COUNTING IN DOZENS

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Our common number system is decimal—based on ten. The dozen system uses twelve as the base, which is written 10, and is called do. for dozen. The quantity one gross is written 100, and is called gro. 1000 is called mo, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 2 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 gro 6 do 5, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

| 94 | 136 | Five ft. nine in. | 5.9' |
| 31 | 684 | Three ft. two in. | 3.2' |
| 36 | 322 | Two ft. eight in. | 2.8' |
| 32 | 1000 | Eleven ft. seven in. | 11.7' |

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 28, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12² (or 144) times the third figure, plus 12³ (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by 12, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or 12².

**Numerical Progression**

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**THE DUODECIMAL SOCIETY OF AMERICA**

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.
THE DUCEDICAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year’s dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island, New York. George S. Terry, Chairman of the Board of Directors, F. Emerson Andrews, President, Ralph H. Beard, Editor. Copyrighted 1949 by the Duodecimal Society of America, Inc. Permission for reproduction is granted upon application. Separate subscriptions $2.00 a year, 50¢ a copy.

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inchoate progress in displacing the comfortable and familiar Roman numbers universally used. “Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn’t it? And why do we need a symbol for nothing? You can’t count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow.”

Yet, although it took D years, the new notation became generally used, and man’s thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

The Duodecimal Bulletin

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Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of number were reexamined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accommodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbersome Roman notation.

The parallel seems tenable. The notation of the dozen base accommodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades easily learn to perform computations in duodecimals, and can then tell why they are better. Literally, the decimal base is unsatisfactory because it has “not-enough-factors.”

Then, shouldn’t we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuable processes of their minds. Duodecimals should be man’s second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into
The Duodecimal Bulletin

THE DUODECIMAL PERPETUAL CALENDAR
by Louis Paul d’Autremont

The Duodecimal Bulletin

All figures in italics are duodecimal.

THE DUODECIMAL PERPETUAL CALENDAR

by Louis Paul d’Autremont

The index of time is the related motion of three bodies: the sun, the earth, and its moon. The travel of the earth in its orbit around the sun determines the year and the seasons. The travel of the moon about the earth is the basis of our concept of the month. The rotation of the earth on its axis clocks the day. Each of these motions varies slightly in its rate, and there is no exact natural period of the relative motions or positions of these bodies. They lack a common denominator.

Man needs a reliable frame of time reference to record and measure his works, and to relate them to natural phenomena. This time measure should be convenient, regular and accurate. The independent motions of the sun, moon, and earth make this a complex problem.

Early efforts in constructing a calendar were unsatisfactory because of inaccurate measure of the year. The present standard Gregorian calendar reflects a very accurate measurement of the sun cycle, but it is divided into twelve months of very uneven lengths.

The concept of the month is directly traceable to the moon, both in name and period. The Moslem calendar is based on twelve lunations, resulting in a year of 354 days. Other calendars intercalate extra days to adjust the length of twelve months to the year and season-cycle of the sun.

The length of the week has no astronomical significance, and weeks of 5, 7, 8, and 10 days have been used. The week of 7 days became general at the beginning of the Christian era.

There have been many proposals to correct the unevenness of the months of the Gregorian calendar, and a number of ideas for the improvement of the calendar have been developed. The League of Nations inaugurated official study of this problem in 1923.

One of the most popular of the suggested calendar improvements is the idea of the perpetual calendar. In this arrangement, the particular dates of the year fall upon the same week day every year. The Gregorian calendar could be made perpetual simply by not giving December 31st a week day name, and the same with February 29th. Thus starting Sunday, January 1st, 1950, if December 31st of that year

general popularity. But no change should be made. Perhaps by the year 2000, or maybe by 1200, which is 14 years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, - eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accomodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Dio-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.
were not given a week day name, then January 1st of 1951
and all succeeding years would fall on Sunday, providing
that February 29th were not given a weekday name in leap
years. Any particular date thereafter would fall upon
the same weekday year after year.

Perpetual calendars would not require annual replacement.
The economy involved is surprising. In addition to their
far greater convenience, and their facilitation of planning,
perpetual calendars could be made part of more perma-
ent structures, such as clocks, desks, wallets, vanity
cases, etc.

Popular support has largely concentrated on two of the
proposed perpetual calendars. The Fixed International
Calendar divides the year into 13 months of 28 days,
arranged in 4 weeks of 7 days each. While this calendar
has the fault of awkward division into half and quarter
year periods, or seasons, the dates of the year would be
easily memorized, as the same date of each month falls
upon the same day of the week. An amusing characteristic
is that the thirteenth day of every month would fall on
Friday, the 13th, in this thirteenth month calendar.

By far the most popular proposal is the perpetual
World Calendar. This advocates a 12-month year of
52 seven-day weeks. The quarters are of equal length and
identical form, containing 13 weeks, 91 days, with months
of 31, 30 and 30 days. Each quarter starts on Sunday
and ends on Saturday. Each month has the same number of week-
days, (26,) the 31 day months including 5 Sundays. Inter-
calary days are Worldday, (December 31st, or W December,)
and Leapeyday, (June 31st, or W June.)

This calendar has been energetically promoted interna-
tionally by the World Calendar Association and its affili-
ates, under the devoted leadership of Elisabeth Achelis.
It is receiving active and serious consideration by most
nations of the world and by the UNESCO. There is a con-
certed drive for general adoption of the World Calendar
by 31 December 1950. A Bill for its adoption by the United
States is now pending in Congress. (S. 1415), and it will
be brought before the General Assembly of the United Na-
tions in the session starting September 1949, on a motion
proposed by Cuba, and seconded by Mexico.

The manifest advantages of the World Calendar involve
only minor changes from our standard Gregorian Calendar,
and it is considered by many that general adoption will
be achieved as planned. Duodecimally, it is far prefer-
able to the 13-month proposal, and its support is advocated.

