COUNTING IN DOZENS

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written 12, and is called doz., for dozen. The quantity one gross is written 100, and is called gros. 1000 is called no., representing the neg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten: that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 2 units, 6 dozen, and 5 dozen-dozen, or gross. This number would be called 2 gross 6 doz. 5, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

<table>
<thead>
<tr>
<th>94</th>
<th>136</th>
<th>Five ft. nine in.</th>
<th>5:91</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>694</td>
<td>Three ft. two in.</td>
<td>3:2</td>
</tr>
<tr>
<td>56</td>
<td>362</td>
<td>Two ft. eight in.</td>
<td>2:8</td>
</tr>
<tr>
<td>39</td>
<td>1000</td>
<td>Eleven ft. seven in.</td>
<td>11:7</td>
</tr>
</tbody>
</table>

You will not have to learn the dozenal multiplication tables since you already know the 12-times-table. Mentally convert the quantities into dozems, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3, so set down 53. Using this ‘which is’ step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2E, which is 2 dozens and 5.

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12² (or 144) times the third figure, plus 12³ (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

<table>
<thead>
<tr>
<th>Numerical Progression</th>
<th>Multiplication Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 One</td>
<td>1 2 3 4 5 6 7 8 9 X E</td>
</tr>
<tr>
<td>10 Do</td>
<td>2 4 6 8 X 12 14 16 18 1X</td>
</tr>
<tr>
<td>100 Gro</td>
<td>3 6 9 10 13 16 19 20 23 26 29</td>
</tr>
<tr>
<td>1,000 Mo</td>
<td>4 8 10 14 18 20 24 28 30 34 38</td>
</tr>
<tr>
<td>10,000 Do-mo</td>
<td>5 12 14 18 20 24 28 30 34 38 42 47</td>
</tr>
<tr>
<td>100,000 Gro-mo</td>
<td>6 18 20 26 30 36 40 46 50 56 60 6X</td>
</tr>
<tr>
<td>1,000,000 Bi-mo</td>
<td>7 25 28 34 40 46 50 56 60 66 72 7E</td>
</tr>
<tr>
<td>1,000,000,000 Tri-mo</td>
<td>8 38 40 46 50 56 60 66 72 7E 82 8E</td>
</tr>
<tr>
<td>and so on.</td>
<td></td>
</tr>
</tbody>
</table>

THE DUODECIMAL SOCIETY OF AMERICA

Office, 20 Carlton Place, Staten Island, N. Y. 10304
Secretary, 11561 Candy Lane, Garden Grove, Cal 92640
is a voluntary nonprofit organization for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

The forms of membership include Honorary, Life, Fellow, and Senior Members, as well as Members, and Student Members. Members and Student Members are not required to pass aptitude tests in base twelve, but are encouraged to do so.

Senior membership with voting privileges requires passing of elementary tests in the performance of twelve base arithmetic. The lessons and examinations are free to those whose entrance application is accepted. Remittance of $6, dues for one year, must accompany application. Forms free on request.

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**The Duodecimal Bulletin**

All figures in italics are duodecimal.

**Chromatic Musical Scales and Notations**

By Erich Kothe

Part One

Hearing and Sound

One of the most important sensory organs of man is his sense of hearing. The usual physical stimuli for hearing are successive pressure waves of the surrounding air. Such pressure waves are called (transversal) vibrations of the air. However, in order to be perceived by the human ear as a sound, those vibrations have to be within the frequency response of man's ear, between approximately 20 and 15000 cycles per second. And their intensity has to be above the so-called "threshold of audibility," yet below the "threshold of feeling" (also called "threshold of pain").

This range of intensity to which man's ear is sensitive is about a millionfold. The graph of Fig. 1 shows these limits. It also indicates that man's ear is most sensitive to sounds of about 2000 cycles per second, which is the approximate resonance frequency of the air within the ear canal (see Fig. 2).

The frequency of a sound is sensed by the ear as its pitch: the higher the frequency, the higher the pitch. Similarly, the intensity of a sound is sensed as its loudness: the higher the

---

Figure 1.
intensity, the greater its loudness. However, there is not necessarily a one-to-one correspondence between the absolute intensity of a sound and its sensed loudness. A simple test can prove this point.

Figure 2.

If someone exposes only one of his ears to a sound of about 800 cycles per second and of medium loudness for about 2 minutes, and then he exposes independently each ear to another sound of about 1200 cycles per second and of the same intensity for about 2 seconds, he will sense different pitches and loudness of this second sound by his two ears. The ear which was exposed to the first sound will sense the second and shorter sound higher (approximately a half tone higher) and distinctively less loud than the other ear which was exposed to the second sound only.

