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The Duodecimal System of Notation

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It is often remarked that the general adoption of the metric system of weights and measures would facilitate calculation because the metric system is based upon the decimal scale of notation. It perhaps has not occurred to the ordinary mind that the decimal scale itself is an arbitrary arrangement that was started because our ancestors counted on their fingers, and after counting to ten they could go no farther without starting over again. Had they been able to see two points beyond the tips of their fingers they would have given us a scale of notation that would have been more convenient than the decimal scale.

What has given to the numbers ten, one hundred, and one thousand their prominence as stations in the scale of notation? Simply the fact that our ancestors when they had counted to ten drove a stake there and hallowed the spot; and succeeding generations have supposed that ten held its commission of nobility from the eternal nature of things instead of from chance. The ten characters that we use in writing numbers are called digits, which is also the name for our fingers.

If you were to build a railroad you would select for stations the points that afford the best natural facilities. Now what natural advantage has the number ten over any other number to fit it for the position it holds? If you are liberal minded enough and sufficiently free from dogmatism that the number ten is not deified in your mind you may admit that it has no special advantages, but may perhaps contend that its advantages are at least equal to any other. You may esteem every number alike and ask how any number can possess properties that fit it any better for a station of nobility than another.

Do numbers have individual characters? Yes, each has a distinct personality. Some are related to each other and others have nothing in common with each other. The traits of character and relations that I refer to are inherent characteristics that could not be changed no matter what system of notation was in vogue. In the first place all numbers belong to two classes, odd and even. Now no odd number is suited for the position held by ten. Its very nature is against it; for then we could have no half way station. We all know how easy it is to count by fives or recite the fives in the multiplication table, or perform numerous

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short methods where the number five is involved. That is not due to any inherent quality of the number five but to the fact that it is a half way station. It is the royal favor conferred upon the number five by its relative ten whom our ancestors crowned in the ages of barbarism and to whom we still pay our allegiance. I am glad that one number enjoys that special favor. It makes easy calculations. I only regret that other numbers cannot come in for royal favor also. But I suppose that if our ancestors had possessed but one arm and an odd number of digits upon it, we would now be burdened with a disjointed system of notation based upon an odd number and we would have thought it the only system and clung to it as faithfully and as blindly as we now do to the decimal system.

Another classification of numbers according to their inherent nature is into prime and composite numbers. A prime number would never do as a basis for a scale of notation for the reason that it has no relatives (factors) on which to bestow its royal patronage. We have seen how nice it is that ten has bestowed its royal favor on its factors two and five but there its patronage ends. It shows no such favors to any other. We can count by twos or by fives and fetch up with a full stop at every decimal station; but when we try three we find ourselves on a through train that runs past two stations out of every three. Try four and we run past every other station. Number four comes in for a little more favor than three because it is indirectly related to his majesty, ten. They have a common relative in the number two. But number three has an extremely peculiar schedule. It stops, perforce, at the royal stations thirty, sixty, and ninety, but it flies past one hundred with a full head of steam. It snubs one thousand as if it were not even a flag station. But it pays dearly for its hostility to the royal family. Its relatives six and nine share its fate. Nine is treated even worse than three perhaps because it is three threes. Six has a common relative with ten in the number two, but his royal majesty cannot overlook its kinship with the hated three and therefore does not extend to it the favors enjoyed by four. Of course four is more closely related to two than six is because it is the product of two twos while six is a cross between a two and a three. Four is a thoroughbred, six is a mongrel and has therefore lost its caste, for no kin of that hated three can expect any royal favors. Three and ten are absolutely and eternally inimical to each other.

Even in the world of fractions the relatives of three, one-third, one-sixth, and one-ninth, share its fate and they and their multiples suffer from royal disfavor. Did you ever try to reduce one of them to a decimal fraction? You can't do it and get rid of your fraction.

In this everlasting hostility between ten and three I am inclined to take the part of the number three. Not that I blame ten, for it is easier for a camel to go through the eye of a needle than for ten to change its nature. Even composite numbers if they have no common prime factors are prime to each other and are then uncongenial spirits. Their dispositions are inimical to each other. Of course numbers must have their enemies just the same as people since they each have their peculiar dispositions; and no matter what number was used as a basis of notation it would have its inimicals. But it is a matter for regret that a number so fundamental in its relation to nature and nature's forms as the number three is should be out of harmony with the system of notation, like a prominent statesman out of harmony with the administration.

Prime numbers are the independent citizens of the numerical world; they have nothing to do with any but their own multiples and unity. Of course every number is in harmony with unity. Unity signifies perfection or completeness and gives the name to the universe. Nothing can be out of harmony with unity. To it and from it proceed all methods of calculation. Unity is the only deity in the realm of numbers. Of it are all numbers created in its likeness. All numbers are but repetitions of unity. The number one is both an odd number and a prime number. It is chief of the prime numbers and all the other prime numbers owe their distinct individuality to their relation to unity.

