The Dozenal Society of America

The Personality of the Integers from One to One Gross

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Introduction

In his fascinating paperback entitled *The Lore of Large Numbers*,[1] author Philip J. Davis devotes a detailed chapter to The Personality of Numbers and provides a list entitled Who's Who Among The Integers from One to One Hundred in the decimal base. The purpose of my paper is to expand his list to the first gross and present additional characteristics of these integers. It is indeed the uniqueness of each integer that lends a special personality to them especially when one resorts to duodecimals. Finally, my goal is to furnish number theoretic characteristics related to each of the integers that are perhaps omitted from the average list. Our initial task is to present the reader with a dictionary which is termed *A Glossary of Symbols and Terms*. These symbols and terms will be illustrated in this section.

A Glossary of Symbols and Terms

1. $n!$ The factorial of the positive integer n. We defined $n!$, known as n *factorial*, recursively as follows: $0! = 1$ and $(n+1)! = (n+1)(n!)$. Hence $1! = (0+1)(0!) = (1)(1) = 1$ and $5! = (4+1)(4!) = (5)(20) = ↊0$.

2. T_n The nth Triangular Number. We defined the nth Triangular Number T_n as follows: $\mathrm{T}_n = 1+2+3+\ldots+n = ((n)(n+1)/2$. Thus $\mathrm{T}_5 = 1+2+3+4+5 = 13 = ((5)(5+1))/2$.

3. S_n The nth Square Number. Define the nth Square Number S_n by the formula $\mathrm{S}_n = 1+3+5+7+\ldots+(2n-1) = n^2$. Hence $\mathrm{S}_4 = 1+3+5+7 = 14 = 4^2$.

4. TH_n The nth Tetrahedral Number. The Tetrahedral Numbers form the sequence $1, 4, ↊, 18, 2↋, 48, \ldots$. The Tetrahedral Numbers are related to the Triangular Numbers as follows: $\mathrm{TH}_n = \mathrm{TH}_{n-1} + \mathrm{T}_n$ for $n \geq 2$ and $\mathrm{TH}_1 = 1$. Hence $\mathrm{TH}_5 = \mathrm{TH}_4 + \mathrm{T}_5 = 18 + 13 = 2↋$.

5. F_n The nth Fibonacci Number. n is a Fibonacci Number if n satisfies the Fibonacci Sequence defined recursively as follows: $\mathrm{F}_1 = 1$, $\mathrm{F}_2 = 1$, and $\mathrm{F}_n = \mathrm{F}_{n-2} + \mathrm{F}_{n-1}$ for $n \geq 3$. Hence $\mathrm{F}_3 = \mathrm{F}_1 + \mathrm{F}_2 = 1 + 1 = 2$.

6. L_n The nth Lucas Number. n is a Lucas Number if n satisfied the Lucas Sequence defined recursively as follows: $\mathrm{L}_1 = 1$, $\mathrm{L}_2 = 3$, and $\mathrm{L}_n = \mathrm{L}_{n-2} + \mathrm{L}_{n-1}$ for $n \geq 3$. Hence $\mathrm{L}_3 = \mathrm{L}_1 + \mathrm{L}_2 = 1 + 3 = 4$.

7. p_n The nth Prime Number. Define a prime number to be any integer $p > 1$ that has only 1 and p as its factors (divisors). Hence $p_3 = 5$; for the only divisors of 5 are 1 and 5.

8. FE_n The nth Fermat Prime. n is a Fermat Prime if n is prime and $n = 2^{2^n} + 1$ for any whole number n. To cite an example, $\mathrm{FE}_2 = 2^{2^2} + 1 = 15$.

9. CO_n The nth Integer for which a regular n-gon is constructible using solely a straight-edge and compass. This idea is related to the geometric notion of *Constructible Regular n-gons* in the following sense as can be demonstrated via abstract algebraic techniques: The sole values of n for which a regular n-gon is constructible using only a straight-edge and compass are such that the only odd primes dividing n are the Fermat Primes whose squares do not divide n. Hence a regular duodecahedron [sic] (10 sides) is constructible in the above sense; for $10 = (2^2)(3)$ and 3 is a Fermat Prime ($F_1 = 3$ as the reader

[1] Philip J. Davis, *The Lore of Large Numbers*. The New Mathematical Library, New York, 1961.

can readily verify) but 3^2 is not a factor of 10. On the other hand, a regular 16-gon is not constructible since $16 = (2)(3^2)$ and while 3 is a Fermat Prime, 3^2 is a factor of 16.

↊. C_n The nth Composite Number. n is designated Composite if $n > 1$ and n is not prime. To cite an example, 10 is composite; for 10 possesses a half dozen factors: 1, 2, 3, 4, 6, and 10 while prime numbers possess only a pair of factors (1 and the prime itself).

↋. HC_n The nth Hypercomposite Number. A positive integer n is termed Hypercomposite if n possesses a greater number of factors than any of its immediate predecessors. Again 10 serves as a near example; for 10 has 6 factors while no integer < 10 has more than 4 factors (6, 8, and ↊).

10. SF_n The nth Square-Free Number. n is Square-Free if n is not divisible by the square of any prime p. To illustrate, $26 = (2)(3)(5)$ enjoys this property while our favorite integer $10 = (2^2)(3)$ does not having 2^2 as a part of its factorization.

11. $\sigma(n)$ The sum of the divisors of a positive integer n including n itself. To cite some example, $\sigma(6) = 1 + 2 + 3 + 6 = 10$, while $(\sigma(10) = 1 + 2 + 3 + 4 + 6 + 10 = 24)$ and $\sigma(11) = 1 + 11 = 12$.

12. P_n The nth Perfect Number. A positive integer n is styled Perfect if it is equal to the sum of all its aliquot (proper) divisors. For example, 6 s perfect since $6 = 1+2+3$. In short, any integer which coincides with the sum of all its divisors not including itself is classified as perfect. Perfect numbers are rare indeed. The next three perfect numbers in order of magnitude are 24, 354, and 4854. Mathematicians often succinctly employ the equation $\sigma(n) = 2n$ to denote that n is perfect; that is, the sum of all the divisors of n including n itself is precisely twice n. Equivalently, one could write $\sigma_0(n) = n$ to connote that the sum of all the proper divisors of n is precisely n if n is perfect.