Besides the just mentioned influence of pre-exposure on the sensed pitch and loudness of a sound, another influence on the sensed pitch has been observed which was due to high intensity. This influence becomes usually noticeable as a decrease of the sensed pitch, and the amount of this apparent change is being affected by the frequency as well as by the intensity of the sound. The graphs of Fig. 3 are from data by Snow.

Many hearing tests have been performed which disregard the already mentioned influences of pre-exposure and of high intensities, and which ignore the widely differing hearing characteristics of different individuals. Therefore, their results have to be interpreted cautiously, and they are valid only for the (statistically) average ear. Nevertheless, these results are helpful guides, as for instance the curves given in Fig. 4 which show sensed loudness levels (expressed in phons).

Figures begin on page 29.
Unfortunately, the sensed loudness of a sound does not increase linearly with an increased sound pressure level. Therefore, another scale based on the average listener's reaction has been devised which uses a "sone" as its fundamental unit; and a sone is defined as the loudness of a simple (pure) tone of 40 decibels at 1000 cycles per second. According to this scale, a sound which appears to be two or three times as loud as a sound of one sone would have its loudness classified as of two or three sones. The actually measured relation of the sensed loudness expressed in millisonses (one millennium equals .001 sone) against the loudness levels expressed in phons is indicated by a heavy line on the graph of Fig. 5. The broken line shows a hypothetical, strictly linear relation.

For musical purposes, the relative intensity of one sound to another one, the change of the intensity level, is much more important than the absolute intensity of each sound. Therefore

Curves showing sensitivity of the ear to changes in sound pressure.

(After Fletcher)
the minimum detectable change in the intensity level is significant, even though it varies greatly with the intensity itself, the frequency, and to some extent with the complexity of the sound. For simple (pure) tones, curves have been drawn (see Fig. 6) which indicate the minimum detectable change in intensity level as a function of intensity and of frequency.

Similarly as to the loudness, the sensed pitch of a sound does not increase linearly with the increased frequency. Therefore, some scales have been proposed which are based on the average listener's reaction. The best of these scales seems to be the one devised by S. S. Stevens and by J. Volkman which uses a "mel" as its measuring unit. A "mel" (derived from the word "melody") is defined as one thousandth of the pitch of a simple (pure) tone at 1000 cycles per second and of 53 phones (see Fig. 4) loudness. The curve appearing in Fig. 7 shows the sensed pitches of various frequencies in the range between 40 and 12000 cycles per second.

For musical purposes, the relative frequency of one sound to another one is much more important than the absolute frequency of each sound. Therefore, the minimum detectable frequency shift is significant, even though it varies greatly with the intensity, the frequency itself, and with the complexity of the sound. For simple (pure) tones, curves have been drawn (see Fig. 8) which indicate the minimum perceptible change in frequency as a function of frequency and of intensity.

Figure 8.

Curves showing sensitivity of the ear to changes in frequency.

(Shepard and Biddulph, Jour. Acous. Soc. Amer. 3, 375 (1931))

(Continued on page fourdo-five)
It is interesting to observe the development of counting habits in children. At kindergarten age they can nearly all count 'by ones'—'one, two, three' and so on, up to twenty, thirty, or perhaps further. At six or seven they know how to count by fives 'five, ten, fifteen, twenty-' or by tens, or even, (getting sophisticated) 'to count to a hundred by hundreds'. This kind of counting is often practiced in such games as 'hide and seek' as 'it' counts while the others hide. We may ask what this practice achieves apart from an illusion of getting somewhere fast. The answer is not very much except practice in the 'five times' table, which is easy in decimals. As much is achieved by counting in ones to twenty as in fives to a hundred.

The situation is rather different however when we employ our counting skills for purposes of tallying, that is, for counting things. If you were asked for instance to count the number of words in this article or to assist with the stock-taking in a shop (both practical needs from time to time), how would you set about it? The chances are that you would find groups of five awkward and rather too big to count at a glance. It is quite possible you might even count them in ones, certainly the slowest and most tedious way, but nevertheless nearly always used by younger children. You might do better counting in twos 'two, four, six, eight...'. Would you count in threes or fours (quite likely), which might be suitable groups to see at a glance, and quite a lot quicker to tally?

The answer, if you are working in decimals, is probably no, because the three and four times tables are not easy to use in this way.

People who have to tally with speed and accuracy have found a way to overcome this problem—by counting in the scale of twelve or dozens. Thus, by 'threes' it becomes 'three, six, nine, a dozen' (one and) three, (one and) six, (one and) nine, two dozen, (two and) three...'; or by 'fours' we have 'four, eight, a dozen...' and so on, where the running count of the dozens themselves (bracketed above) may be omitted in a rapid verbalization.

This latter scheme also makes it easy to count by 'sixes' if the pattern of objects is sufficiently regular to permit it, or to fall back to pairs if use of larger groups is inconvenient. It is this fact, just as much as the greater economy in packaging by multiples of a dozen and their greater convenience for handling or other purposes, which has led to the use of dozens groupings of goods for a very high proportion of commercial transactions.