Composite numbers are the products of prime numbers and, true to the principles of heredity, they inherit all the traits of their prime factors. The number three being composed of three units, is divisible into three equal integral parts. That is its distinct characteristic which it imparts to all of its multiples. Therefore every composite number having three as a factor is capable of division into three equal integral parts. And so it is with all prime numbers; they each derive their individual character from unity the common father of all, and convey it to their offspring.

Now as the most successful governor of a state is one who can harmonize the greatest number of factions, or the one with the most friends and fewest enemies; so the most suitable number for the basis of a scale of notation is the one in harmony with the greatest number of factors; particularly those factors that are fundamental in their relation to nature. I claim that advantage for the number twelve over the number ten as a basis for a scale of notation. In other words, the duodecimal scale would give us a more convenient system for the reason that the number twelve and its multiples are capable of division into a greater number of integral aliquot parts. Although custom has established a station at ten, yet more trains stop at twelve, for whether you count by twos, threes, fours, or sixes, you are sure to stop at twelve. Custom can't change nature. Twelve has a wider range of patronage than ten, and fewer inimicals; more friends and less enemies.

By the duodecimal scale of notation I mean a system in which the value of the unit of each order shall be twelve times the value of the unit of the next lower order. As in the decimal scale we have ten units in a ten, ten tens in a hundred, etc., so in the duodecimal scale we would have twelve units in a twelve, twelve twelves in a gross, etc. The value of the orders of units would increase from right to left at the ratio of twelve.

This scale may seem hard and confusing, but that is only because we have become accustomed to think in the decimal scale. It would be easier than the decimal scale if we had only got started that way. But now if a number were written in the duodecimal scale we would have to translate it into the decimal scale before we would have any conception of its value. In fact to adopt the duodecimal scale into general use would be such a radical change and would necessitate such overwhelming changes in our very methods of thought that I am not brave enough (perhaps I should say foolish enough) to advocate it seriously as a reform. Yet I would like to call attention to its advantages. I am satisfied that in adopting the decimal scale we got started wrong; but we are such creatures of habit that to get out of the rut we are in is the next thing to impossible.

In order to use the duodecimal system it would be necessary to have two more characters or digits to represent ten and eleven. Suppose we were to adopt the system and should use X to represent ten and Y to represent eleven. Then 10 would be twelve (i. e., one twelve and 0 units). What we now call thirteen would be represented by 11 (1 twelve and 1 unit). We would call it onedeen. The next could be twodeen and so on to tendeen and elevendeen. Then would come two twelves which we might call duoty since two tens are now called twenty. Then would come duoty-one, etc. In due order we could have terty, quarty, quinty, sexty, septy, octy, nonty, tenty, eleventy, and one gross. I know that sounds silly, but couldn't we say one-gross quinty-six (written 156) as easily as we could say one hundred fifty-six? Translated into the decimal scale that number would be:

One-gross (twelve twelves)	144
Quinty (five twelves)	60
Six units	6
Total	210

Foolishness! did you say? Well perhaps, but let us go a little farther and see some of the advantages. Six would enjoy all the privileges as a half way station now conferred upon five. Its multiples would all end in 6 or 0. We would have the conveniences of quarter way stations and even third way stations. All multiples of four would end in 4, 8, or 0, and all multiples of three would end in 3, 6, 9, or 0. Under the decimal system such uniformity is only enjoyed by two and five. Two would enjoy privileges under the duodecimal system the same as it does under the decimal system and all its multiples would end in 2, 4, 6, 8, X, or 0. Only the number five would lose its caste and be expelled from court. In its place we would have three, four, and six as direct favorites and eight and nine would come in for indirect favors. Seven and eleven would not be favorites but neither are they under the decimal system, so we would lose nothing by their enmity. Ten of course would lose by its dethronement but twelve would gain all that ten would lose.

In the realm of fractions the most natural division of any unit of measure is into halves. The next most natural division is quarters. Then it comes natural to us to divide each quarter into eighths. Take the foot rule for instance and see how each inch is divided into halves, quarters, eighths, and sixteenths. Some are divided into quarters and then each quarter is cut into three pieces giving us twelfths. In my opinion the latter arrangement is better because we can then measure thirds or sixths of an inch, while with a rule divided into sixteenths we could not measure a third of an inch without guessing a little. Such a rule would be constructed on the duodecimal scale. Why is it convenient to divide a foot into twelve inches? Because it can then be divided into halves, thirds, fourths, or sixths with integral results. But we cannot divide it into fifths and we would seldom want to do so if we were not burdened by the decimal system, because a fifth is an unnatural division. Why do we have half dollars and quarter dollars? Because they are natural divisions of the unit. We could have a third dollar coin if the number three were not out of harmony with the system.