13. A_n The nth Abundant Number. A positive integer n is termed Abundant if it is greater than the sum of all its aliquot divisors. For example, 10 is abundant, since $1+2+3+4+6 = 14 > 10$. The equation $\sigma(n) > 2n$ indicates that n is abundant; that is, the sum of all the divisors of n including n is larger than twice n. Equivalently, one often writes $\sigma_0(n) > n$ to signify that the sum of all the proper divisors of n is greater than n if n is abundant. It can be shown that any integer multiple of a perfect number is necessarily abundant (10 fits this description) while most abundant numbers are even. In fact, the initial odd abundant number is 669 (decimally nine hundred forty-five), and we observe that $\sigma(669) = 1140 > (2)(669) = 1116$. In fact, $1+3+5+7+9+13+19+23+2↋+39+53+89+↋3+139+223 = 693 > 669$.

14. D_n The nth Deficient Number. A positive integer n is classified as Deficient if it is less than the sum of its aliquot divisors. To illustrate, 2↋ is deficient, since $1+5+7 = 11 < 2↋$. The equation $\sigma(n) < 2n$ signifies that n is deficient; that is, the sum of all the divisors of n including n is smaller than twice n. Equivalently, one generally writes the inequality $\sigma_0(n) < n$ to indicate that the sum of the proper divisors of n is less than n if n is deficient. It can be shown with relative ease that any prime number is deficient. Moreover, so is any power of a prime and any integer save for $6 = (2)(3)$ that is expressible as the product of two distinct primes.

15. M_n The nth Mersenne Prime. n is a Mersenne Prime is n is prime and $n = 2^p - 1$, where p itself is prime. Mersenne primes lead to perfect numbers. In fact, the form of an even perfect number according to the Greek Mathematician Euclid is $p = (2^{p-1})(2^p - 1)$ where both p and 2^{p-1} are prime. To illustrate, if we take $p = 3$, then 3 is prime and so is $2^3 - 1 = 7$. This leads to the second perfect number $P = (2^{3-1})(2^3 - 1) = (2^2)(7) = 24$.

16. $\tau(n)$ The number of divisors of a positive integer n. To cite some examles, $\tau(10) = 6$ (1, 2, 3, 4, 6, 10 are the divisors of 10) while $\tau(6) = 4$ (1, 2, 3, and 6 are the divisors of 6) and $\tau(5) = 2$ (1 and 5 are the divisors of 5). It is readily apparent that if p is a prime, then $\tau(p) = 2$, while for any power of a prime p, say p^2, $\tau(p^2) = n + 1$. For example, $8 = 2^3$ and $\tau(8) = 4$; for 1, 2, 4, and 8 are the divisors of 8.

17. $\phi(n)$ The number of positive integers $< n$ that are relatively prime to n. To illustrate, $\phi(10) = 4$ since 1, 5 7, and ↋ are < 10 and have no factors in common with 10 other than 1, which is equivalent to saying that the integers are relatively prime to 10. In addition, $\phi(7) = 6$; for 1, 2, 3, 4, 5, and 6 are all < 7 and relatively prime to 7. It is readily apparent that if p is prime, then $\phi(p) = p - 1$. Moreover, if $t = p^2$

(p a prime), then $\phi(p^n) = p^n - p^{n-1}$. Hence $\phi(8) = \phi(2^3) = 2^3 - 2^{3-1} = 2^3 - 2^2 = 8 - 4 = 4$. Observe that this agrees with enumeration; for the integers 1, 3, 5, and 7 are < 8 and are relatively prime to 8. This number-theoretic function known as the *Euler-phi function* or *totient function* possesses extremely important applications in higher algebra and the theory of numbers. In fact, recalling our discussion earlier concerning constructible regular polygons utilizing only a straight-edge and compass, one can demonstrate that the values of n for which a regular n-gon is constructible using only a straight-edge and compass are those values for which $[\phi(n)/2]$ (and thus $\phi(n)$) is an integral power of 2. Hence a regular duodecagon is constructible since $\phi(10) = 4 = 2^2$ as is a regular octagon ($\phi(8) = 4 = 2^2$) while a regular 16-gon is not ($\phi(16) = 6$ and 6 is not an integral power of 2).

18. $\pi(n)$ The number of positive primes $\leq n$. To cite an example, observe hat $\pi(10) = 5$, since 2, 3, 5, 7, and ↋ are primes not exceeding a dozen. Morever, $\pi(11) = 6$; for 2, 3, 5, 7, ↋, and 11 are primes ≤ 11.

19. $\Pi d|n$ The product of the divisors of a positive integer n. To illustrate, $\Pi d|10 = (1)(2)(3)(4)(6)(10) = 1000 = 10^3$, while $\Pi d|23 = (1)(3)(9)(23) = 509$ and $\Pi d|2↋ = (1)(5)(7)(2↋) = 861$. Observe in the latter pair of examples that 23 and 2↋ coincide with the product of their proper divisors.

1↊. PSK_n The nth Perfect Number of the Second Kind. The idea of a Perfect Number of the Second Kind generalizes the notion of a perfect number. n is styled Perfect of the Second Kind if the product of the divisors of n coincides with the square of the number n. Symbolically, one would express this fact via the equation $\Pi d|n = n^2$. Citing the examples 23 and 2↋ from 19 above illustrates this type of behavior; for $509 = 23^2$ and $861 = 2↋^2$. Moreover, it is immediate that if $n = (p)(q)$ for distinct primes p and q or if $n = p^3$ for some prime p, then the product of all the aliquot divisors of n coincides with n and the Product of all the divisors of n is n^2. (Note that $(1)(p)(q)(pq) = (p^2)(q^2) = (pq)^2$ and $(1)(p)(p^2)(p^3) = (p^6) = (p^3)^2$.)