If indeed we were to make greater use of base twelve for arithmetic, counting could be a substantial aid to the learning of multiplication tables and also to a better understanding of their nature than sometimes seems to be the case at present. For those who still feel the need for an anatomical aid to calculation, even one of these is available—use the twelve joints (Continued on page fourdo-ten)

"If a slow-moving country such as England can finally change its monetary system, it ought to be more than time for the U.S. to move our whole system of weights and measures from the Middle Ages into modern times." I quote above what is being said by the philadecometrics every day now. But let us go further.

Is it not time for the whole world to improve its Eighteenth-century, outdated, base-ten metric system, and, eventually, the international inch and mile, to bring both opposing systems into ONE WORLDWIDE, TWENTY FIRST CENTURY wholly correlated system of weights, measures, time, angle, and base? The base-twelve metric system could integrate all of these!

These United States of America have been reluctant to adopt an eighteen century metric system until that concoction shall have been brought to date and so improved as to include time, angle, and navigational metric units—not the half-baked, indigestible cake sometimes offered to us at an undetermined price.

The French metric system of measurements is pointing the way. But, at a time when any scientist could understand how tenuous is the tie of his head to his torso, with the guillotine standing in Concorde Place to welcome anyone, erudite or ignorant, whose conduct offended a powerful rabble, thoughtful mathematicians, sitting in Paris, considered and abandoned base-twelve (the necessary ingredient to create a universal metric system) as too advanced for the people of France.

Fortunately, Frenchmen today do not live in fear of that ancient anesthetic, and some are working out a complete system to change their own measuring scheme into a base-twelve potentially worldwide metric system. These efforts have produced a system in which time, angle, and navigational units of measurement are correlated with base-twelve millimètre, mètre, and kilomètre, and all other metric units in use, to form a complete, thus far unachieved, modern metric system.

When the international metric union adopts these changes, and takes steps to implement them, the people of the United States and Canada, without any statutory coercion, might reasonably be expected not only to live with the improved metric system, but to embrace it eventually in its entirety.

In fact, every day, Americans are employing multiples and submultiples of the metron (75 000 00 krypton atomic wave lengths) and the edon (75 000 000; 0 kr. 86 wave lengths), which are the equals of the French newly suggested duodécimètre and the hectomètre duodécimal, under the cloak of 3/23 inches and the Canadian one-tenth land mile, respectively. The edon is only two and a fraction inches greater than the length of 528 international feet. The metron is 1/3000th part greater than 3/23 International or Canadian inches—requiring a microscope to see the difference.

Navigationally, one edon multiplied by the fifth power of twelve is the equal of one great circle of the earth. Edons (one-tenth of an English land mile) are not unsuitable for mo-
tor vehicle use, ships at sea, and air jetliners— an identical unit for land, sea and air. all three employing the aeronecle (twelve edons), also called the Navimout6 or Kilometre Duodecimal. The single, universal unit of length could replace kilometre, land miles, and nautical miles completely, all out the window together when people begin to think and act.

If pounds and shillings are obsolete, then so are degrees and minutes of angle, and minutes and seconds of time, since all are compound denotive numbers alike. In any properly improved metric system, hours and minutes must follow pounds and shillings to Sheol. The Eighteenth Century baseten, metric system has seemed unable to supplant the second of time, which is quite unmetric. Today we find the second in the metric MKS and egs formulas. Nothing is metric about seconds in their relation to the hour or the day, in base-ten.

In the United States of America, in one square statute mile (a Congressional Section of farm lands) there can be said to be one hundred square edons or 100 square hectometres duodecimal. The variation is one in three thousand parts— about the same dissimilarity as between one section of land and perhaps an adjoining section in the same township. The U.S. Department of Agriculture might be able to work in percentages instead of 1/640 parts of a Section of farm lands. Iowa could he said to have rested 5937.50 Sections or square miles (3,800,000 acres) of crop land in 1968. An inadequate conservation of farm land!

One dozen meters are equal to the dimension of fourty-four inches (one metre duodecimal) in a framework of reference differing by one in 3000 parts. By definition, the meter is exactly equal to seven dozen and five great gross (72,000,0) kr. 86 wave lengths. The international meter equals 1,659,763.73 kr. 86 wave lengths today. The N.B.S. can measure either exactly.

The costs of a changeover, voluntary or involuntary, in the United States of America from our present inches and miles to the metric system, either ten-base or twelve-base, is entirely prohibitive. But, if we let the Frenchmen lead in reforming their present metric system into base-twelve units of length, volume, weight, angle, time, and all related or derived measurements, and join with France in setting up the base-twelve system on a voluntary, or auxiliary, worldwide scale, the costs might be half as much as they must be if we change first to the base-ten metric system and later to the base-twelve metric.

