 34

PATENT NOTES
SOMETIMES CALLED BUCKWHEAT NOTES

The Duodecimal Bulletin

It should be noted that conventional musical notation uses only seven letters for naming the notes within one hon- nance (or octave), and it provides only seven locations on the staff for each hon- nance. Both conventions originated during the Middle Ages when church music was based on several diatonic scales as shown in Fig. 16, and these diatonic scales had only seven notes within one hon- nance. Yet as shown in Fig. 17, the Greater Perfect system had already one exception (called one accidental), the b-flat, although this accidental was regarded at first as a variant of the note b rather than a chromatic half- step down.

Figure 35

The developing polyphonic style of music increasingly emphasized the harmony, and thus a greater number of accidentals were introduced. These accidentals were expressed as sharpened or flattened or as restored (natural) notes. When using letters, it became customary to place the already mentioned sharp, flat, or natural signs (also known as chromatic signs) immediately after the letters. When using staff notation, the chromatic signs are placed immediately before the notes. In Fig. 35 are shown sharpened and flattened notes as examples.

Figure 36

The major mode, or scale, has its half step in a harmonically strong position. When sounded together, as after the double bar, they form the basis for the dominant-tonic progression called V7. This is probably the strongest harmonic progression for establishing a key or for ending phrases and entire compositions.

Another change occurred during the seventeenth century due to the polyphonic style. Instead of the former church modes as shown in Fig. 16, two new modes known as major and as minor modes (also called "major and minor scales" or "major and minor forms") became dominant, and their influence is still observable. From the Ionian mode on C as shown in Fig. 16, there developed the major mode. However, the major mode could start from any other note as well. In Fig. 36 is seen the C major mode.
Similarly, the Aeolian mode on A as shown in Fig. 16 became the base for developing the minor modes. However, not only can the minor mode act as a contrast to the major and achieves its structural clarity primarily by borrowing tones, such as the raised seventh step, or leading tone, from the major mode. As a result, three forms are recognized: pure (or natural), melodic, and harmonic minor.

Figure 37

Although the developing polyphonic style tended to increase the number of accidentals as just illustrated, musical instruments with keyboards tended to restrain such increases. Keyboards with only two accidentals as shown in Fig. 18 were still in use during the lifetime of Michael Praetorius (1571–1621). Yet, Praetorius described an organ built in 1560 by a priest, Nikolaus Faber, for the Harlestadt cathedral as being the first one to have a complete scale of semitones. And the masterpiece "Worship of the Lamb" by Van Eyck (painted not later than 1426) includes the first authentic representation of a keyboard having seven long (white) and five short (black) keys for each honance. (Ref. : "Encyclopedia Britannica", 1962)

At first the seven white keys of the keyboard were tuned according to the diatonic scale as shown in Fig. 15, and any black key was tuned as the "flat" of the next higher white key (see Fig. 19). Besides having the many shortcomings of the diatonic scale as already discussed in the second part of this writing, this tuning perpetuated the old idea that a diatonic scale would be the "ideal musical scale".

As soon as the keyboard with seven white (long) and five black (short) keys for each honance (or octave) had been accepted as standard, many attempts were made to tune them satisfactorily. Finally, the tuning according to the 12-note chromatic scale (equally tempered or well-tempered scale) proved to be most satisfying. In Fig. 39 are shown the keys of the so-called third or one-lined honance (or octave) as they are arranged on a conventional keyboard, and their pitches are indicated by conventional notes. It should be noted that each black key may be represented by a flat or sharp note, but usually are the black keys expressed as sharp notes while descending the scale and as flat notes while ascending.

During the 19th century the just mentioned well-tempered tuning succeeded as the standard tuning of keyboard instruments. Thus the 12-note chromatic scale became the most representative musical scale of the Western world, and a number of persons (like Mozart, etc.) started to come up with ideas for a so-called chromatic keyboard.

The chromatic keyboard as invented by the Hungarian pianist and mathematician, Paul von Janko, in 1852, originally built by...
the disposition of its keys, it reduces the honance span from
the standard 16.5 cm (6 2/3 inches) to 13 cm (5 15/127 inches
or about 5.12 inches), and it opens up new possibilities of fin-
gering by permitting each key to be played from three differ-
ent positions.