1↋. MP_n The nth Multiply Perfect Number. A second generalization of a perfect number is that of a multiply-perfect number. We define n to be multiply-perfect if $\sigma(n) = (k)(n)$ for some positive integer $k \geq 3$. (A perfect number is in essence a multiply perfect number with $k = 2$.) Our initial example of a 3-perfect number is ↊0. $\sigma(↊0) = 1+2+3+4+5+6+8+↊+10+13+18+20+26+34+50+↊0 = 260 = (3)(↊0)$. The reader is invited to show that 480 is a second 3-perfect prime number; 4-perfect numbers are also known although the smallest one is quite large.

20. $\text{A}(m,n)$ An amicable or friendly number pair. A pair of numbers m and n are termed amicable or friendly if the sum of the aliquot divisors of one is equal to the other. In essence, we are asserting that $\text{A}(m,n)$ is an amicable pair if $\sigma_0(m) = n$ and $\sigma_0(n) = m$. Moreover from this, one deduces that if $\text{A}(m,n)$ is an amicable pair, then $\sigma(m) = \sigma(n) = m + n$. If $m = 164$ and $n = 1↋8$, then $\sigma_0(164) = 1+2+4+5+↊+↋+18+1↊+38+47+92 = 1↋8$ while $\sigma_0(1↋8) = 1 + 2 + 4 + 5↋ + ↋↊ = 164$, so that (164, 1↋8) is an amicable number pair, the smallest of its kind. Note that $\sigma(164) = 360 = \sigma(1↋8) = 164 + 1↋8$. The reader is invited to check that the number pair (828, 84↊) is a second amicable number pair, discovered in 10↋6, by a 14; year old Italian school boy. The pair seemingly escaped the great mathematicians of their day.

It should be noted in closing that $\sigma(n)$, $\tau(n)$, and $\phi(n)$ are multiplicative number-theoretic functions in the sense that for relatively prime pairs of integers r and s, $\sigma(rs) = \sigma(r) \times \sigma(s)$, $\tau(rs) = \tau(r) \times \tau(s)$, and $\phi(rs) = \phi(r) \times \phi(s)$. This result is extendable to more than two integers provided none of the integers has a common factor apart from 1 among them. To cite an example, we calculate $\sigma(↊0)$, $\tau(↊0)$, and $\phi(↊0)$. Initially observe that $↊0 = (2^3)(3)(5)$. Now $\sigma(↊0) = \sigma[(2^3)(3)(5)] = (2^3) \times \sigma(3) \times \sigma(5) = [(2^{3+1} - 1)/(2-1)] \times (3+1) \times (5+1) = [(2^4 - 1)/(2-1) = (14-1)/(2-1) \times (4) \times (6) = (13)/(1) \times (4) \times (6) = (13) \times (4) \times (6) = 260$, $\tau(↊0) = [(2^3)(3)(5)] = \tau(2^3) \times \tau(3) \times \tau(5) = (3+1) \times (1+1) \times (5+1) = 4 \times 2 \times 2 = 14$, and $\phi(↊0) = [(2^3)(3)(5)] = [2^3 - 2^{3-1}] \times (3-1) \times (5-1) = [2^3 - 2^2] \times 2 \times 4 = (8-4) \times (3-1) \times (5-1) = 4 \times 2 \times 4 = 28$.

Our next goal is to provide an enumeration of a number of essential dozenal sets of numbers in the range from 1 to 100.

Dozenal Sets of Numbers (Range 1–100)

1. **Perfect Numbers** {6, 24}. (Total: 2)

2. **Deficient Numbers** {1, 2, 3, 4, 5, 7, 8, 9, ↊, ↋, 11, 12, 13, 14, 15, 17, 19, 1↊, 1↋, 21, 22, 23, 25, 27, 28, 29, 2↊, 2↋, 31, 32, 33, 35, 37, 38, 39, 3↊, 3↋, 41, 42, 43, 44, 45, 47, 49, 4↊, 4↋, 51, 52, 53, 54, 55, 57, 58, 59, 5↊, 61, 62, 63, 64, 65, 67, 69, 6↊, 6↋, 71, 72, 73, 75, 77, 78, 79, 7↊, 7↋, 81, 82, 83, 85, 87, 89, 8↊, 8↋, 91, 92, 93, 95, 97, 98, 99, 9↊, 9↋, ↊1, ↊2, ↊3, ↊4, ↊5, ↊7, ↊8, ↊9, ↊↊, ↊↋, ↋1, ↋2, ↋3, ↋4, ↋5, ↋7, ↋9, ↋↊, ↋↋}. (Total: 91)

3. **Abundant Numbers** {10, 16, 18, 20, 26, 30, 34, 36, 40, 46, 48, 50, 56, 5↊, 60, 66, 68, 70, 74, 76, 80, 84, 86, 88, 90, 94, 96, ↊0, ↊6, ↋0, ↋6, ↋8, 100} (Total: 29)

4. **Multiply Perfect Numbers** {↊0} (3-Perfect Number) (Total: 1)

5. **Square-Free Numbers** {1, 2, 3, 5, 6, 7, ↊, ↋, 11, 12, 13, 15, 17, 19, 1↊, 1↋, 22, 25, 26, 27, 29, 2↊, 2↋, 31, 32, 33, 35, 36, 37, e↊, 3↋, 43, 45, 47, 49, 4↊, 4↋, 51, 52, 55, 56, 57, 59, 5↊, 5↋, 61, 62, 65, 66, 67, 6↊, 6↋, 71, 72, 73, 75, 77, 79, 7↊, 7↋, 81, 85, 86, 87, 89, 8↊, 8↋, 91, 92, 93, 95, 96, 97, 9↊, 9↋, ↊2, ↊3, ↊7, ↊9, ↊↊, ↊↋, ↋1, ↋2, ↋5, ↋6, ↋7, ↋9, ↋↊, ↋↋}. (Total: 75)

6. **Prime Number** {2, 3, 5, 7, ↋, 11, 15, 17, 1↋, 25, 27, 31, 35, 37, 3↋, 45, 4↋, 51, 57, 5↋, 61, 67, 6↋, 75, 81, 85, 87, 8↋, 91, 95, ↊7, ↊↋, ↋5, ↋7}. (Total: 2↊)