Yet further improvements in the calendar are possible
and desirable. These are of a type perhaps not now of
pressing concern, but they should be considered. For in-
stance the civil year starts on a date without astronomical
significance, which is out of step with the seasons,
and celebrates no acknowledged anniversary. It differs
widely from the start of the astronomical year, which is
the day of the vernal equinox.

There is a noticeable trend in industry toward a shorter
work week, and this might enter into our considerations.
Since weeks of 5, 6, 7, 8, and 10 days have already been
used, it is possible that a calendar based on a 6-day
week and designed for maximum factorability might find
popular acceptance, - especially in that time when duode-
cimal numbering has come into general use. The Duode-
cimal Perpetual Calendar, first promulgated in 1912,
attains certain of these desired improvements not con-
sidered in the World Calendar.
The Duodecimal Calendar proposes that the year start with the spring equinox, as the astronomical year does. The first day of the year is Sunday. The year is divided into twelve months of 30 days each. Each week is of 6 days, starting Sunday and ending Saturday, with Thursday omitted. There are 60 normal weeks in the year, 5 in each month, 15 in each quarter.

Between the last month of the year, and the beginning of the new year, there is a week of 5 days. This week begins on Sunday and ends on Friday. For leap years the week has 6 days, ending on a Saturday. This week is by itself, and is not attached to any month. It could be called “New Year Week,” and be a holiday week.

The Duodecimal Calendar is made perpetual by the omission of the Saturday of New Year’s Week in ordinary years. In leap years no omission is necessary.

The outstanding characteristic of this calendar is its complete regularity. Each month is like every other month, and so are the quarters. Because of the formal regularity, all dates are readily memorized. For this reason, too, intervals can be calculated with ease. All planning is greatly facilitated.

With March as the first month of the year, the months coincide with the signs of the zodiac. The correlation of dates with astronomical events is much simpler. Once again the months of September, October, November, and December are in their proper places, as the 7th, 8th, 9th, and 10th months of the year. The seasons fall into their proper groupings in entire accord with the months of the calendar.

The element of factorability has not been unduly stressed, as it is not well to attempt to predict the form that the work week will take. But, since all of the periods of the calendar are highly factorable quantities, much of the awkwardness of the current calendars is eliminated.

Greater detail on the Duodecimal Calendar, as well as on other proposed calendars will be found in three brochures published by the author: The Duodecimal Perpetual Calendar, 1926, The Calendar of the Future, 1931, and tables of the Calendars, 4th edition 1944.
DATES
by Ralph H. Beard

Events that happen in regular order are normally well suited for representation by numerical series. Yet this is not true about dates. Our peculiar habits in designating the days of the year, and dates in general, introduce awkward complications in fitting normal numbering procedures to this application.

In planning a duodecimal dating practice we have certain suggestions to offer, which, if they be found acceptable, will inaugurate an adequate and satisfactory arrangement.

American practice differs from that of most of the rest of the world in the way that dates are stated. We state a date as September 2, 1947. In other countries this is generally written 21 September 1947. We find the latter form preferable. We have used it for some time, mainly for its economy of punctuation.

But when we state a date entirely in numbers, as 21-9-1947, we are exactly inverting what is our standard numerical practice. The order in which we write the figures of our numbers is in descending magnitudes from left to right. To state a date in full accordance with this principle, we would write 1947 921. And the following day would be 1947 922. Corresponding duodecimal statements would be 1163 919 and 1163 912. In decimal statements of this form, some months will require the use of two places for the designation of the month. But it must be noted that two places must always be used to state the day of the month. For the early days of the month, a zero must occupy the second column; the 6th of September would be written 1163 906.

However, we depart from our standard numbering practice in the way in which we number the months. January is the first month and we number it “one”, December is the twelfth month and we number it “twelve”, or “do”. There is no “zero” month, nor “zero” day. This is probably a relic of the days when Roman numerals were used. This practice is ill suited to modern number systems, but it would be extremely difficult to change. For duodecimal use, the alternative is to adopt a single symbol for representation of the “do”, and use it to represent the last month of the year. Suppose we use “D”. Then on the first day of the next month, (January, or “1”), we must remember to carry one over into the next column to advance the year. As an illustration of this, the day after 1164 D27 would be

1165 101. This is about as near to standard numeration as is possible under the circumstances, and the practice is recommended.

In line with this dating practice, it would be possible to express simply a definite instant in time. For instance the minette after noon of the day mentioned last above would be stated as 1165 101.601, and the time fractional could be extended to designate as fine a division of time as might be desired.

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THE APPEAL OF DUODECIMALS
by Frederick Condit

In these days of high pressure campaigns for social reforms, debates about labor conditions and peace problems, struggles to accommodate ourselves to the changing values of the dollar, and so on, - even temporary escape from all controversial questions and drafts on one’s purse would seem to be a blessed relief. To be able to bury oneself in a subject which does not require immediate solution, - one which requires only as much concentration as one wishes to give, yet yields rich rewards in opening unlimited fields for exploration within the scope of everyone from elementary to highly technical levels, - is an inviting prospect. If it appeals to you, then go to the technical section of the nearest public library and look up the literature on the Duodecimal System.

Here is a subject that is as close to the interest of everyone as a man’s vest, and yet very few even know of its existence. If you ask a group of one hundred people if they have ever heard of the duodecimal system, perhaps a dozen will confuse it with the Dewey Decimal System, which is used for classifying books in our libraries. One of the entire hundred may know of it, having heard that efforts to put it into use were made in Sweden during the reign of King Charles XII.