But if we move in now and embrace the base-ten metric system, then when nearly the whole world is using a twelve-base metric system we might be the only nation, plus a few aboriginal Pygmies in outback Australia, stuck with a ten-base metric system— simply because of the stubborn resistance of any people, any people, to two changes in their measures in ten generations.

*Read DOUZE NOIRE DIX FUTUR, by Jean Essig, 1955, Dunod, Paris, with a foreword by M. Albert Canout, Member de l’Institut.

Modern computers have made it possible for man to extend the decimal representation of mathematical constants by a factor of about six gross. Longer decimal representations provide a better basis to analyze the randomness of these fractions and also to show where these fractions nearly terminate. Because fractions are easier to analyze when they are expressed duodecimally, there is a similar need to extend the duodecimal representations of these constants. The subsequent computer conversions are the first step in this analysis.

Because the IBM 360 Model 75 computer that I used can not accurately store more than a ten digit number, the doubly-checked decimal fractions were stored in eight digit blocks in a linear array. To multiply the fraction or array by twelve, the computer was instructed to multiply the last block of eight digits by twelve, add a carry (initialized to zero for the last block), and store the result in a dummy variable. Because the computer absolutely truncates its intergals in division, that is to say, 1199999999 divided by 100000000 gives simply 11, the computer was then instructed to divide the dummy variable $10^8$ and store the quotient in the carry to be used for next block up in the array. Finally, to get the last block ready to be multiplied again by twelve, the product of the carry and $10^8$ was subtracted from the dummy variable and the result stored back in the last block of the array. The computer then carried upon the whole set of operations successively up to the blocks of the array until the initial block was completed. The carry was then the first digit in the duodecimal representation.

Mechanically carrying out this procedure, one sees that it is essentially multiplying the array by twelve. After eight such "multiplications by twelve," the computer was told to ignore the last block of eight digits because these digits had lost all significance in computing the result and because this procedure saved computer time. The "multiplications" were repeated until all decimal significance was gone.

The conversions shown on the following five pages are Pi, Fuller's Constant, Khintchine's Constant, e, and l/e. The duodecimal representation of Khintchine's constant is as accurate, I found, as the decimal representation. All of the rest of the constants have been truncated to save space. Fuller's constant was converted to $10^4.0$ duodecimal places; $1/e$ to $1498.0$, pi and e to $10,068.0$, and I have the natural log of two converted to $1888.0$ places.

I would like to thank Robert Childress, James Roman, James Dixon, Janette Birkner, and Bruce Moon for their help in assisting me with my program. A special thanks goes to Bruce Moon for testing the program and suggesting a programming technique which saved 500 e g computer time.

[In the following five tables X equals ten and E represents 10]
### Euler's Constant Concluded.

<table>
<thead>
<tr>
<th>Euler's Constant</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approximation</td>
<td>3.1416</td>
</tr>
</tbody>
</table>

### Khinchine's Constant

<table>
<thead>
<tr>
<th>Khinchine's Constant</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approximation</td>
<td>2.3036</td>
</tr>
</tbody>
</table>

### Duodecimally Euler's Constant Equals

<table>
<thead>
<tr>
<th>Duodecimally Euler's Constant</th>
<th>Duodecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal to</td>
<td>28.25791723</td>
</tr>
</tbody>
</table>

### Duodecimally \( e \) Equals

<table>
<thead>
<tr>
<th>Duodecimally ( e )</th>
<th>Duodecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal to</td>
<td>28.25791723</td>
</tr>
</tbody>
</table>
A number, of three digits, in base seven, when expressed in base nine has its digits reversed; find the number.

I found this question in an old algebra book, circa 1910, and solved it (the number is 503 base seven). While thinking out the solution, I wondered if there were any other numbers in any other bases which could be treated in the same way. I present the following results for your interest.

<table>
<thead>
<tr>
<th>Number</th>
<th>Base</th>
<th>Number</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>Five</td>
<td>102</td>
<td>Seven</td>
</tr>
<tr>
<td>503</td>
<td>Seven</td>
<td>305</td>
<td>Nine</td>
</tr>
<tr>
<td>302</td>
<td>Nine</td>
<td>203</td>
<td>Eleven</td>
</tr>
<tr>
<td>705</td>
<td>Eleven</td>
<td>507</td>
<td>Thirteen</td>
</tr>
</tbody>
</table>

Might one find a pattern? For example, in base five, if 201 obeys the rule, then so should 402.

Whence, we find:

<table>
<thead>
<tr>
<th>Number</th>
<th>Base</th>
<th>Number</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>402</td>
<td>Five</td>
<td>204</td>
<td>Seven</td>
</tr>
<tr>
<td>604</td>
<td>Nine</td>
<td>406</td>
<td>Eleven</td>
</tr>
</tbody>
</table>

"A number consisting of three digits is doubled by reversing the digits. Show that the same will hold for the number formed by the first and last digits."