7. **Triangular Numbers** {1, 3, 6, ↊, 13, 19, 24, 30, 39, 47, 56, 66, 77, 89, ↊0, ↋4} (Total: 14)

8. **Tetrahedral Numbers** {1, 4, ↊, 18, 2↋, 48, 70, ↊0}. (Total: 8)

9. **Fibonacci Numbers** {1, 1, 2, 3, 5, 8, 11, 19, 2↊, 47, 75, 100}. (Total: 10)

↊. **Lucas Numbers** {1, 3, 4, 7, ↋, 16, 25, 3↋, 64, ↊3}. (Total: ↊)

↋. **Composite Numbers** {4, 6, 8, 9, ↊, 10, 12, 13, 14, 16, 18, 19, 1↊, 20, 21, 22, 23, 24, 26, 28, 29, 2↊, 2↋, 30, 32, 33, 34, 36, 38, 39, 3↊, 40, 41, 42, 43, 44, 46, 47, 48, 49, 4↊, 50, 52, 53, 54, 55, 56, 58, 59, 5↊, 60, 62, 63, 64, 65, 66, 68, 69, 6↊, 70, 71, 72, 73, 74, 76, 77, 78, 79, 7↊, 7↋, 80, 82, 83, 84, 86, 88, 89, 8↊, 90, 92, 93, 94, 96, 97, 98, 99, 9↊, 9↋, ↊0, ↊1, ↊2, ↊3, ↊4, ↊5, ↊8, ↊9, ↊↊, ↋0, ↋1, ↋2, ↋3, ↋4, ↋6, ↋8, ↋9, ↋↊, ↋↋, 100}. (Total: 91)

10. **Perfect Numbers of the Second Kind** {6, 8, ↊, 12, 13, 19, 1↊, 22, 23, 29, 2↊, 2↋, 32, 33, 3↊, 43, 47, 49, 4↊, 52, 55, 59, 62, 65, 6↊, 71, 72, 73, 77, 79, 7↊, 7↋, 8↊, 93, 97, 9↊, 9↋, ↊2, ↊3, ↊5, ↊9, ↋1, ↋2, ↋9, ↋↊, ↋↋}. (Total: 3↊)

11. **Set of Values such that a Regular n-gon** is constructible using only a straight-edge and compass: {3, 4, 5, 6, 8, ↊, 10, 13, 14, 15, 18, 20, 26, 28, 2↊, 34, 40, 43, 50, 54, 58, 68, 71, 80, 86, ↊0, ↊8, ↋4}. (Total: 24)

12. **Fermat Primes** {3, 5, 15}. (Total: 3)

13. **Hypercomposite Numbers** {2, 4, 8, 10, 20, 30, 40, 50, ↊0}. (Total: 9)

14. **Mersenne Primes** {$M_2 = 3$, $M_3 = 7$, $M_5 = 27$, $M_7 = ↊7$}. (Total: 4)

15. **Square Numbers** {1, 4, 9, 14, 21, 30, 41, 54, 69, 84, ↊1, 100}. (Total: 10)

The Catalogue of Integers from One to One Gross Characteristics

In what follows, we shall make use of the symbols described in our glossary above. Hence F_n will denote the nth Fibonacci Number and C_n will connote the nth Composite Number. Thus $\mathrm{F}_5 = 5$, since the fifth Fibonacci Number in the sequence is five, while $\mathrm{C}_{↊} = 16$, as the tenth composite integer in the sequence is one dozen six. The other codes are obtained in similar fashion, and we cap the article with the catalogue.

1. Unit; Multiplicative Identity; Neither Prime nor Composite. F_1; F_2; 1^n for all counting integers n; 0!; 1!; T_1; TH_1; L_1; S_1; SF_1; D_1; $\tau(1) = 1$; $\phi(1) = 1$; $\sigma(1) = 1$; $\pi(1) = 1$; $\Pi d|1 = 1$.

2. p_1; Binary Base; $1^n + 1^n$ for all counting integers n; HC_1; F_3; 2!; $\mathrm{F}_1 + \mathrm{F}_2$; D_2; SF_2; $\tau(2) = 2$; $\phi(2) = 1$; $\sigma(2) = 3$; $\pi(2) = 1$; $\Pi d|2 = 2$.

3. p_2; Ternary Base; M_2; FE_1; C_1; F_4; T_2; L_2; $1! + 2!$; $F_1 + F_3$; SF_3; D_3; $\tau(3) = 2$; $\phi(3) = 2$; $\sigma(3) = 4$; $\pi(3) = 2$; $\Pi d|3 = 3$.

4. 2^2; C_1; HC_2; TH_2; L_3; S_2; CO_2; $F_1 + F_2 + F_3$; $F_2 + F_4$; D_4; $L_1 + L_2$; $\tau(4) = 3$; $\phi(4) = 2$; $\sigma(4) = 7$; $\pi(4) = 2$; $\Pi d|4 = 8$.

5. p_3; $2^2 + 1^2$; FE_2; F_5; CO_3; SF_4; D_5; $L_1 + L_3$; $\tau(5) = 2$; $\phi(5) = 4$; $\sigma(5) = 6$; $\pi(5) = 3$; $\Pi d|5 = 5$.

6. 2×3; P_1; C_2; PSK_1; SF_5; CO_4; T_3; $3!$; $\tau(6) = 4$; $\phi(6) = 2$; $\sigma(6) = 10$; $\pi(6) = 3$; $\Pi d|6 = 30$.

7. p_4; L_4; $F_1 + F_2 + F_3 + F_4$; M_3; SF_6; D_6; $\tau(7) = 2$; $\phi(7) = 6$; $\sigma(7) = 8$; $\pi(7) = 4$; $\Pi d|7 = 7$.

8. 2^3; C_3; PSK_2; Octal Base; F_6; $2! + 3!$; $2^2 + 2^2$; CO_5; $F_1 + F_3 + F_5$; D_7; $L_1 + L_2 + L_3$; HC_3; $\tau(8) = 4$; $\phi(8) = 4$; $\sigma(8) = 13$; $\pi(8) = 4$; $\Pi d|8 = 54$.