There is a group with headquarters in New York, the Duodecimal Society of America, which is interested in spreading information as to the advantages in using twelve instead of ten as the basis of our number system, and in collating the records of the work that already has been
and is being done on the subject. In its publication, The Duodecimal Bulletin, will be found material covering the historical background of the origin and development of our present number system, and the parallel development of the use of twelve as the basis of grouping, measuring, etc. These alternate methods cause us double work in our everyday computations in converting from one scale to the other. We buy our wares in the twelve system, and pay for our purchase in the ten system. There is no necessity for this complex duplication.

In addition to interested average people, the membership of the Society includes mathematicians, teachers, and engineers, who have had, and are having, a field day working out the mathematical tables of duodecimals to facilitate their work. One of the enthusiastic members has had a computing machine modified to perform operations in duodecimals. The design of a duodecimal slide rule is being elaborated by an active committee. A new duodecimal musical notation has been proposed, which would eliminate all accidentals, assigning instead a single definite pitch to each line and space of the staff. There being twelve tones and half-tones in our "octave," and the musical staff having five lines, four internal spaces, two external spaces, and an interstave line, all of this falls naturally into the duodecimal notation.

The period of transition from the general use of the decimal base to the duodecimal will present many problems, especially to the teachers. They are the ones who are now faced with the problem of making the study of arithmetic interesting. Their successors will reap the reward of their contributions to the change. They have always realized that there must be some serious lack in the subject to cause the aversion which most children show toward it. Many methods of arousing interest have been tried, such as card tricks, emphasizing the so-called magic of the numbers nine and eleven, or cutting a small piece of paper so that a piano can be passed through it. All of these tricks help for a time, but when the novelty wears off, the drudgery is still present.

The writer never taught school and cannot speak with authority on problems of education. In fact the whole subject of number systems is no more than academic to him, and his interest in figures is mainly limited to efforts to keep those of the red variety out of his business. However, pending the arrival of the numerous contributions which may be expected from the educators after they become aware of this most interesting field, it will not be out of order to assemble a few obvious facts bearing on why arithmetic is non grata to children, and what, if anything, can be done about it.

To help out in this discussion let us call on the musician. He is not ordinarily considered a person of science. Music as an art is great, but as a science it is a poor step-child. The physicist looks down on the science of music because its scale does not follow a regular mathematical pattern. Granted, but it should be realized that our ancestors acquired their ideas of pitch and of the musical scale long before Sir Isaac Newton watched the apple drop and made his contribution to the science of physics. Music may be a step-child, but its basis is enough of a science to make apparent the inherent fault of the ten-system, and that is that it has no rhythm, or if you prefer, that it has an unnatural rhythm.

Rhythm is everywhere in nature, - in the seasons, in astronomy, chemistry, physics, and (except for the ten system) in mathematics. It finds a response in humans, even those who have no ear for melody or harmony. Music is said to be the universal language, but the element which contributes most to its universal appeal, is rhythm. With one recallable exception, no composer has used a ten-base rhythm. Tchaikowsky set the second movement of his sixth, Pathetic, symphony in five-four time. In spite of the fact that this movement has two excellent themes, no one ever asks to have it repeated. The rhythm is so disturbing that even the orchestra conductors have difficulty with it. When popular orchestras play it, the five-four time is changed to one of the natural rhythms.

The rhythm of the ten-system can hardly be called disturbing when music is not involved. Instead, it is of negative value, as that it cannot be set to music without providing for hiatuses, such as occur in hymns. For this reason, learning the numbers of the ten system is like memorizing prose, and that, for a child, is real work. No one who has witnessed the agony a child goes through in reciting a speech in prose, after succeeding magnificently with a poem, will need to be convinced of this. It lacks the points of support which are available through the use of rhythm, as expressed in music, marching, dancing, etc., which perform a real function in the association of ideas.

Let us hear from the department of psychology for a time. Much is being heard of the importance of educating
the "whole child." The more "pegs" a child has on which information can be hung, the more easily he can absorb it and the longer he can retain it. It is found that the coordination of physical activities including singing, marching, and dancing, with verbal and visual instruction, furnishes such pegs.

The musician has pointed out at least one serious fault of the ten-system, and the psychologist has given us a hint as to what use can be made of replacing the negative quality of the ten-system by the positive quality of the twelve-system.

The twelve-system is rhythm in essence. It contains the masculine form, the martial four-four march time enjoyed by boys, and the feminine three-four waltz time preferred by girls. One of the favorite meters of composers of band music is twelve-eight time, or four-four time with each quarter beat containing three eighth notes. Nearly every child will have marched to this rhythm before reaching first grade.

Ditties can be arranged which will contain the numbers of the twelve system. They can be sung and marched to. A group of three stanzas, each containing four lines, with twelve notes to a line, will provide a note for each number of the GHO, now expressed as 144. A child starting a study which uses a rhythm familiar to him together with physical activities which he enjoys, would feel as much at home as in his mother's pantry. The job would be undertaken with interest and enthusiasm. All the timidity now felt by young entrants into the strange world of arithmetic would be absent. Instead, it will have been turned into a game.

When he is ready to start graphs, the staves of his music, with notes corresponding to his numbers will furnish him a ready made graph, and this graph will be useful not only for learning, but will be a tool which he can carry through life, using it in every computation. Each child, knowing that all other children use the same that he does, will talk about it freely, as about other things shared in common.

Many other advantages, not mentioned here, are to be had by changing to the twelve system. The problem of getting the idea accepted and started should be a prime interest to the educators of the young. They are hereby invited to turn their imaginations loose in this most interesting field of research.

ON THE GRADUATION OF SCALES

by William Shaw Crosby

The slide rule is one of the greatest arithmetical short cuts ever invented. Since the most obvious purpose and effect of duodecimal arithmetic is to simplify the paperwork of figuring, with which the slide-rule expert has learnt to dispense altogether, he might object that for him a change to dozens would be superfluous. He would be wrong. The slide rule is one application of the graduated scale, and uncial graduations can be more nicely adapted to any interval, and more logically patterned, than the divisions now in use, yet easy to read.