I knew one solution (in base eight) to be $2 \times 25 = 52$. For three digits the solution, in base eight, is $2 \times 275 = 572$.

And upon investigating other bases, I found, for example:

In base five, $2 \times 143 = 341$.

In base eleven, $2 \times 3x7 = 73x$.

---

(Mr. Ferguson will have more on this subject in the next issue of the Bulletin. His delight is to be with figures and to work among digits in any base, purely in exploratory manner and without limiting himself to base twelve or base ten alone. He is an asset to any duodecimal society. His address is 58, Scothby Village, Mr. Carlisle, Cumberland, England. He will be pleased to hear from you, if you are unable to reach same results as he, or when you find other examples. --Editors.)

The Duodecimal Bulletin

More on douze notre dix futur

On page 106 of DOUZE NOTRE DIX FUTUR*, loosely translated, we find the following:

"Now note that one sees at once, in duodecimal numeration, how all these new units of length are derived one from another, by an extremely simple formula:

A Great Circle of Earth

\[
\begin{align*}
10000;0 & \quad \text{equals} \\
100000;0 & \\
(1;0 \text{ km}) = \\
(10;0 \text{ km}) & = \\
(100;0 \text{ dam}) & = \\
(1000;0 \text{ m}) & = \\
(10000;0 \text{ dm}) & .
\end{align*}
\]

Since there could be as many different Great Circle lengths as there are physicists or authors (each great circle divided by 100 000 000;0 will equal 75 000;0 krypton 86 wave lengths, MORE OR LESS), it is suggested that one duodécimètre duodécimal be legally defined by the world's governments as the exact equal of 75 000;0 krypton 86 light waves to achieve familiarity.

This dimension could be determined by the National Bureau of Standards of the United States of America, or, for that matter, anywhere in the heavens or on earth, now or in any regroup of years hereafter, in the most exact manner and in whole numbers of krypton 86 light waves (now used to define the present international meter as equal to 1,650,763.73).

(However, a stick in that dimensional unit for one dozen such units) might be held in the palm of one's hand (or seen at your arm's length) in a most practical manner.

Iowa farmers, in moving from decimal to duodecimal dimensions at some future date, might be surprised to learn that one hundred hectares duodécimaux and one congressional section (one square statute mile) of farm lands are sisters under the skin.

H.C.C.

*Published by JEAN ESSIG, 1955, Dunod, Paris.
EXHIBIT OF A COMPUTER PROGRAM

by Robert R. McPherson.

Here is shown a printer page an 80-80 listing of (1) one-digit data input in data cards, and (2) one-digit data output data cards. These two items constitute an exhibit of a computer program which accepts data in decimal format and produces an output card containing a statement of equivalent decimal and duodecimal numerals. In (1), two or more is shown by the symbol X and el or eleven by the symbol E. Also note that base twelve numbers are preceded by an asterisk in lieu of a duodecimal point. The decimal numbers, excepting the tens, hundreds, thousands, etc., have been chosen at random, and include whole numbers, decimal fractions, and mixed decimal numbers.

Item (1)

<table>
<thead>
<tr>
<th>1234567890</th>
<th>4758.456</th>
</tr>
</thead>
<tbody>
<tr>
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<td>7654.321</td>
</tr>
<tr>
<td>0000123456</td>
<td>00000123</td>
</tr>
<tr>
<td>0000004567</td>
<td>00000045</td>
</tr>
<tr>
<td>1111111112</td>
<td>12345678</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
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<td>1000</td>
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<td>10000</td>
<td>10000</td>
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<td>100000</td>
<td>100000</td>
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</tbody>
</table>

Item (2)

<table>
<thead>
<tr>
<th>123456789000</th>
<th>X355971E2</th>
</tr>
</thead>
<tbody>
<tr>
<td>475845600000</td>
<td>2906557E</td>
</tr>
<tr>
<td>98765431000000</td>
<td>33837131</td>
</tr>
<tr>
<td>00001234</td>
<td>0000030X</td>
</tr>
<tr>
<td>00000945</td>
<td>00000214</td>
</tr>
<tr>
<td>00000005</td>
<td>00000002</td>
</tr>
<tr>
<td>11111111000000</td>
<td>387050E0</td>
</tr>
<tr>
<td>1010000000</td>
<td>X690</td>
</tr>
<tr>
<td>1000000000</td>
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</tr>
<tr>
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<td>34230540</td>
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</tbody>
</table>

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Senior Member Robert McPherson operates the McPherson Laboratories for employment of Hindu-Arabic-American Base-Twelve Numerals, at 1030 N. W. 3rd Street, Gainesville, Florida, 32601. Space permits the showing of only two of his exhibits. --76.