9. 3^2; C_4; S_3; $1^3 + 2^3$; $1! + 2! + 3!$; D_8; $\tau(9) = 3$; $\phi(9) = 6$; $\sigma(9) = 11$; $\pi(9) = 4$; $\Pi d|9 = 23$.

↊. 2×5; C_5; CO_6; PSK_3; SF_7; Decimal Base; $1^2 + 3^2$; T_4; TH_3; D_9; $L_2 + L_4$; $\tau(↊) = 4$; $\phi(↊) = 4$; $\sigma(↊) = 16$; $\pi(↊) = 4$; $\Pi d|↊ = 84$.

↋. p_5; SF_8; L_5; $D_↊$; $\tau(↋) = 2$; $\phi(↋) = ↊$; $\sigma(↋) = 10$; $\pi(↋) = 5$; $\Pi d|↋ = ↋$.

10. $2^2 \times 3$; Duodecimal Base; HC_4; C_6; CO_7; A_1; $2! \times 3!$; $F_1 + F_2 + F_3 + F_4 + F_5$; $F_2 + F_4 + F_6$; $\tau(10) = 6$; $\phi(10) = 4$; $\sigma(10) = 28$; $\pi(10) = 5$; $\Pi d|10 = 1000$.

11. p_6; F_7; $2^2 + 3^2$; $D_↋$; SF_9; $\tau(11) = 2$; $\phi(11) = 10$; $\sigma(11) = 12$; $\pi(11) = 6$; $\Pi d|11 = 11$.

12. 2×7; C_7; $SF_↊$; PSK_4; $1^2 + 2^2 + 3^2$; D_{10}; $\tau(12) = 4$; $\phi(12) = 6$; $\sigma(12) = 20$; $\pi(12) = 6$; $\Pi d|12 = 144$.

13. 3×5; C_8; PSK_5; CO_8; T_5; D_{11}; $L_1 + L_2 + L_3 + L_4$; $\tau(13) = 4$; $\phi(13) = 8$; $\sigma(13) = 20$; $\pi(13) = 6$; $\Pi d|13 = 179$.

14. 2^4; Hexadecimal Base; 4^2; S_4; C_9; $2^3 + 2^3$; CO_9; D_{12}; $L_1 + L_3 + L_5$; $\tau(14) = 5$; $\phi(14) = 8$; $\sigma(14) = 27$; $\pi(14) = 6$; $\Pi d|14 = 714$.

15. p_7; $1^2 + 4^2$; $1^4 + 2^4$; FE_3; $CO_↊$; D_{13}; SF_{10}; $\tau(15) = 2$; $\phi(15) = 14$; $\sigma(15) = 16$; $\pi(15) = 7$; $\Pi d|15 = 15$.

16. 2×3^2; $C_↊$; A_2; $3^2 + 3^2$; L_6; $\tau(16) = 6$; $\phi(16) = 6$;$\sigma(16) = 33$; $\pi(16) = 7$; $\Pi d|16 = 3460$.

17. p_8; D_{14}; SF_{11}; $\tau(17) = 2$; $\phi(17) = 16$; $\sigma(17) = 18$; $\pi(17) = 8$; $\Pi d|17 = 17$.

18. $2^2 \times 5$; A_3; $C_↋$; score; TH_4; $2^2 + 4^2$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6$; $\tau(18) = 6$; $\phi(18) = 8$; $\sigma(18) = 36$; $\pi(18) = 8$; $\Pi d|18 = 4768$.

19. 3×7; C_{10}; PSK_6; F_8; SF_{12}; T_6; $F_1 + F_3 + F_5 + F_7$; D_{15}; $\tau(19) = 4$; $\phi(19) = 10$; $\sigma(19) = 28$; $\pi(19) = 8$; $\Pi d|19 = 309$.

1↊. $2 \times ↋$; C_{11}; PSK_7; SF_{13}; D_{16}; $\tau(1↊) = 4$; $\phi(1↊) = ↊$; $\sigma(1↊) = 30$; $\pi(1↊) = 8$; $\Pi d|1↊ = 344$.

1↋. p_9; SF_{14}; D_{17}; $\tau(1↋) = 2$; $\phi(1↋) = 1↊$; $\sigma(1↋) = 20$; $\pi(1↋) = 9$; $\Pi d|1↋ = 1↋$.

20. $2^3 \times 3$; HC_5; C_{12}; A_4; CO_{10}; $4!$; $\tau(20) = 8$; $\phi(20) = 8$; $\sigma(20) = 50$; $\pi(20) = 9$; $\Pi d|20 = 14\,0000$.

21. 5^2; $3^2 + 4^2$; C_{13}; D_{18}; S_5; $\tau(21) = 3$; $\phi(21) = 18$; $\sigma(21) = 27$; $\pi(21) = 9$; $\Pi d|21 = ↊5$.

22. 2×11; SF_{15}; C_{14}; $1^2 + 5^2$; PSK_8; D_{19}; $L_1 + L_2 + L_3 + L_4 + L_5$; $\tau(22) = 4$; $\phi(22) = 10$; $\sigma(22) = 36$; $\pi(22) = 9$; $\Pi d|22 = 484$.

23. 3^3; PSK_9; C_{15}; $D_{1↊}$; $\tau(23) = 4$; $\phi(23) = 16$; $\sigma(23) = 34$; $\pi(23) = 9$; $\Pi d|23 = 509$.

24. $2^2 \times 7$; P_2; C_{16}; $1^3 + 3^3$; T_7; $L_2 + L_4 + L_6$; $\tau(24) = 6$; $\phi(24) = 10$; $\sigma(24) = 48$; $\pi(24) = 9$; $\Pi d|24 = 1\,0854$.

25. $p_↊$; $2^2 + 5^2$; L_7; SF_{16}; $D_{1↋}$; $2^2 + 3^2 + 4^2$; $\tau(25) = 2$; $\phi(25) = 24$; $\sigma(25) = 26$; $\pi(25) = ↊$; $\Pi d|25 = 25$.