The fewness of the divisors of ten is as much a handicap in graduating scales as in any other field of applied decimal arithmetic. Often an interval to be subdivided is too small to hold ten graduations. Then the division could be into fifths. But if the interval is not quite big enough to contain fifths, it cannot be split into useful, easily recognized quarters, nor into thirds--these fractions are too hard to deal with in decimal arithmetic--instead, the subdivision can only be into coarse halves. Note the awkward change at point 2 of the decimal slide-rule scale, Figure 1.

Duodecimal arithmetic is more adaptable. Where twelfths of a scale interval would be too crowded it permits us to graduate in sixths, quarters, thirds or halves, whichever best fits the case.

But when there are as many as five different ways of dividing intervals it is important to prevent the mistakes that could result from confusing one sort of

1. Must I apologize to members of a Duodecimal Society for using this word? By "twelve" I mean the number that you get by doubling six or by adding five to seven. To most of us the good old word means primarily that--the number rather than the verbal rendering of the symbol 12. "Do?" "Go?" "Me?" NO! 2. Cam. in "Bul." Vol. 4, No. 2, p. 10, terms graduation by sixths or thirds "confusing" and proposes to use only twelfths, quarters and halves as recognized divisions on the slide rule. I agree that in particular cases he may be right, but a general refusal to use thirds and sixths seems to me a needless sacrifice of one of the advantages of duodecimal arithmetic. Although tradition has limited our practice in the use of scales marked with other than binary or decimal graduations, we should yet be able to stretch our imagination to make us the equals of someone whose experience will not have been so limited, and who can take thirds in his stride. Actually there is something to be said in favor of preferring a division into sixths or thirds to a division into quarters. When we must visually estimate between the lines, let the instrument maker perform the comparatively tricky trisections and leave the easy halvings for our eyes!
division with another. A standard and easily legible
pattern should distinguish each mode of graduation—so
that, for example, a division into quarters will look un-
mistakably different from a division into sixths, and the
eye will need no second glance to assure itself of the
value of each graduation.

Logic and system can easily be carried too far in mak-
ing such patterns. The most notable example of this un-
derirable extreme is furnished by the ordinary desk ruler
marked off into inches and binary fractions of an inch.
Figure 2 shows part of such a ruler graduated to 1/32''.

Repeat divisions into halves is the simplest method of
graduation and makes possible a very logical ranking of
the different fractions according to simplicity. That is,
the line marking the half inch is longer than the lines
at the quarters, the quarters are accentuated in compar-
is on with the eighths, and so forth. But note that when
this process has been carried as far as to 16ths or 32nds
the graduations become confused. In the forest of lines
within each inch one is at a loss to pick out some frac-
tion like 27/32'' without first doing a little quick mental
arithmetic, such as 24/32'' = 3/4'', and then actually count-
ing divisions from a reference point like 3/4'' that can
be recognized at a glance. Binary fractions smaller than
eighths are troublesome to use not merely because our
arithmetic is not based on 2, but especially because their
pattern of graduation is so repetitious that it is easy
to get lost in it.

Except for the inch, most of our scale intervals are
divided to accord with the decimal system, which is to
say that they are graduated either in halves, fifths,
or tenths.

Halves are unmistakable and need no distinguishing pat-
tern. Fifths are more difficult. Five marks are about
as many as can be counted at a glance—too many in some
cases. A pattern would help us recognize that the inter-
val is divided into five and not some other number of
parts, and instantly and positively identify each mark,
but five is a prime number and no logical pattern sug-
gests itself. So fifths are usually marked by gradu-
ations of even length.

Occasionally an "illogical" pattern is used. Figure 3
shows a type of scale introduced by the Ford Motor Company,
on which inches are divided into numbered tenths, each
tenth is subdivided into fifths, and the second and third
fifth of each tenth are accentuated. The pattern is arbitrary, of course -- .04" and .06" are not highly important scale divisions in the sense that 1/4" or 1/8" were in figure 2 -- but it is distinctive. The user is less likely to confuse 1/5 of a tenth with 2/5, or 2/5 with 3/5, than he would be if all these small divisions were of the same length, and he could hardly fall into the absent-minded mistake of supposing that any of the marks within each tenth represented a half or a quarter of the interval. The eye is quickly set free for the task of estimating hundredths.

Tenths are almost always graduated as fifths of halves, as in the scale of centimeters, Figure 4. This doesn't form much of a pattern. The five-tenths' mark is stressed but the wide spans to either side are just as patternless as the ordinary way of showing fifths, and indeed can too easily be read as fifths of a basic unit half as large.

One never sees the division into tenths made in the other possible order: first into five parts, then each fifth into halves. It would be just as sound arithmetically, but we are not accustomed to it; it is not standard practice, and a first encounter with it would be confusing. Duodecimal graduations can also be approached in more than one way. What shall we establish as standard practice? If an interval has to be marked into sixths, for instance, shall we accustom ourselves to stressing the thirds, or the half?

My suggestion is "stress both." Stress both equally. The result (Figure 5, a scale of Do-Metric "quans"), with its clumps of three long marks in the middle, is at first sight funny-looking. But it is not more so than the Ford scale of Figure 3, which as we have seen has a cluster in the middle for two very good reasons: to diminish the chance of mistaking one mark for its neighbor and to make a distinctive pattern that will be peculiar to a division into fifths. These reasons apply with equal force to this pattern of sixths. Furthermore, the elongated divisions in the pattern of sixths do stand for something important in their own right: the useful fractions 1/3, 1/2, and 2/3.

Figure 6, a scale of "centiells," shows a distinctive pattern for a graduation into twelfths. Its two clusters of two accentuated marks, paired on either side of the long accentuated center-line, stamp it instantly as something different from a pattern of sixths, and each of the stressed divisions represents a frequently needed fraction: 1/4, 1/3, 1/2, 2/3, or 3/4.