As illustrated in Fig. 40a the keys are arranged in six ter-
raced rows, sloping slightly towards the front, and they are
tuned in whole-tone sequence as indicated in Fig. 40b. To
accommodate conventional notation, all keys representing natural
notes are white and all keys representing accidentals are black
as on the conventional keyboard (see Fig. 39). (Ref.: S. Mar-
cuse, "Musical Instruments", New York, 1964; "Der Grosse Brock-

The acceptance of the 12-note chromatic scale also ini-
tiated a review of the presently used pitch indications, since the
conventional notation accommodates only seven letters and only
seven staff locations with unequal successive intervals for
each honance (or octave), and it employs three different clefs.

While doing so, some reformers like Angel Menchaca, with his
"Nouveau système de notation musicale" went so far as to dis-
card the conventional five (or four) line staff and to rely on
only one reference line.

As shown in Fig. 41, he indicated the pitches of the notes by
different positioning and/or orienting of their characteristic-
ally formed note heads. Obviously, such musical notations fail
to show graphically the flow of a melody. (Ref.: J. Wolf,

Reformers like Dr. Karl Chr. Fr. Kreuse went to the other ex-
treme by proposing to utilize only the staff locations between
the lines (1811). Unfortunately, the required large amount of
staff lines makes the recognition of an indicated pitch very
difficult.

All chromatic musical notations (staff notations) which have
been proposed so far, and are worth being considered seriously,
can be divided into two major groups. As representative of the
first group there is shown in Fig. 42 the notation by Roualle
de Boisseljou (1764) which had its lines separated by a whole-
note step, thus any succession from a staff location on a line
to one between the lines equaled a half-note step and no sharp
or flat signs were required.
In Fig. 43 are shown four proposed notations which illustrate the transition from the first to the second major group of chromatic musical notations. The first notation as shown under a) had been proposed 1892 by Hugo Riemann; it left each second staff line, but when needed it used a short additional line. Sacher's notation (1892) as shown under b) only left out each second staff line. Already further went Leopold Engelke's notation (1893) as shown under c); besides leaving out each second staff line it also modified uniformly the note heads and limited the staff to four lines.

Still further went the so-called "Viennese notation" as shown under d); it relied no longer on the distinction between note heads being free between the lines and note heads touching either the upper or the lower staff lines, but it used four differently formed note heads. By so doing it already blended into the second major group.

The characteristic of the second major group of chromatic musical notations is the whole-note step of each succession from a staff location on a line to one between the lines, and thus the staff lines are separated by a major third interval.

Therefore only 6 staff locations or 3 staff lines are required for each location (or octave), and this produces a relatively compact graphical representation of a melody. However, some additional means for indicating half-note steps have to be supplied.

In Fig. 44 are illustrated three proposed notations which used different means for indicating half-note steps. The first notations as shown under a) had been proposed 1792 by Johannes Rohleder and it used black or white note heads to distinguish notes a half-note step apart. Since this principle is so sim-
ple, it had been used by many other reformers like Charles Lemme (1829), Gambale (1840), Giuseppe Borio (1862), Bartolomeo Grassi-Landi (1880), K. M. Huyghofer (1896), or Paul Riesen (1902). However, black and white note heads are already employed for indicating duration values of the notes and therefore such notations require additional innovations.

**Figure 45**

![Image of Figure 45](image)

In Fig. 45 are illustrated three different proposals, although all three used C-clefs. The first proposal by Alexis Azevedo (1868) as shown under a) could be applied to any staff location. It used a 90° rotated "C" to mark the "one-lined c" and all other honances are indicated by added attachments.

**Figure 46**

By Velizar Godjevatz

![Image of Figure 46](image)

Similar to the first proposal is the second one by Joseph Lanz (1842) as shown under b); however, it was applied only to the center staff line. And the third proposal, by Charles Meerens (1873), as shown under c) had its C-clef applied to the lowest staff line; but instead of showing the C-clef it only indicated the referred honance by numbers 7 to 16. —Ref.: J. Wolf, "Handbuck der Notationskunde", Billedheim (1963).

Attempts to simplify the naming of the notes for the 12-note chromatic scale by using the first twelve letters of the alphabet A - L instead of the conventional seven letters A - G proved to be undesirable because of the conflict with the conventional naming of the notes.