26. $2 \times 3 \times 5$; C_{17}; A_5; CO_{11}; $1^2 + 2^2 + 3^2 + 4^2$; SF_{17}; $3! + 4!$; $\tau(26) = 8$; $\phi(26) = 8$; $\sigma(26) = 60$; $\pi(26) = ↊$; $\Pi d|26 = 33\,0900$.

27. $p_↋$; M_5; SF_{18}, D_{20}; $\tau(27) = 2$; $\phi(27) = 26$; $\sigma(27) = 28$; $\pi(27) = ↋$; $\Pi d|27 = 27$.

28. 2^5; C_{18}; $2^4 + 2^4$; $4^2 + 4^2$; CO_{12}; D_{21}; $2! + 3! + 4!$; $\tau(28) = 6$; $\phi(28) = 14$; $\sigma(28) = 53$; $\pi(28) = ↋$; $\Pi d|28 = 16↋68$.

29. $3 \times ↋$; SF_{19}; C_{19}; $PSK_↊$; D_{22}; $1^5 + 2^5$; $1! + 2! + 3! + 4!$; $F_2 + F_4 + F_6 + F_8$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7$; $\tau(29) = 4$; $\phi(29) = 18$; $\sigma(29) = 40$; $\pi(29) = ↋$; $\Pi d|29 = 769$.

2↊. 2×15; $SF_{1↊}$; D_{23}; $C_{1↊}$; $PSK_↋$; $3^2 + 5^2$; F_9; CO_{13}; $\tau(2↊) = 4$; $\phi(2↊) = 14$; $\sigma(2↊) = 46$; $\pi(2↊) = ↋$; $\Pi d|2↊ = 804$.

2↋. 5×7; $SF_{1↋}$; D_{24}; $C_{1↋}$; PSK_{10}; $1^2 + 3^2 + 5^2$; $2^3 + 3^3$; TH_5; $\tau(2↋) = 4$; $\phi(2↋) = 20$; $\sigma(2↋) = 40$; $\pi(2↋) = ↋$; $\Pi d|2↋ = 861$.

30. $2^2 \times 3^2$; C_{20}; HC_6; $1^3 + 2^3 + 3^3$; 6^2; S_6; T_8; A_6; $\tau(30) = 9$; $\phi(30) = 10$; $\sigma(30) = 77$; $\pi(30) = ↋$; $\Pi d|30 = 346\,0000$.

31. p_{10}; SF_{20}; D_{25}; $1^2 + 6^2$; $\tau(31) = 2$; $\phi(31) = 30$; $\sigma(31) = 32$; $\pi(31) = 10$; $\Pi d|31 = 31$.

32. 2×17; SF_{21}; C_{21}; D_{26}; PSK_{11}; $\tau(32) = 4$; $\phi(32) = 16$; $\sigma(32) = 50$; $\pi(32) = 10$; $\Pi d|32 = ↊04$.

33. 3×11; SF_{22}; C_{22}; D_{27}; PSK_{12}; $\tau(33) = 4$; $\phi(33) = 20$; $\sigma(33) = 48$; $\pi(33) = 10$; $\Pi d|33 = ↊69$.

34. $2^3 \times 5$; C_{23}; CO_{14}; $2^2 + 6^2$; A_7; $\tau(34) = 8$; $\phi(34) = 14$; $\sigma(34) = 76$; $\pi(34) = 10$; $\Pi d|34 = ↊3\,5594$.

35. p_{11}; SF_{23}; D_{23}; $4^2 + 5^2$; $\tau(35) = 2$; $\phi(35) = 34$; $\sigma(35) = 36$; $\pi(35) = 11$; $\Pi d|35 = 35$.

36. $2 \times 3 \times 7$; A_8; SF_{24}; C_{24}; $\tau(36) = 8$; $\phi(36) = 10$; $\sigma(36) = 80$; $\pi(36) = 11$; $\Pi d|36 = 106\,0900$.

37. p_{12}; SF_{25}; D_{29}; $\tau(37) = 2$; $\phi(37) = 36$; $\sigma(37) = 38$; $\pi(37) = 12$; $\Pi d|37 = 37$.

38. $2^2 \times ↋$; C_{25}; $D_{2↊}$; $L_1 + L_2 + L_3 + L_4 + L_5 + L_6$; $\tau(38) = 6$; $\phi(38) = 18$; $\sigma(38) = 70$; $\pi(38) = 12$; $\Pi d|38 = 4\,1368$.

39. $3^2 \times 5$; C_{26}; $D_{2↋}$; $3^2 + 6^2$; T_9; $L_1 + L_3 + L_5 + L_7$; $\tau(39) = 6$; $\phi(39) = 20$; $\sigma(39) = 66$; $\pi(39) = 12$; $\Pi d|39 = 4\,4899$.

3↊. $2 \times 1↋$; C_{27}; D_{30}; SF_{26}; PSK_{13}; $\tau(3↊) = 4$; $\phi(3↊) = 1↊$; $\sigma(3↊) = 60$; $\pi(3↊) = 12$; $\Pi d|3↊ = 1284$.

3↋. p_{13}; D_{31}; SF_{27}; L_8; $\tau(3↋) = 2$; $\phi(3↋) = 3↊$; $\sigma(3↋) = 40$; $\pi(3↋) = 13$; $\Pi d|3↋ = 3↋$.

40. $2^4 \times 3$; C_{28}; A_9; HC_7; CO_{15}; $\tau(40) = ↊$; $\phi(40) = 14$; $\sigma(40) = ↊4$; $\pi(40) = 13$; $\Pi d|40 = 7140\,0000$.

41. 7^2; C_{29}; D_{32}; S_7; $\tau(41) = 3$; $\phi(41) = 36$; $\sigma(41) = 49$; $\pi(41) = 13$; $\Pi d|41 = 247$.

42. 2×5^2; $5^2 + 5^2$; $1^2 + a7^2$; $3^2 + 4^2 + 5^2$; $C_{2↊}$; D_{33}; $\tau(42) = 6$; $\phi(42) = 18$; $\sigma(42) = 79$; $\pi(42) = 13$; $\Pi d|42 = 6\,0408$.