When twelfths or sixths would be packed too close for easy reading, an interval will have to be divided into quarters, thirds, or even into halves. Danger of confusing these should be slight. Quarters can be distinguished by stressing the middle division, as is done at present, and it would hardly be possible to mistake thirds for halves.

It should be possible to compound the pattern of twelfths with any of the other patterns and still have a scale that can easily be read. Figure 7 is a scale of "egrogages" graduated to .06 egrogage; Figure 8, "tums" to .04 tum; and Figure 9, inches to .03 inch. With a little practice any of them can be read much more quickly than the scale of Figure 2 with a lot of practice.

Finally, all the uncial patterns I have discussed are exhibited in a single logarithmic scale, Figure 7. Although this is intended more as a tour de force than as an example of a practical slide rule scale, I believe that compared with the decimal scale of Figure 1 it fulfills the promise of my first paragraph: its graduations are "more nicely adapted to any interval, and more logically patterned, yet easy to read."

3. A "centiell" is a unit of a system proposed by the writer in Dd. Bul. Vol.3 No.2 pp. 4, 20. It is defined as exactly 8 4/16 millimeters, and it is approximately double the distance a freely falling body drops from rest in the first 0.00 001 day of its fall. In this system the writer has used the decimal metric prefixes with duodecimal meanings, so that a centiell is 1/100 of an "all" of approximately 46 35/64 inches.

This system is not to be confused with a proposal by Prof. Carl Forsell in the Swedish Magazine Norden (cited in Newsweek Nov. 5, 1945 p. 291), which involves an "all" of 50 inches decimally divided into "centiells," "milliells," etc., a scheme which seems to combine the advantages of the English and metric systems.

4. The "egrogage" is a centimeter in deep disguise. See Dd.Bul.Vol.1 No.2 p.53, in which Handy advocates a measure system based on the standard railroad gage slightly readjusted to make 1/100"gage" equal precisely 1 centimeter.

5. The "tum" is a unit of the "tweciell" measures proposed by Norland (see Dd.Bul.Vol.3 No.3 p.3), and would be the distance traversed in free space by a ray of light in 0.000 000 000 000 000 000 000 000 of a day-well, in practically no time at all. Lien independently proposed (in Dd.Bul.Vol.1 No.2 pp.21-22) a unit based on the speed of radiation: the "mark", equal to Norland's "link" of 10 tums, although inaccurately computed. It should not be forgotten that in the lore of dozerny the name "mark" stands also for Fitman's 10- shilling unit of currency.
MATHEMATICAl RECREATIONS

D. M. Brown, Editor

One of the aims of the editor of this department is to provide problems which will give practice in manipulation of numbers expressed in various bases. As one becomes familiar with the mechanics of such manipulations, he begins to see what a number system is, an artificial device. Although such a device is necessary, practice in the use of a variety of bases enables one to more fully appreciate the advantages of particular bases, especially the dozenal base. So ideas any of you readers have for new kinds of problems will be welcome.

In the last (Dec. 1948) issue of the Bulletin, we posed problems involving number \((xyzw)_B\) in which \(x, y, z,\) and \(w,\) are digits, and \(B\) is the base, so that:

\[(xyzw)_B = xB^3 + yB^2 + zB + w.\]

The solutions to the problems are as follows:

1. \((xy)_{10} + (xy)_{12} = (xy)_{32}\)  \(\text{Ans. } x = 0, y = 0.\) (A trivial case)

Ye Editor apologizes; he still thinks decimally, and the problem should have read:

\[(xy)_{10} + (xy)_{12} = (xy)_{32}\]  \(\text{Ans. } x = 1, y = 8.\)

2. \((xyz)_4 + (xyz)_2 = (xyz)_{10}\)  \(\text{Ans. } x = 1, y = 2, z = 1.\)

Harry Robert submitted several similar problems, some with more than one solution. He says that:

1. \((abc)_4 + (abc)_5 = (abc)_7\) has one solution, and that

2. \((abc)_5 + (abc)_6 = (abc)_8\) has 9 solutions for each value of \(n.\) Can you find them?

George S. Terry suggests the type

\[
[(xy)_a]^2 + [(xy)_b]^2 = [(xy)_c]^2
\]

Try the following:


5. \([(xy)_3]^2 + [(xy)_{15}]^2 + [(xy)_{20}]^2 = [(xy)_{25}]^2

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NUMBERS AND THE OPEN MIND

by Paul Van Buskirk

The word "Arabian" calls up thoughts of Ali Baba and the Forty Thieves and that delightful world in which Aladdin could obtain his slightest wish by merely rubbing his lamp and repeating the magic words "Open-Sesame".

But, these same Arabians carried another lamp, the lamp of learning, for centuries after it fell from Greek hands. They not only preserved the old light, but prevailed on the lamp to shed new light, not by the magic words "Open-Sesame", but by the use of the method of "Open Mentality". The studious Arabs applied their mind to a minor branch of Greek mathematics and produced that major branch which we know as Algebra.

What is meant by the method of open mentality? If I hold my watch up and ask you the function of the long hand you will reply that it measures the sixty minutes of
the hour. But, many men of open mentality have observed
that it also divides the hour into twelve fractions and
that each of these can be divided into twelve parts. If
the watch be then regarded as the right hand driving unit
in a series of recording dials, as in the familiar gas
meter, we have a device that registers in terms of dozens
and dozens of dozens. In order to set down in figures
the reading from such a meter, in which 10 must be used
to mean twelve, we must open our minds to the idea that
"ten" can be a "digit", if it has a separate symbol. That
is not hard to do since we often see "X" used for ten in
Roman numerals. It is a little harder to accept the idea
that eleven can be a "digit" since we unconsciously as-
associate the word "digit" with the fingers.