EXCERPTS FROM LETTERS AND COMMENTS OF OUR CORRESPONDENTS.

BRIAN R. BISHOP, 155, Leighton Avenue, Leigh-on-Sea, Essex, England, writes: I was absolutely delighted to receive the September Bulletin... I feel honoured by the flattering references to me on page six [but] I did not resign for reasons of health. I found myself quite unable to give proper attention to my duties, and I saw no point in half-doing a job. At that time I found I had too much at home and, being recently married, with our first home to set up, I thought my prime duty lay there. My job was also changing and becoming more exacting and, being my bread and butter, could not be allowed to suffer. Please let those concerned know, lest the incorrect documentation be accepted.

Equally concerned, Mr. Bishop further writes: I read with interest your page 15. You may be interested to know that I was certainly dislike the printers' lower case letter I to represent one [and] I am not alone in this. This is a type-writing convention to save a character. Although characters on my machine were at a premium because I had to eat on it for dozenth ten and eleven, and all the accents in French and Spanish, nonetheless I insisted on having a 1.

S. FERGUSON, 53 Scothy Village, Scothby, Nr. Carlisle, Cumberland, England, writes: In your Dozenal Essays you have used 1 for the base sixteen point. I think this is a mistake. The use of the exponent mark for the factorial function makes this confusing. Factorial 6 = 6! and I read 10! to mean factorial 10, whatever base the 10 is.

STAN BUMPUS, 534 Townsend St., Urbana, Illinois 61801, writes: I strongly disagree on the use of (!) for the hexadecimal identification point. This symbol is now almost exclusively used to mean a factorial. Thus, 3! = 3 × 2 × 1. I think (!) as the h.d.p. would cause too much confusion.

[Editors' note: There was a time when ½ was the factorial sign to indicate the continued product up to 6. The use of (1) by so many mathematicians simply indicates that the science of mathematics is not standing still.]

THOMAS H. GOODWIN, 3218 Shelburne Road, Baltimore, Maryland 212 05, writes: The article [see September 1968 Bulletin] on hexadecimal system particularly interested me because I just finished IBM orientation to the relatively new 360 computer, ALC language, which heavily used hex. ...the more these binary-type computers take over, the less obvious will be the lack of the ten-system of counting. The big use I see for the duodecimal system in the elementary mathematical curriculum—it will enhance the idea of number systems and give better conceptions of quantities.

F. EMERSON ANDREWS, 34 Oak Street, Tenafly, New Jersey 07670, writes: Many thanks for Dozenal Essays of 1968 and all your efforts to do that most needful—get our publications going again. One suggestion, in the light of historic accuracy. The
use of \( \varepsilon \) for duodecimal value eleven derives from my original Atlantic Monthly article, October 1934, where I said (pp 462-3) "First, we must invent the two additional symbols which the Hindus and Arabs forgot. For 11, let us 'use \( \varepsilon \), and call it elf.' I had the printer use an italic \( \varepsilon \), and was already distinguishing duodecimal values by italicizing. On the typewriter we used simply capital \( \varepsilon \). It was never a reversed \( \varepsilon \). As a capital \( \varepsilon \) it stood for Eleven, or the German Elf. See also the first (1035) edition of NEW NUMBERS, Appendix B, Notes on Nomenclature. "The \( \varepsilon \), which is not an epsilon, but a 'fancy' form of the italic \( \varepsilon \) known to printers as 'swash \( \varepsilon \),' seemed to avoid confusion with the regular letter \( \varepsilon \) and at the same time be easily identified as a symbol for the old 11. For names, for the same reasons of simplicity, I adopted 'dec'---a one syllable word coined from Latin \textit{decem}---and 'elf'---German for eleven."

Equally interesting, Mr. Andrews further writes: When the Society adopted its official seal (I cannot be sure of precise date, but before 1945) that eminent typographer Wiggins designed the 'Flat' \( \varepsilon \) which has been put on some of our typewriters and generally used where possible. See also the William S. Crosby letter on this subject in the 1945 Duodecimal Bulletin, Vol. 1, No. 2, page 9.

RALPH H. BEARD, 20 Carlton Place, Staten Island, New York 10304 writes: I wish I had the courage to explore the 24- and the pentadozenal 60-base. Harry Robert said that he had done a lot of work in it. We really should have some one become fully familiar with its advantages and faults. I have enough on my plate. I may have to learn the Boolean Algebra of the Ternary Base.

And the farseeing Mr. Beard further writes: I am fully convinced that the ultimate computer base will be the 12-base, and that the computer language will be Esperanto.