43. 3×15; $C_{2↋}$; SF_{28}; D_{34}; PSK_{14}; CO_{16}; $\tau(43) = 4$; $\phi(43) = 28$; $\sigma(43) = 60$; $\pi(43) = 13$; $\Pi d|43 = 1609$.

44. $2^2 \times 11$; C_{30}; D_{35}; $4^2 + 6^2$; $\tau(44) = 6$; $\phi(44) = 20$; $\sigma(44) = 82$; $\pi(44) = 13$; $\Pi d|44 = 6\,9454$.

45. p_{14}; SF_{29}; D_{36}; $2^2 + 7^2$; $\tau(45) = 2$; $\phi(45) = 44$; $\sigma(45) = 46$; $\pi(45) = 14$; $\Pi d|45 = 45$.

46. 2×3^3; $3^3 + 3^3$; C_{31}; $A_{↊}$; $2^2 + 3^2 + 4^2 + 5^2$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8$; $\tau(46) = 8$; $\phi(46) = 16$; $\sigma(46) = ↊0$; $\pi(45) = 14$; $\Pi d|46 = 2↊2\,0900$.

47. $5 \times ↋$; C_{32}; $SF_{2↊}$; D_{37}; PSK_{15}; $F_{↊}$; $1^2 + 2^2 + 3^2 + 4^2 + 5^2$; $T_{↊}$; $F_1 + F_3 + F_5 + F_7 + F_9$; $\tau(47) = 4$; $\phi(47) = 34$; $\sigma(47) = 60$; $\pi(47) = 14$; $\Pi d|47 = 1901$.

48. $2^3 \times 7$; C_{33}; $A_{↋}$; $2^2 + 4^2 + 6^2$; TH_6; $\tau(48) = 8$; $\phi(48) = 20$; $\sigma(48) = ↊0$; $\pi(48) = 14$; $\Pi d|48 = 336\,3314$.

49. 3×17; C_{34}; $SF_{2↋}$; PSK_{16}; D_{38}; $\tau(49) = 4$; $\phi(49) = 30$; $\sigma(49) = 68$; $\pi(49) = 14$; $\Pi d|49 = 1↊69$.

4↊. 2×25; C_{35}; SF_{30}; D_{39}; PSK_{17}; $3^2 + 7^2$; $\tau(4↊) = 4$; $\phi(4↊) = 24$; $\sigma(4↊) = 76$; $\pi(4↊) = 14$; $\Pi d|4↊ = 1↋44$.

4↋. p_{15}; SF_{31}; $D_{3↊}$; $\tau(4↋) = 2$; $\phi(4↋) = 4↊$; $\sigma(4↋) = 50$; $\pi(4↋) = 15$; $\Pi d|4↋ = 4↋$.

50. $2^2 \times 3 \times 5$; HC_8; C_{36}; CO_{17}; A_{10}; Sexagesimal Base; $\tau(50) = 10$; $\phi(50) = 14$; $\sigma(50) = 120$; $\pi(50) = 15$; $\Pi d|50 = 90\,6100\,0000$.

51. p_{16}; SF_{32}; $D_{3↋}$; $5^2 + 6^2$; $\tau(51) = 2$; $\phi(51) = 50$; $\sigma(51) = 52$; $\pi(51) = 16$; $\Pi d|51 = 51$.

52. 2×27; C_{37}; SF_{33}; D_{40}; PSK_{18}; $\tau(52) = 4$; $\phi(52) = 26$; $\sigma(52) = 80$; $\pi(52) = 16$; $\Pi d|52 = 2284$.

53. $3^2 \times 7$; C_{38}; D_{41}; $\tau(53) = 6$; $\phi(53) = 30$; $\sigma(53) = 88$; $\pi(53) = 16$; $\Pi d|53 = 10\,0853$.

54. 2^6; 4^3; 8^2; $2^5 + 2^5$; C_{39}; CO_{18}; D_{42}; S_8; $\tau(54) = 7$; $\phi(54) = 28$; $\sigma(54) = ↊7$; $\pi(54) = 16$; $\Pi d|54 = 85\,1768$.

55. 5×11; $C_{3↊}$; SF_{34}; D_{43}; PSK_{19}; $1^2 + 8^2$; $4^2 + 7^2$; $1^3 + 4^3$; $1^6 + 2^6$; $\tau(55) = 4$; $\phi(55) = 40$; $\sigma(55) = 70$; $\pi(55) = 16$; $\Pi d|55 = 2541$.

56. $2 \times 3 \times ↋$; $C_{3↋}$; A_{11}; SF_{35}; $T_{↋}$; $\tau(56) = 8$; $\phi(56) = 18$; $\sigma(56) = 100$; $\pi(56) = 16$; $\Pi d|56 = 643\,0900$.

57. p_{17}; SF_{36}; D_{44}; $\tau(57) = 2$; $\phi(57) = 56$; $\sigma(57) = 58$; $\pi(57) = 17$; $\Pi d|57 = 57$.

58. $2^2 \times 15$; C_{40}; $2^2 + 8^2$; D_{45}; CO_{19}; $\tau(58) = 6$; $\phi(58) = 28$; $\sigma(58) = ↊6$; $\pi(58) = 17$; $\Pi d|58 = 13\,1↊68$.

59. 3 × 1↋; C_{41}; SF_{37}; D_{46}; $PSK_{1↊}$; $\tau(59) = 4$; $\phi(59) = 38$; $\sigma(59) = 80$; $\pi(59) = 17$; $\Pi d|59 = 2909$.

5↊. 2 × 5 × 7; C_{42}; SF_{38}; A_{12}; $\tau(5↊) = 8$; $\phi(5↊) = 20$; $\sigma(5↊) = 100$; $\pi(5↊) = 17$; $\Pi d|5↊ = 805\,↊814$.

5↋. p_{18}; SF_{39}; D_{47}; $\tau(5↋) = 2$; $\phi(5↋) = 5↊$; $\sigma(5↋) = 60$; $\pi(5↋) = 18$; $\Pi d|5↋ = 5↋$.