Employing conventional (decimal) numbers was also unsatisfactory because at least two of the twelve notes within a honance (or octave) would have to be expressed as two-digit numbers.

However, since the duodecimal system has twelve (called "do") digits available (zero to eleven, called "el"), it permits the expression of each note within a honance by a single digit ("relative pitch").

Moreover, since only ten (called "dek" in the duodecimal system) honances encompass the total range of notes, two digits of the duodecimal system can indicate the "absolute pitch" of a note (that is its fixed position in the entire range of notes).

The first chromatic musical notation which utilized duodecimal numbers for naming the notes had been proposed 1948 by Velizar Godjevatz. As shown in Fig. 46 and under b) in Fig. 48, it belongs to the first major group of chromatic musical scales (see Fig. 42) and thus is very simple in design. Its clefs indicate the referred honances and thus they are similar to those
shown under c) in Fig. 45. ——(Ref.: "The Duodecimal Bulletin" October 1868).

Being aware of the "graphical stretching" by the first group of chromatic notations, the author tried the second group. It allowed him to use a 4-lined staff for the range of 3/2 harmonies (or octaves) instead of the 5-lines staff for the conventional notation.

Figure 47
The author's notations

a)

b)

c)

\[\begin{align*}
\text{\texttt{\textbackslash 0}} & \quad \text{\texttt{\textbackslash 1}} \\
\text{\texttt{\textbackslash 2}} & \quad \text{\texttt{\textbackslash 3}} \\
\text{\texttt{\textbackslash 4}} & \quad \text{\texttt{\textbackslash 5}} \\
\text{\texttt{\textbackslash 6}} & \quad \text{\texttt{\textbackslash 7}} \\
\text{\texttt{\textbackslash 8}} & \quad \text{\texttt{\textbackslash 9}}
\end{align*}\]

\[\text{\texttt{\textbackslash 0}} \quad \text{\texttt{\textbackslash 1}} \quad \text{\texttt{\textbackslash 2}} \quad \text{\texttt{\textbackslash 3}} \quad \text{\texttt{\textbackslash 4}} \quad \text{\texttt{\textbackslash 5}} \quad \text{\texttt{\textbackslash 6}} \quad \text{\texttt{\textbackslash 7}} \quad \text{\texttt{\textbackslash 8}} \quad \text{\texttt{\textbackslash 9}} \quad \text{\texttt{\textbackslash 0}} \quad \text{\texttt{\textbackslash 1}}
\]

d)

For better recognition he employed in his first attempt four different note head forms as shown under a) in Fig. 47 (like the "Viennese notation" as shown in Fig. 43d), and he even designed new numerals (number symbols) for the digits "one" to "el" (eleven).

Later he tried to simplify his notation by reducing the number of note head forms. At first he reduced it from four to three as shown under b), and then to only two different note head forms as shown under c) in Fig. 47. The latter is similar to the notation as shown under c) in Fig. 44.

Recognizing that the available manuscript (sometimes called

duodecimal numbers in italics, the author modified his notation as shown in Fig. 47 under d).

In closing Part Three, an excerpt from Chopin's Etude op. 25, no. 9, is shown in Fig. 48, written in three different notations. These are: under a) the conventional notation, under b) the Notation Godjevats, and under c) the last notation of the author as shown under d) of Fig. 47.

End.
Fractions and Decimal and Duodecimal Equivalents

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimals</th>
<th>Duodecimals</th>
</tr>
</thead>
<tbody>
<tr>
<td>one-half</td>
<td>.5</td>
<td>8</td>
</tr>
<tr>
<td>third</td>
<td>.33333''</td>
<td>4</td>
</tr>
<tr>
<td>fourth</td>
<td>.25</td>
<td>3</td>
</tr>
<tr>
<td>fifth</td>
<td>.2</td>
<td>2</td>
</tr>
<tr>
<td>sixth</td>
<td>.16666''</td>
<td>2</td>
</tr>
<tr>
<td>seventh</td>
<td>.142857''</td>
<td>186256''**</td>
</tr>
<tr>
<td>eighth</td>
<td>.125</td>
<td>16</td>
</tr>
<tr>
<td>ninth</td>
<td>.11111''</td>
<td>14</td>
</tr>
<tr>
<td>tenth</td>
<td>.1</td>
<td>1</td>
</tr>
<tr>
<td>eleventh</td>
<td>.090909''</td>
<td>719111''</td>
</tr>
<tr>
<td>twelfth</td>
<td>.08333''</td>
<td>7</td>
</tr>
</tbody>
</table>