GEORGE S. TERRY, Box 101, Sonora, Arizona 85379 writes: Also a tribute to Ralph Beard for his discovery in 1963 that 101229 + 1 has the factor 221225 (two figures). Since the power has 1229 figures in Duo and 12249 has about three thousand or more it was small discovery.

If we did not have these pages, this periodical would lose much of its usefulness. Speak your piece. Without your cooperation serious faults might creep into any man-created article and any machine-created work as well. Of course we look for constructive comments particularly and bits of interesting and informing dozenal culture, as well as individual preferences. We reserve our right to cut without damage to an idea. --The Editors.

The March 1969 bulletin is being put together. It will contain many scintillating subjects of interest to all dozeners. And begin to plan now to attend, in the Los Angeles area in the Spring of 1969, the annual membership meeting.

CHROMATIC MUSICAL SCALES (Cont'd from page three)

These curves reveal clearly that the average listener's ear is most sensitive to the shifts, expressed as fractions of the frequencies, of those above about 1000 cycles per second. However, the ability of the ear to detect an off-pitch note in the lower register of the piano is not necessarily as poor as the data might suggest, since the low piano tones are rich in harmonics.

Although the frequency response of man's ear stretches from about 20 to about 15000 cycles per second, a musical sound (or tone) ranges only from about 60 to about 4000 cycles per second. Below and above these frequencies a sound (or tone) loses its musical character. Within this musical range man can recognize more or less accurately the frequency region of a given sound without any pre-exposure to another one ('regional hearing'). Similarly, he can sense the distance (interval) between two consecutive sounds ('relative hearing').

Such a recognition, within a modest degree of accuracy, is universally necessary for understanding human speech, and man quite generally possesses this ability. However, for musical purposes, especially for musical performance, a greater than usual accuracy is required, and many persons need additional hearing training. While practicing this hearing, some people can memorize the pitch of one particular tone (or even more than one); and then with the aid of this reference they can identify quite accurately the pitch of any given sound by judging the interval between the reference pitch and the given one ('relative hearing with reference pitch').

Another rare form of musical hearing is the 'absolute pitch' (or 'perfect pitch'), where the listener can recognize immediately and very precisely the pitch of a given tone. Certainly, these last two forms of musical hearing are very helpful for musical performance as well as for musical analysis. However, they have no obvious bearing on musical appreciation, since this is a 'conditioned response.' (Ref.: G. Révész, "Einführung in die Musikpsychologie," Basel, 1946).

When a vibrating body produces a sound, especially a musical instrument, this sound is usually not a tone of a single frequency but is a complex sound consisting of several different frequencies, which are called "partial frequencies." Depending upon the distribution of the partial frequencies (when the partial frequencies are whole-number multiples of a fundamental frequency and thus follow the simple harmonic series), they can
be called "overtones" or "harmonics") and their relative intensities, the sound of a particular body or musical instrument assumes its characteristic quality, its timbre.

With the aid of a frequency analyzer such as the heterodyne type analyzer shown schematically by Fig. 9 (or similar instruments such as an arrangement of tuned mechanical reeds, etc.) or with the Fourier's series applied to the oscillographic record (Fig. 10).

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ord of a wave shape, a complex sound can be broken down into its partial frequencies.

While listening to a complex sound man may hear, in addition to the existing partial frequencies, some frequencies which are not present in the vibrating air. They could have been created within his ears due to some distortion or due to mixing of existing partial frequencies, called "combination frequencies" (which are the sum and difference frequencies).

The easiness with which different partial frequencies can be recognized, out of a complex sound, varies widely. For the harmonic overtones of the c-tone, Figure 10 indicates, in row e, the approximate order by which they are recognizable. Row e expresses the usually given notes of the fundamental and its harmonics as conventional letters; row b gives their names; row d lists their theoretical, exact frequency ratios; and row e names the successive intervals. (Subharmonics can usually not be heard and are therefore omitted.) (Ref.: G. Révész, "Einführung in die Musikpsychologie," Basel, 1946).

As already mentioned, the tones of most musical instruments are rich in partial frequencies, especially in harmonics, some of which may be even more prominent than the fundamental. Now, if the correct harmonics are presented, man will hear the pitch associated with the fundamental frequency, even though the fundamental may be missing. This phenomenon is probably a result of the difference in frequency between the various terms in a harmonic series which, of course, is identical with the fundamental of that series.

When two sounds of nearly the same frequency and the same intensity occur simultaneously, they alternate reinforce and cancel each other, as shown in row b of Figure 11. The sensation is of a tone associated with the slightly higher frequency and with periodic variations of its intensity known as beats.