You need not feel chagrined that you have never before
known that there could be a system of numbers other than
the decimal. The Romans, for centuries, used the uncial
or twelve-part system for fractional numbers, even using
a single name for ten-twelfths and another for eleven-
twelfths, without ever arriving at a true twelve system
of numbers, as far as their written records show. If
Caesar did not see it, why should you?

The first man to leave a written record of the twelve
system was the 17th century French scholar, Blaise Pascal.
Other French scholars explored the system which his open
mentality had revealed, and urged its adoption by the new
French Government when the old monarchy was overthrown.
Had their counsel prevailed, the whole world would now
be counting by dozens, measuring in feet, inches and
twelfths of inches, and using volume and weight units
related to each other in terms of twelve as the decimal
system units are related in terms of ten.

We cannot have the thrill of discovery that rewarded
Pascal for his work, but we can have the thrill of ex-
ploring this system and working out its application to
our own modern problems of figuring. Pioneering in
figures is fun. You can have the stimulus of company
by joining the Duodecimal Society of America, from whom
you can get publications bringing you up to date on what
has been done, how to go about the number work, and what
particular fields other members are exploring.

*Talk delivered at the Engineering Society of Detroit
Speakers Club, January 25, 1949, by Paul VanBuskirk*

THE ANNUAL MEETING

The fifth Annual Meeting of the society, held January
27th, 1949, at the Gramercy Park Hotel, in New York,
proved to be even more interesting and enjoyable than its
predecessors. Those interested in music and the dance,
who had come to hear Mr. Godjevatz talk, lent color and
gayety to the occasion.

The formal business of the meeting was briskly handled.
President Andrews welcomed the members and friends of
the society, and reviewed the outstanding features of the year's
work: the work of Mr. Seelbach in extending the duodecimal
bibliography, which was marked by the discovery of Simon
Stevin's advocacy of duodecimals in his "L'Arithmetique"
of 1585; the proposal of a new musical notation on the
duodecimal base by Velizar Godjevatz, of which we would
hear more in the course of the evening; and the publica-
tion of the index of the first four volumes of the Bul-
etin in the issue just coming off the press.

The financial summary for 1948 showed an initial balance
of $625.37, normal revenues of $301.75, and donations of
$892.00, for a total of $1,819.12. Expenses were $1,248.31,
leaving a balance of $570.81 to start the new year. Mem-
bership figures continue their gradual increase with a
gain of 15 for the year, for a total of 55. A better
index than the number of members for measuring the growing
interest in duodecimals, is that our present printing of
1,000 copies of each issue of the Duodecimal Bulletin has
become insufficient.

The Nominating Committee suggested the re-election of
the two directors whose terms were expiring, and the addi-
tion of three new directors to the Board, - H. K. Humphrey,
Nathan Lazar, and Kingsland Camp, for the classes of 1950,
1951, and 1952 respectively. For the new Nominating Com-
mittee, the names of Paul Van Buskirk, Chairman, Paul
Adams, and H. F. Stevens were proposed. These nominees
were unanimously elected, and President Andrews announced
the committee appointments for 1949, as shown in the
attached list.

The Secretary informed the meeting that our active group
of members in Baltimore had formed a local chapter, and
elected officers. He submitted the formal application of
that group for recognition as the Baltimore Chapter of
the Society, with a copy of their proposed by-laws. En-
thusiastically, the application was approved and the
Baltimore Chapter recognized. The by-laws were referred to Vice-President Friedemann for review and approval.

The establishment of the first branch chapter is an event of outstanding importance in the history of the Society. It is hoped that the initiative of our Baltimore members will stimulate member groups in other large cities to similar action.

The necessary business of the meeting having been covered, President Andrews introduced Velizar Godjevatz, who gave an interesting exposition of the advantages of his new duodecimal musical notation. The simplicity and lucid rationality of his method is attractive. It opens up new ground for musical development, in that it easily accommodates the notation of the finer musical intervals, those less than half-tones.

Equally important is the fact that the “Notation Godjevatz” is completely reasonable. The present notation is another of those conventional disciplines which hamper the free ideation of the mind with the constant necessity to translate a thought into an irregular and complex transcription. In performance, playing such a transcription involves the reverse of the same complex process. The new notation is simpler and more natural, which will make sight reading easier to learn, and more enjoyable in escape from restraint.

A talk by Kingsland Camp on The Duodecimal Slide Rule followed. Mr. Camp covered the current stage of the plans for the design of the rule, and explored the preference of those in the audience who were familiar with slide rules, as to folding the scale at π, or at the square root of the base. A show of hands indicated a surprising evenly-divided opinion. More ample statement of these details will be found elsewhere in this Bulletin.

Next was a talk by Frederick Condit, who approached the advantages of duodecimals from an entirely novel angle. Using a special form of motor-driven metronome, Mr. Condit proved, beyond a doubt in mind of anyone present, that one could count much more rapidly in duodecimals than in decimals. The reason for this surprising facility was demonstrated to lie in the natural introduction of a convenient rhythm into the duodecimal counting process. Decimal counting does not lend itself to a similar subdivision into groups because of the awkwardness of the five-factor.

Mr. Condit progressed from this basis to the exposition of the advantage to the school child, in having the rhythm-sense include in its development the numbers that he uses increasingly every day. The child mind accepts easily those things that fit naturally into familiar patterns, and when he finds a rhythm in numbers to accord with his marching, singing, dancing and pulse, he will find his numbers pleasant to learn, easy to use, and soon familiar. We can thus avoid the approach that demands of our children another thing that has to be learned by rote, a natural thing that has to be accepted in awkward form as being right because we say it is right.

Refreshments and general discussion followed Mr. Condit’s talk. Among the comments noted was that scripts for dancing used a three-dimensional duodecimal notation for the delineation of motion, of posture, and gesture. The discussions were at least as refreshing as the excellent refreshments. There was so much of interest in the independent thinking expressed, so much of talent and charm in the personalities present, that all were reluctant to have the meeting end.