Five is in harmony with the system yet we have no fifth dollar coin because we don't want any. We have had a twenty cent coin which in fact was a fifth of a dollar but it was not called such by name, while the 25 cent piece and 50 cent piece are specifically called quarter dollar and half dollar respectively. Why did we once have a two and one-half dollar coin? Because of the natural desire for a quarter eagle. This shows that halves and quarters are the most fundamental divisions of the unit. Thirds are not quite so fundamental as quarters, but they are more fundamental than fifths. Three outranks five; it is nearer unity and closer to nature. Four has more favors than three because it is a composite relative of two, but five is a still more remote prime number and it is only due to administrative nepotism that five is blessed and three is accursed; while three in its inherent nature merits more favors than five.

Look at the face of your watch and see if you could divide it into a more convenient number of spaces than twelve. How else could it be resolved into halves, quarters, or thirds? If it were divided into ten spaces for hours when would it be quarter past six? Our measure of time as well as the foot rule is out of joint with the decimal system but it is in harmony with the nature of all measures. Even in France, the home of the metric system, they use the same measure of time as we; their clocks and watches are made like ours and they figure angles and circles in degrees the same as we do. By dividing the hour into sixty minutes we accommodate even the factor five. No other division would give so many facilities. For this reason there are twenty-four hours in a day (twelve for the day and twelve for the night), and twelve months in the year, twelve signs in the zodiac and twelve signs in a circle.

Let us see how important a factor three is in its relation to space. Draw a circle and inscribe the hexagon; then from the alternate corners of the hexagon, or points of division, of the circle, draw three radii. Now look at your figure and you will find yourself looking directly at the corner of a cube. A cube has three dimensions, six faces, eight corners, and twelve edges. The number five seems to have no part in the plan of construction. Even to find the area of a pentagon we would divide it into triangles. and we find the cubic contents of a pyramid or cone by multiplying the area of the base by one-third of the altitude.

Equilateral triangles, squares, and hexagons match perfectly when grouped together. Octagons fit together leaving small squares; but pentagons do not match. Who has not observed the hexagonal shaped columnar rocks in geological formations, or the shape of the cells in a honeycomb?

Divide a circle into quarters and you have four right angles at the center. The right angle is the fundamental angle. What would the carpenter or the mason do without his square? The simplest plane figure is the square or rectangle which has four right angles and two dimensions. This shows the fundamental nature of two and four. There are four cardinal points to the compass. Wagons have four wheels; a fifth would be a superfluity. Yet we have vehicles with three wheels, and some with two. Animals have two legs or four. Five is represented in most animals by the number of digits on each limb; yet with many of the lower animals the fifth toe has degenerated into a useless appendage. Fowls usually have four toes on each foot. We have five senses but we know not if that completes the lot: indeed there is so much that is a mystery to us that it would seem that they do not. Because we are blessed with more faculties than the lower animals is not proof that we have a full complement.

Thus it is seen that ten is the real hybrid in the numerical world inheriting its even nature from its factor two and its eccentricities from the factor five. Ten is the usurper of honors it does not merit. The truly royal blooded numbers are the multiples of two and three.

Now let us see how the decimal system treats the fractional world. We have already seen that the most fundamental fractions are halves, quarters, and thirds. Now of these the half is the only one that can be expressed by one decimal place. Fourths require two decimal places and eighths three, and so on every time the fraction is bisected another decimal place must be added. Thirds cannot be expressed decimally at all.

How would it be with duodecimals? One-half would be .6 (six twelfths), one-fourth would be .3 (three twelfths), one-third would be .4 (four twelfths) and one-sixth .2 (two twelfths). Besides these are two-thirds (.8, eight twelfths), three fourths (.9, nine twelfths), and five-sixths (.X, ten twelfths), all of which could be expressed by a single duodecimal place. Eighths would require only two duodecimal places. One-eighth would be .16 (sixdeen grossths, translated 18/144) and one-sixteenth (one fourdeenth duodecimal) would be .09 (nine grossths).

Fifths, sevenths, tenths, and elevenths would make circulating duodecimals, but as they are less important divisions, and as sevenths and elevenths produce circulating decimals anyway, there would not be much loss.

But what is the use of all this? The decimal system is so thoroughly established that we cannot hope to change it, and besides if we could, would it be worth while? Perhaps not; but this should teach us independence of thought. It should teach us to get down to fundamentals in laying our premises. The fact is that the decimal system is but the arbitrary invention of man, a graven image set on a pedestal. We are such slaves to habit, slaves to precedent, slaves to custom, that we are prone to mistake customary forms for axioms. We are such slaves to authority that we are prone to take things as they are presented to us as matters of fact without going behind the scenes to investigate. Thousands before Newton's time saw apples fall from trees without asking why.

It is well to bow to authority but it should be the bow of courtesy and not the obeisance of the slave. Let us bow to authority but let us not kneel to any save the authority of the Eternal, for He has commanded that we have no other gods before Him.

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