If two simultaneously occurring sounds are fairly strong, and if their frequencies differ by 50 or more cycles per second, other sounds may be heard besides the primary ones. These are the combination sounds, or "combination frequencies," which were heretofore mentioned in describing "complex sounds." They are either the sum or the difference of frequencies between the one primary frequency or its harmonics, and the other primary frequency, or its harmonics. This relation can be expressed by the following two formulas, where \( f_1 \) and \( f_2 \) are the two primary frequencies, "a" and "b" are whole numbered factors, and \( f_{cs} \) and \( f_{cd} \) are the combination frequencies:

\[
\begin{align*}
    f_{cs} &= (a)f_1 + (b)f_2 \\
    f_{cd} &= (a)f_1 - (b)f_2
\end{align*}
\]

Combination frequencies between the primary frequencies (where the factors "a" and "b" equal one) are of the first order, and they are usually the strongest and the most easily noticed ones. They are best heard when they are well below or
well above the primary frequencies; otherwise, the masking effect of the primary frequencies makes it difficult to recognize these combination frequencies. For example, frequencies of 2000 and of 2500 cycles per second will give a frequency difference of 500 cycles per second, which can be readily recognized. However, frequencies of 1000 and of 2500 cycles per second will give a difference frequency of 1500 cycles per second, which can be noticed only by a careful observer.

The just mentioned phenomenon of masking a useful sound by an other sound is a familiar one. Studies of this phenomenon with simple tones show that the masking effect is greatest for tones whose frequencies are close to the disturbing tone; it falls off rapidly for more distant frequencies. Thus a loud tone of 1000 cycles per second will have a profound effect upon a tone of 900 cycles per second. A very similar effect results from background noise on a useful simple tone. However, although noise usually consists of a heterogeneous mixture of many frequencies, only a certain band width of frequencies adjacent to the desired simple tone frequency causes masking, and this band width varies with the frequency of the desired tone as shown by Figure 12.

Furthermore, in order to be audible, the intensity of the desired simple tone has to be higher by a certain amount than the effective pressure of the interfering noise band (spectrum level). Since this amount is related to the band width of the interfering background noise, that critical pressure level increase has also been indicated on Figure 12.

<table>
<thead>
<tr>
<th>frequency ratio</th>
<th>conventional term of tone combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 : 1</td>
<td>octave or 8th. or &quot;homesonance&quot;</td>
</tr>
<tr>
<td>3 : 2</td>
<td>quint or 5th.</td>
</tr>
<tr>
<td>4 : 3</td>
<td>quart or 4th.</td>
</tr>
<tr>
<td>5 : 4</td>
<td>large or major 3rd.</td>
</tr>
<tr>
<td>6 : 5</td>
<td>small or minor 3rd.</td>
</tr>
<tr>
<td>5 : 3</td>
<td>large or major 6th.</td>
</tr>
<tr>
<td>8 : 5</td>
<td>small or minor 6th.</td>
</tr>
<tr>
<td>9 : 5</td>
<td>small or minor 7th.</td>
</tr>
<tr>
<td>15 : 8</td>
<td>large or major 7th.</td>
</tr>
</tbody>
</table>
As a last item of Part One, we should mention the pleasantness of tone combinations, their consonance or harmony. Both musicians and physicists have long discussed what constitutes a pleasant combination of frequencies (called consonance) and what is an unpleasant combination (called dissonance). However, since there is a strong individual subjective element that often determines the final impression, and since this subjective element is influenced by the changing musical fashions, it is questionable if this problem will ever be solved in absolute terms.

Nevertheless, Figure 13 lists the major combinations of two simultaneous musical sounds expressed in conventional terms (although these terms may be misleading), it arranges them according to their degree of consonance, and it gives their frequency ratios as related to the loudest tone (called tonic). (Ref.: G. Révész, "Einführung in die Musikpsychologie," Basel, 1946).

Of course, "hearing and sound" (the topic of Part One of this discourse) could include many more items than mentioned thus far, like the transient features of a sound (its change in loudness or timbre) or the different theories of hearing, etc., but that would go beyond the scope of this writing.

(In the next issue will appear Part Two, Musical Scales, by Mr. Kotho, based upon this introduction. Lack of space makes this interlude necessary. Ed.)

COUNTING AND TALLYING

(Continued from page 32)

of the fingers (not thumb) with the hand palm up. This method is used today in the bazaars of the Middle East where theoretical knowledge of arithmetic may be limited today but where hard practical experience had led to the discovery of the practical usefulness of dozens.

The lesson these examples can teach us is not to feel bound to use our improving knowledge of the theory of arithmetic to bolster up the use of a structure for numbers based in the most primitive way on any particular anatomical feature. We should, on the contrary, examine the many practical uses of number in daily life, and then aim to learn the number skills best suited to making those practical tasks simpler or more efficient. We may find, as a result that many of the assumptions upon which current teaching of arithmetic is based are indeed themselves questionable and subject to improvement.

Look at the Bankers in line ahead of us—the best seats will be sold out!

107 1m00 256 080m 6 12m 11m 31