Officers and Committees for 1949

Chairman of the Board: George S. Terry, 507 Main Street, Hingham, Mass.
President: F. Emerson Andrews, 34 Oak St., Tenafly, N.J.
Vice-President: Paul E. Friedemann, 904 Mifflin Avenue, Pittsburgh 21, Pa.
Secretary: Ralph H. Beard, 20 Carlton Place, Staten Island 4, N.Y.
Treasurer: H. K. Humphrey, 520 Ash St., Winnetka, Ill.

Members of the Board:
Class of 1950: Ralph H. Beard, H. K. Humphrey, George S. Terry
Class of 1951: F. Emerson Andrews, William S. Crosby, Nathan Lazar
Class of 1952: Kingsland Camp, Paul E. Friedemann, Harry C. Robert, Jr.

Committee on Awards: George S. Terry, Chairman, Ralph H. Beard, Harry C. Robert, Jr.
To decide whether a 1950 Award is to be made, and if so, to recommend for that award a person of outstanding achievement in mathematical research, as related to duodecimals.
THE MAIL BAG

Louis Paul D'Autremont, author of the Duodecimal Perpetual Calendar, is named for an earlier Louis Paul d'Autremont, who came to America in 1792. They lived for a while at Asylum, Pa., a settlement originally established for French royalist refugees, as interestingly recorded in Mildred Jordan's recent novel, "Asylum for the Queen." His father, Hubert, was guillotined in the French revolution. It is from the brother of this Louis, Alexander Hubert d'Autremont, that our Louis Paul d'Autremont is descended.

He has been keenly interested in duodecimals and in calendar reform for many years, as is attested by the date (1912) of his proposal for the Duodecimal Perpetual Calendar. He is one of the older members of the World Calendar Association, and ably supports their proposed revision, The World Calendar, as the best of the current proposals. The adoption of the Duodecimal Perpetual Calendar could not well be considered under present conditions.

Stewart J. Ogilvy, Editor of World Government News, writes: "If UN is going to set a standard 'A', I don't see why they don't adopt the duodecimal notation suggested in the Bulletin."

They certainly should. The Notation Godjevatz marks a distinct advance in technique, and provides advantages not otherwise available. It will be interesting to note the reception accorded this new musical notation as the composers and musicians become familiar with it.

There is another element involved, an approach from quite a different angle. The Do-Metric System bases its measures of time on the duodecimal subdivision of the day. The .000 002 day is called the Vic, and there are, in decimal terms, 34.56 Vics per second. Part of the argument about standard pitch is involved with International Pitch with A4 given a frequency of 435 vibrations per second, and American Standard Pitch with the same A4 given a frequency of 440 per second. If A4 were given a frequency of 439 vibrations per second, then the frequency for C', is one vibration per Vic.

Paul Van Buskirk comments: "I wish to thank Dallas H. Lien for his article on the circle which appeared in the December Bulletin. I struggled with that problem during the depression, but never solved it. Circular measure is very poorly handled in present engineering practice. The gradual accumulation of evidence as to the superiority of
Base-Twelve on more and more fronts, will ultimately become incontrovertible and conclusive.

"May I suggest that 'double pi' be named 'dopit', so that we can have one of the seven dwarfs on our side. It could be symbolized by simply doubling the horizontal stroke in the present symbol for pi: π."

This refers to Dallas Lien's suggestion of "A Better Ratio for Pi" in the December Bulletin. The use of a double π in radian measure is excellent, and brings that form of circular measure into exact conformance with the duodecimal measure of time and angle. Mr. Van Buskirk's suggested "dopit" involves a confusing application of the prefix "do", which should refer exclusively to the dozen. The use of the prefix "du", as in "duor", might avoid this fault, the name of the double π becoming "dopit".

THE DUODECIMAL SLIDE RULE

Comments and Suggestions

Eugene M. Scifres.

I have read both articles on the Slide Rule in the Bulletin with great interest. On many points I agree perfectly, but on some details I wish to suggest changes from the tentative plan. The idea of having a reciprocal scale along each log-log scale is especially good. In fact, I wonder why that hasn't been applied to slide rules before now. On numerous occasions I have had to take the negative power of some number, and, with an arrangement like this it would have been reduced to one operation.

It is well, as was said, to put the traditional arrangements of a slide rule on the defensive. Even if this is done though, many of the arrangements will have a pretty good defense. They are there because through years of change it has been found that the greatest number of people in many different lines of work (many of which have nothing in common) can use the slide rule best the way it is.

Of the several rules I have, the best is a K&E #4081-3 Log-log Duplex Decitrig. The general arrangement of this rule is very satisfactory and, while some changes are necessary, I believe we should follow its general pattern as most universally adapted to many different types of problems.

On several points I agree with Mr. Humphrey. I avoid the use of the cross-hair whenever possible, and I will not have a magnifier on the indicator. It upsets my judgment for interpolation. But I do not agree that folding the scale at π is just a talking point. Quite frequently in my work I start a problem on the CF or DF scale, divide by π by dropping down to the bottom scale, go through any necessary proportions, and refer to the log-log scale to get the natural logarithm of the result.

As to details, the total length of the log-log scales should be 4 feet instead of 2 feet as shown in the October Bulletin, and should be referred to the D scale instead of the A scale. The increased length will assist in avoiding visual error. You will notice that the log-log scale in the range from $e^{0.001}$ to $e^{0.01}$ uses exponents small enough to say that $e^x$ is approximately equal to $1 + x$. So for values of $e^x$ less than 1.01, we can use the D scale, insert the correct number of zeros and add one. For values less than 1.001 there will be no detectable difference. Example:

\[
e^{0.00345} = \text{approximately } 1.00345
\]

\[
e^{0.000000878} = 1.000000878
\]

For this reason we can use just two feet of scale to the "left" of $e$ and, by eliminating one foot on the lower end, we can add one foot on the upper end, which will come in handy in finding powers or roots of large numbers.