**Note that duodecimal counting requires the use of two additional symbols, corresponding to the decimal values of ten and eleven. The new symbols are X and E, which are called 'dek' and 'el', respectively. The number which looks like an italic decimal 10, and is equal in value to decimal twelve, is called 'do'. The names of numbers immediately above twelve, or 'do', are 'do-one', 'do-two', etc., 'do-dek', 'do-el', 'two-do', 'three-do', etc.

Future Possibilities

Future generations, in search of outlets for their creative energies, if nothing more, may very well undertake the world-wide implementation of duodecimal counting. Perhaps this will come about when man, at long last, will have overcome the perennial plague of war, by the extension of his system of law to current areas of void. (There is also the thought that we must leave something constructive for them to do, an additional reason for this generation not to try to solve everything mentioned in this paper.)

The serious point is made, nevertheless, that it is a big project to achieve a broad base of international coordination for such a project, particularly in the midst of an atmosphere of extensive international conflict. Energies that are thus disposed, then, are better directed toward projects which will bear higher rates of return on existing material problems—of education, communication, and trade, with resulting effects in alleviating problems of food production, population distribution, and general international conflict.

Computer Applications

It might be asked, in reference to the activities of future generations, suggested above: Why duodecimal counting, why not base eight? Base eight would put man in step with his computers, it might be suggested. This is a possibility. On the other hand, computers might be mechanized to work on the base-twelve as easily as base-eight, though not so easily on base-ten. Beyond this thought, it is conceivable that computers could one day actually lead the way to the over-all conversion to base-twelve. It is possible that, at some stage of the ever-growing computerization trend, the mathematical advantages of base-twelve might be seen to be of sufficient magnitude to dictate the conversion of computers.
The Duodecimal Bulletin

QUO VADIS

The parliaments of New Zealand, Australia, South Africa, Pakistan, Kenya, Uganda, Tanzania, and Great Britain (in that order) have quite recently listened to the testimony in the matter and, acting as would a court of law and justice, did pass sentence, NOT for ten years NOR for life, but that their peoples and their children's children for at least ten generations shall work at hard labor under the burdens of the obsolete 18th century, base-ten, metric system—which France, herself, might be the first to abandon.

In addition they have fined them, in effect, such sums of money as run into the billions, even trillions, of dollars, under cover of what has been called "conversion expenditures". These fines, they suavely tell their people, are a peculiar kind of bird which feed and clothe themselves, and the people who buy them will be practically nothing in decimal money. We will find otherwise. The misdemeanor? Loitering—yes, loitering at the customary corner of Weights and Measures.

Since no major nation was ever known to change its whole system of weights and measures twice within any 200-year period, it is fair to assume that these sentences will run for a period that long. (See chart at left for Canada and U.S. if we buy.)

France will be free, by 1995, to adopt the sound proposals of the late and brilliant M. Jean Essig, who in 1955 suggested the change by his native land from the 18th century system to a 12-base metric system, using duodecimals, meters, liters, and grams. Essig rose to become the Inspector General of Finances for all France; but he died without seeing his proposed duodecimal metric system adopted by his confreres as a reprieve for his beloved people from their incompetent decimal measures, simply because the minimum 200 years had not run their course.

Our Congress is conducting a similar trial of the people of these United States on the same misdemeanor and will need all of the pleas of Friends of the Court possible, if we are not to be sentenced to a like term and much larger fines. Will the U. S. be first or last on the list to adopt a duodecimal metric system? One conversion, once and for all, seems less costly.

The Vice President of one of our largest aircraft corporations, discussing his company's needs for various machine tools in a base-ten metric system, is reported to have said: "A recent Stanford Institute study estimates that the average cost will be $250 to convert each control requiring an accuracy within one-thousandth of an inch. This amount can probably cover only the cost of replacing the inch graduated dials with metric dials. For the majority of machines such as milling machines, jig borers, and lathes requiring metric lead screws, metric nuts, and metric graduated dials, a cost of some $2,000 per control is more likely. A milling machine with six controls would probably run approximately $12,000 to convert."