The proposal that all of the trig scales be "full length" meets with my whole hearted approval. When the sine and tangent scales are both referred to the same scale, it becomes quite easy to solve any problem involving vectors at right angles to each other. The idea of graduating the length of the division lines in proportion to their spacing is indeed good, but may prove to involve expense out of proportion to the value gained. We should endeavor to hold the cost of the rule within reason.

Chairman Kingsland Camp

Discussion at the Annual Meeting, and experiments with a crude outline model rule, have brought this subject to a somewhat more definite stage. The following matters seem not entirely clear as yet to some practiced slide rule users among us; their views, of course, we most particularly desire.

I. Pairing of Scales with Reciprocals

This proposed innovation especially calls for general understanding and wider comment. It entails what seems to some of us a disadvantage, that no scales will then appear along edges where slide and frame bear against each
other. On account of parallax - the small distance between the crosshair of the indicator and the surface of the rule - such numbers as are exactly indicated on the scales can be more accurately set against each other without using the crosshair at all. For most of us, probably, only a small minority of calculations involve such numbers; we far more often work with longer numbers that require interpolation, and therefore the crosshair, when setting the scales against each other.

Suppose that we pair scales with their reciprocals, the latter to be lettered in red as is the usage with the present form of separate inverted scales. This arrangement will have the following advantages:

1. Space is saved, with a gain in clarity. Inverted scales need not be separately designated as their functions will be obvious from their red lettering and the scales to which they are attached. The nature of the so-called CIF scale, especially, will be less likely to be mistaken than on current models.

2. It will promote among all users the habit (now, probably, confined mostly to those called "wizards") of always conceiving the separation of two marks on any scale as representing the quotient of the indicated quantities; and the separation between a mark on a scale and one on its inverted scale as representing the product of the numbers (or the reciprocal of such product, in readily recognizable and easily handled cases).

3. Square or cube scales, when paired with their reciprocals, save further space by dispensing with the need for similar scales on the opposite member of the instrument. For, just as with the full-length scales, quotients and products are represented by the separation of the appropriate scale marks; this separation, transferred to a full-length scale, represents the square root or cube root of such quotient or product. After such transfer, any further steps of the calculation may usually be performed and the answer found on a full-length scale.

Multicycle scales, by the way, have another feature worth mentioning: any unity point may be taken as the cardinal one in an operation of this nature, thus in effect treating them as folded scales.

II. Arrangement of Scales

This follows my original essay "A Duodecimal Slide Rule" (Vol. 4, No. 2, page 7 of our Bulletin), except that a cube scale with reciprocal is now added on the slide, and the inverted decimal square scale (dec'1 BI) is omitted from the slide as it would seldom be useful. It may be good practice hereafter to engrave on the instrument and use generally really descriptive names, such as the square, cube, log (of the full-length factor scale) rather than the cryptic traditional symbols. In the outline below, all scales are duodecimal except where otherwise noted.

<table>
<thead>
<tr>
<th>Position</th>
<th>Name</th>
<th>Details</th>
<th>Traditional Symbols Scales</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>POWERS SIDE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Frame:</td>
<td>Powers Scales</td>
<td>$e^{-10}$ to $e^{10}$ with reciprocals</td>
<td>LL 4</td>
</tr>
<tr>
<td>Slide:</td>
<td>Square</td>
<td>Two cycle scale with reciprocal</td>
<td>B, BI 2</td>
</tr>
<tr>
<td></td>
<td>Cube</td>
<td>Three cycle scale, reciprocal</td>
<td>K, KI 2</td>
</tr>
<tr>
<td>(On Edge) Dec'1 Square</td>
<td></td>
<td></td>
<td>(Dec'1) B 1</td>
</tr>
<tr>
<td>Lower Frame:</td>
<td>Dec'1 Powers</td>
<td>$e^{1/12}$ to $e^{12}$ with reciprocals</td>
<td>LL 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$e^{1/1728}$ to $e^{1/12}$ with reciprocals</td>
<td></td>
</tr>
<tr>
<td><strong>OTHER SIDE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Frame:</td>
<td>Factors Folded Factors</td>
<td>Full-length scale with reciprocals</td>
<td>D, DI 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Same as last line, but folded</td>
<td>DF, DIF 2</td>
</tr>
<tr>
<td>Slide:</td>
<td>Factors Folded Factors</td>
<td>Full-length scale with reciprocal</td>
<td>C, CI 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Same as line above but folded</td>
<td>CF, CIF 2</td>
</tr>
<tr>
<td>Lower Frame:</td>
<td>Sine</td>
<td>Full-length sine scale</td>
<td>S 1</td>
</tr>
<tr>
<td></td>
<td>Tan</td>
<td>Full-length tangent scale</td>
<td>T 1</td>
</tr>
<tr>
<td></td>
<td>Small-Angles at $1^\circ$</td>
<td>Full-length factor scale, folded</td>
<td>ST 1</td>
</tr>
<tr>
<td></td>
<td>Log</td>
<td>Foot-long scale, 40 subdivisions to the inch. For measuring</td>
<td>L 1</td>
</tr>
</tbody>
</table>

In decimal notation, a total of 25 scales. 19 (21) seems to be the greatest total number of scales on any currently manufactured instrument; its illustration at least looks more crowded than the instrument proposed above probably will, because of the pairing of so many scales on the latter. Nevertheless it would probably be a real improvement in nearly all polyphase rules, to widen them somewhat and separate scales more generously, whether paired or not. To a user, it sometimes seems that if additional scales are wanted, the manufacturers merely squeeze them into an existing model, producing a crowded instrument rather hard for some of us to get used to.