H.C.C.
Shed a tear for the fate of one royal English Inch, victim in fact of a broken home and of the corollary: "A house divided against itself can not stand."

Now deserted by you and me and by his kingless mother who has to go out of her home to seek daily work abroad (after the demise of her father the great hearted king whose foot, now divided by engineers into ten parts, stood then as a measure of the power of his twelve children), that universally beloved and reliable little Inch, nephew of Uncle Sam, is likely to be found without a foot as of old to stand on.

Especially after his self-indulgent uncles and aunts, following the earlier example of Sam, the eldest, broke up the old domain and each took "his share" of the patrimony leaving their sister, the queen, to go it alone.

None seems to feel remorse as problems mount for this unfortunate minor, now in fact and circumstances an orphan with this French guardian near Paris who every few years determines the number of millimeters and fractions of millimeter to be allotted him (with power to cut off his entire allowance).

And only Heaven can protect his working mother, the queen, as she now seeks to offer strong hands alone to serve on the Continent among foreign peoples, unfamiliar measures, and strangely recorded winter temperatures as the cold north winds blow.

The estate of her father, the late king, is believed by some to have been pillared by his late majesty's advisers who are at this moment boldly dissipating his last shillings, dishonoring his measures and weights, and tossing his good will to the five winds.

But save a few tears for yourselves. For it is here written, when the nineteenth is upon us many will shed tears copiously as they long for the "good old days" back in the "sloppy-sixties."

(Will Little English Inch surmount his problems, terminate an unnecessary guardianship, and return to save his mother? Will he arrive in time to snatch his mother the queen from the claws of those scheming Continental buzzards who believe she is part of a decadent and deteriorating people very ripe and ready as a smorgasbord for their table? Watch at this same time and place tomorrow as his problems mount to discourage even the stout of heart.)

-o-o-o-

Is U. K., the high born, eccentric, dear Old Lady, now to act as an ordinary commoner and to "take her place" in the milling mob at the market? That's an order, you know. Sam. SAM!

-o-o-o-

Nectares, meters, kilograms, Measure feed for sheep and rams; Measure too the length of fence—
Everything except new pence. ---Anon.

The Duodecimal Bulletin
NUMBER-BASE ODDMENTS
By Shaun Ferguson

Using reverse notation (also called "two-way" notation), we can show:

<table>
<thead>
<tr>
<th>Base five</th>
<th>Base eight</th>
<th>Base eleven</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 = 22</td>
<td>25 = 33</td>
<td>37 = 44</td>
</tr>
</tbody>
</table>

This led me to consider whether there were any cases of trebling and quadrupling by reversal of digits in a given base. In fact we can have any multiple by reversing the order of the digits, but the multiple concerned depends on the base used. Here are the results in tabular form:

<table>
<thead>
<tr>
<th>x = Double Treble</th>
<th>4 Times</th>
<th>5 Times</th>
<th>6 Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x - 1)</td>
<td>(4x - 1)</td>
<td>(5x - 1)</td>
<td>(6x - 1)</td>
</tr>
<tr>
<td>1 base 2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2 5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>3 8</td>
<td>2</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>4 2</td>
<td>13</td>
<td>17</td>
<td>12</td>
</tr>
</tbody>
</table>

and so on.

In each case the basic form for x = 2 is 27 x 11 for the three-figure solution.

This led me to ask if there were solutions if one reversed the digits and used another base. For your interest the results obtained so far:

<table>
<thead>
<tr>
<th>232 four</th>
<th>is half of</th>
<th>232 six</th>
</tr>
</thead>
<tbody>
<tr>
<td>354 seven</td>
<td>&quot;</td>
<td>453 nine</td>
</tr>
<tr>
<td>253 nine</td>
<td>&quot;</td>
<td>352 eleven</td>
</tr>
<tr>
<td>476 ten</td>
<td>&quot;</td>
<td>674 twelve</td>
</tr>
</tbody>
</table>

And another problem: Same digits, different base:

<table>
<thead>
<tr>
<th>121 four</th>
<th>is half of</th>
<th>121 five</th>
</tr>
</thead>
<tbody>
<tr>
<td>130 seven</td>
<td>&quot;</td>
<td>132 nine</td>
</tr>
<tr>
<td>130 six</td>
<td>one-third</td>
<td>130 nine</td>
</tr>
<tr>
<td>110 four</td>
<td>&quot;</td>
<td>110 six</td>
</tr>
<tr>
<td>101 three</td>
<td>&quot;</td>
<td>101 five</td>
</tr>
<tr>
<td>101 five</td>
<td>one-half</td>
<td>101 seven</td>
</tr>
<tr>
<td>101 four</td>
<td>one-eighth</td>
<td>101 eleven</td>
</tr>
<tr>
<td>101 five</td>
<td>one-fifth</td>
<td>101 eleven</td>
</tr>
</tbody>
</table>

I have assumed in most cases that there should be a difference of two between the bases; perhaps others who are interested in number bases other than ten would like to try the cases where the difference between the bases is three or more?
ELUSIVE PEACE AND BASE TWELVE

Almost simultaneously the United Nations Organization and the Duodecimal Society of America were born. In the third issue of Volume 7 of The Duodecimal Bulletin, p. 7, the formation of the United Nations Standards Coordinating Committee was quickly called to the attention of DSA. At that early date it was expected that representations would be made to the UNSCC urging adoption of the French Metric System as the world’s coerced standard.

To meet that pressure it was promptly urged in the October 1945 Bulletin that DSA expedite our agreement upon a duodecimal metric system and present it to the National Bureau of Standards and the American Standards Association, and that steps be inaugurated looking towards legislation for permissive and simultaneous use of the duodecimal standards. The mills of the gods grind slowly, ‘tis often said.

In two dozen years DSA has found itself unable to secure favorable permissive legislation from the Congress or acceptance by the NBS. It is perhaps time, if we value the advancement of science, to seek a different avenue. In coming issues will be found an approach to the beginning of a study now being undertaken by direction of the fruitful Santa Barbara DSA meeting.

There are not less than a half dozen potential duodecimal metric systems upon which to concentrate our attention, any one of which might be hammered into shape for international acceptance by the sanction of its permissive adoption. None of any one system will prevent, from time to time, the exposition by this editor of others believed by their authors to contain improvements vital to an increased use of base twelve.

One of these systems was ably advanced very early from the fertile mind of Ralph H. Beard, another by M. Jean Essig, still another by Horatio W. Hallwright, another from the early thinking of Shaun Ferguson, one by Admiral G. Elbrow R. N., one by the Alsomogordo geodesist Charles S. Bagley, another by T. Pendlebury, another by Henry C. Churchman, and by others in more or less detail.

If a concerted program be now inaugurated involving the least change of terms (merely adding the word "dozenal" or a small ‘d’, such as "md" for "mètre duodécimal"), and the least reorientation (merely recasting of the base ten metric system into a base twelve metric system), then possibly the Essig suggestions with slight modifications might furnish the most practical approach to a beginning of a permissive dozenal metric system.

We might thereby achieve an international agreement for its voluntary use alongside our customary weights and measures and the base-10 metric system over a period of the next two dozen years, culminating hopefully in an improved 21st century international metric system of measurements. A duodecimal metric system need not be as elusive as peace is believed to be since the end of World War II. Should we not prepare now for both?

H. C. C.

JEAN ESSIG

Not quite 70, Jean Essig died suddenly on the second day of June, 1969. He was born 5 October 1899.


Joined Inspection des Finances 1926. Held post of financial adviser (conseiller financier) on French Railways (1930), then on Tourism (1938). In 1939–1940 on the Lorraine front he held the rank of Captain of Field Artillery in the French army. It was there, during the black gathering storm, he began to think about the advantages of a duodecimal metric system in contrast to all base-ten measurements in France and in the entire metric world.

He headed the financial delegation for armed forces in Algiers (1942), to help alleviate the lot of France in her captive state.

After World War II he applied himself to problems of national defense. In this capacity from 1948 to 1960, he was Assistant Director, then Director of the Institute for Advanced Studies in National Defense. It was during this period and in relation to the national defense that he marshalled all of his ideas on a duodecimal metric system. Since 1960, as Inspecteur-Général des Finances he carried on until his death.

Jean Essig wrote Douze Notre Dix Futur (Twelve, Our Modern Ten), published by Dunod, Paris, in 1955. In that effort he extended generous credit to the work of two of the founding fathers of The Duodecimal Society of America, F. Emerson Andrews and George S. Terry, in the same field.

The Duodecimal Society of America, in recognition of his profound thinking, granted him the society’s Annual Award for 1957. He delivered many lectures on duodecimals both in France and in Belgium.

In 1958, he published a work entitled "Les Aspects Civils et Militaires de le Défense Nationale (Civil and Military Considerations of National Defense)."

He was Commandeur de la Légion d’Honneur and held Croix du Combattant, as well as many foreign decorations.

In another facet of his life, he was a member of the Board of Directors of the Club Alpin Français, from whom he received the Gold Medal.

He was married and the father of five.

—Brian Bishop
The Duodecimal Bulletin

ACCOLADE TO THE NEA

It has been brought to our attention that the Representative Assembly of the NEA is said to have passed the following resolution in July 1969:

"The National Education Association recognizes the importance of the Metric System of weights and measures in contemporary world commerce and technology. The Association believes that a carefully planned effort to convert to the metric system is essential to the future of American industrial and technological development and to the evolution of effective world communication. It supports federal legislation which would facilitate such a conversion. The Association believes it is imperative that those who teach and those who produce instructional materials begin now to prepare for this conversion by urging teachers to emphasize the use of the Metric System in regular classroom activities."

That statement is to be highly praised and might have gone much further. What is being done in schools to make a study of base-12 metric time, angle, navigation, money, communications?

A metric timespace ever so gently might be given space on the wall of every classroom alongside the current clock, dividing the whole day (still employing Grandfather-clock Roman numerals if you will—for metric time is ageless) into a dozen equal parts, and each such arc into a dozen subdivisions so that the longer hand can count the "Moments" of each day and a "seconds" hand the metric time of "Dots" (1/3 second, or 25/72 EXACTLY). See Metric Time, p. 8, April 1969 Bulletin.

There is nothing metric about hours, minutes, and seconds of time now displayed in our classrooms. Imagine its far-reaching and silent teaching effect on millions and tens of millions of pupils throughout the five days of a school week.

H. C. C.

EXCERPTS FROM LETTERS AND COMMUNICATIONS

ET TU BRUTUS, in the spirit, to be found at Via Appius Unum, or The Forum, is aroused and sends the following communication: "I must object to the last few lines on page 28 of the April 1969 Bulletin in which your editor says that 'the year in which Julius Caesar (the traitor who deserved to be and) was struck down was a leapyear...'. The leapyear obtained in B.C. 45 (-44), the year in which that rascal confused every one, threw our accountants off balance with his so-called calendar reform, and began to act like an imperialist. The year in which we patriots took him apart on the Idea of March is now called B.C. 44 (or -43), dozannally -27, and was a regular year, without the bissextile. Therefore, the anniversary moment at the end of the second millennium after our brave action, came on the Idea of March, 1957 (1171)—not 1956, certainly not 1955. Please correct. BRUTE."

Note: Brutus is dead right. Happy to know he is still serving time on earth. --Ed.

Editorially speaking—

WE COULD GO METRIC PLEASANTLY AND PERMANENTLY

In 1955, the late M. Jean Essig, afterwards Inspecteur Général des Finances for all France, a native Frenchman, published in Paris "DOUCE NOTRE DIX FUTUR" (Twelve, Our Modern Ten) to point out the natural obsolescence of 18th century metrication. It was purposely prepared for study in the base-10 metric world.

Essig conceived a dozen metric system to replace France's base-10 system, using the same terms of meter, liter, and gram, merely adding the adjective "duodecimal" and recalculating accordingly. He described these new sizes in their relationship to the 18th century metric dimensions. They were later defined by this writer completely independent of the base-10 meter, in the base-twelve wavelengths of orange-red krypton 86 light. On page 2, April 1969 Bulletin, linear comparisons may be found.

Dimensions and Areas

As an indication of the more intelligent earlier English government, notice that one square duodecimal hectometer (one duodecimal hectare), and the square one-tenth English or Canadian mile, are quite interchangeable in our modern planting and gathering of crops by machinery in North America.

One might readily imagine an alert U.S. Agricultural Stab. Service modernizing its pre-measurement and estimates of crop acres by employing "duodecimal hectares." It could, initially, describe areas of wheat or corn land in Kansas or Iowa by percentages of our present Congressional Sections (12% equated with one hectare) and give the totals by states in square miles.

In Saskatchewan (Canada), too, one square statute mile might be considered the equal of one hundred "duodecimal hectares" for all agricultural purposes in describing areas of farms in crops, creeks, roads, runways, lakes, pasture, etc.

Try to imagine pleased farmers in Illinois or Alabama or Oregon (or Saskatchewan) employing the obsolete 18th century hectare—one is said to equal, let's look it up, in the conversion tables, 0.003 861 022 part of a square mile.

18th Century Faults

But areas in this or that crop, while exceptionally important to the Central, Southern, and Western States, are a minor part of the duodecimal metric show in the United States.

The 18th century metric system seems powerless to deal with metric time, metric angles, navigation, communications, and new metric money that can be subdivided as more intelligent British people divided their shillings in the Goode Olde Days.

Money

Duodecimal metric money employs INTEGERS equal to exact third or sixth parts, as well as halves and fourths of a dollar, half dollar, quarter dollar, or BIME. No one is robbed or enriched or confused during this changeover since a penny remains a pen-
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The other symbol is the familiar Arabic eleven, shown 11, but
having two horizontal lines across her waist, thus ¥, to in-
dicate a single symbol equal to eleven but NOT TO BE CALLED ELE-
VEN (perhaps the English diminutive "el"), nor written as 11.
one-one is a different number from ¥. Just touch the digits in
the order assigned to your party—simple as yesterday.

If you call X ten or ¥ eleven, some listener may, from long
training, write down 10 or 11 and dial your number incorrectly.
It would seem more practical to drop "ten" and "eleven" from
all duodecimal counting eventually, perhaps substituting tek
dek and el to achieve a greater efficiency.

American know how Bell Laboratories and Western permitted
this advancement. Business demand caused it (not government
decree), and therefore the expense can be met from profits.

Your E-Z Zip Number

Now that Bell has moved to twelve touch-tone buttons, shall we
not soon see watchmakers permissively going duodecimal met-
cric; farm lands voluntarily measured in duodecimal hectares;
and Eumecical-Zone (E-Z) worldwide zip numbers, as advanced in
Britain by Brian R. Bishop (April 1969 Bulletin, page 2) and in
America by Kingsland Camp and Henry Churchman, displacing paro-
chial zips?

E-Z numbers, employing only eight duodecimal angle digits,
can designate any point on the globe a dozen duodecimal hecto-
meters from another. Fancy being able to pinpoint any spot on
Earth not more than 6/5 Canadian statute mile, either easting or
northing, from another—using but 8 digits.

Other Possibilities

The Russians, some believe, are already far ahead of France
with the duodecimal hectometer, as well as duodecimal metric
angles and time, to control worldwide navigation of their sub-
maries. Each Russian ship is ordered to its new post on the
face of the globe, within one duodecimal kilometer it is sug-
gested, by adding only eight digits to its code name. You will
not believe it? Degrees, minutes, and seconds of angle could
be as tabular as the pterodactyl was when it disappeared.

Russian experts, a few years ago, directed one of their rock-
ets to a spot in the Pacific ocean, bringing it down within a
pre-designated ring equal to six duodecimal kilometers in radi-
um. Newspapers at that time reported the size of the guarded
circle in customary measures, but the conversions tell us much.

If it ever becomes necessary for the USSR (do you believe no
such plan exists?) to send occupation forces into Canada or the
United States, their base-ten meters would confuse them because
no line fences here are laid out in patterns of the 18th cen-
tury metric system. But if their army employed their navy's du-
odecimal hectometers, each unit would be quite equal to the one-
tenth Canadian or U. S. statute mile. And ten such hectometers
might agree with the mile on the ground—each hectometer with
the one-tenth mile on odometers of automobiles and trucks which
their armies might confiscate in the United States or Canada.

N.C.C.