60. $2^3 \times 3^2$; C_{43}; A_{13}; $6^2 + 6^2$; $2^3 + 4^3$; $\tau(60) = 10$; $\phi(60) = 20$; $\sigma(60) = 143$; $\pi(60) = 18$; $\Pi d|60 = 230\,0000\,0000$.

61. p_{19}; $SF_{3↊}$; D_{48}; $3^2 + 8^2$; $L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7$; $\tau(61) = 2$; $\phi(61) = 60$; $\sigma(61) = 62$; $\pi(61) = 19$; $\Pi d|61 = 61$.

62. 2 × 31; C_{44}; $SF_{3↋}$; D_{49}; $PSK_{1↋}$; $5^2 + 7^2$; $\tau(62) = 4$; $\phi(62) = 30$; $\sigma(62) = 96$; $\pi(62) = 19$; $\Pi d|62 = 3204$.

63. 3×5^2; C_{45}; $D_{4↊}$; $L_2 + L_4 + L_6 + L_8$; $\tau(63) = 6$; $\phi(63) = 34$; $\sigma(63) = ↊4$; $\pi(63) = 19$; $\Pi d|63 = 18\,4183$.

64. $2^2 \times 17$; C_{46}; $D_{4↋}$; L_9; $\tau(64) = 6$; $\phi(64) = 30$; $\sigma(64) = ↋8$; $\pi(64) = 19$; $\Pi d|64 = 19\,2054$.

65. 7 × ↋; C_{47}; D_{50}; SF_{40}; PSK_{20}; $4^2 + 5^2 + 6^2$; $\tau(65) = 4$; $\phi(65) = 50$; $\sigma(65) = 80$; $\pi(65) = 19$; $\Pi d|65 = 3521$.

66. 2 × 3 × 11; C_{48}; A_{14}; SF_{41}; T_{10}; $\tau(66) = 8$; $\phi(66) = 20$; $\sigma(66) = 120$; $\pi(66) = 19$; $\Pi d|66 = 1049\,0900$.

67. $p_{1↊}$; SF_{42}; D_{51}; $\tau(67) = 2$; $\phi(67) = 66$; $\sigma(67) = 68$; $\pi(67) = 1↊$; $\Pi d|67 = 67$.

68. $2^4 \times 5$; C_{49}; A_{15}; $CO_{1↊}$; $4^2 + 8^2$; $\tau(68) = ↊$; $\phi(68) = 28$; $\sigma(68) = 136$; $\pi(68) = 1↊$; $\Pi d|68 = 7\,7548\,8368$.

69. 3^4; 9^2; S_9; $C_{4↊}$; D_{62}; $\tau(69) = 5$; $\phi(69) = 46$; $\sigma(69) = ↊1$; $\pi(69) = 1↊$; $\Pi d|69 = 2\,↊209$.

6↊. 2 × 35; $C_{4↋}$; SF_{43}; D_{53}; PSK_{21}; $1^4 + 3^4$; $1^2 + 9^2$; $\tau(6↊) = 4$; $\phi(6↊) = 34$; $\sigma(6↊) = ↊6$; $\pi(6↊) = 1↊$; $\Pi d|6↊ = 3↊84$.

6↋. $p_{1↋}$; SF_{44}; D_{54}; $\tau(6↋) = 2$; $\phi(6↋) = 6↊$; $\sigma(6↋) = 70$; $\pi(6↋) = 1↋$; $\Pi d|6↋ = 6↋$.

70. $2^2 \times 3 \times 7$; C_{50}; A_{16}; TH_7; $1^2 + 3^2 + 5^2 + 7^2$; $\tau(70) = 10$; $\phi(70) = 20$; $\sigma(70) = 168$;p $\pi(70) = 1↋$; $\Pi d|70 = 581\,0100\,0000$.

71. 5 × 15; C_{51}; SF_{45}; D_{55}; PSK_{22}; $6^2 + 7^2$; $2^2 + 9^2$; $CO_{1↋}$; $\tau(71) = 4$; $\phi(71) = 54$; $\sigma(71) = 90$; $\pi(71) = 1↋$; $\Pi d|71 = 4221$.

72. 2 × 37; C_{52}; SF_{46}; D_{56}; PSK_{23}; $3^2 + 4^2 + 5^2 + 6^2$; $\tau(72) = 4$; $\phi(72) = 36$; $\sigma(72) = ↋0$; $\pi(72) = 1↋$; $\Pi d|72 = 4344$.

73. 3 × 25; C_{53}; SF_{47}; D_{57}; PSK_{24}; $\tau(73) = 4$; $\phi(73) = 48$; $\sigma(73) = ↊0$; $\pi(73) = 1↋$; $\Pi d|73 = 4469$.

74. $2^3 \times$ ↋; C_{54}; A_{17}; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9$; $F_2 + F_4 + F_6 + F_8 + F_{↊}$; $\tau(74) = 8$; $\phi(74) = 34$; $\sigma(74) = 130$; $\pi(74) = 1↋$; $\Pi d|74 = 1810\,0714$.

75. p_{20}; SF_{48}; D_{58}; $F_{↋}$; $5^2 + 8^2$; $\tau(75) = 2$; $\phi(75) = 74$; $\sigma(75) = 76$; $\pi(75) = 20$; $\Pi d|75 = 75$.

76. $2 \times 3^2 \times 5$; C_{55}; A_{18}; $3^2 + 9^2$; $2^2 + 3^2 + 4^2 + 5^2 + 6^2$; $\tau(76) = 10$; $\phi(76) = 20$; $\sigma(76) = 176$; $\pi(76) = 20$; $\Pi d|76 = 86↋\,↋662\,3000$.

77. 7 × 11; C_{56}; SF_{49}; D_{59}; PSK_{25}; $3^3 + 4^3$; $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$; T_{11}; $\tau(77) = 4$; $\phi(77) = 60$; $\sigma(77) = 94$; $\pi(77) = 20$; $\Pi d|77 = 4961$.

78. $2^2 \times$ 1↋; aC_{57}; $D_{5↊}$; $\tau(78) = 6$; $\phi(78) = 38$; $\sigma(78) = 120$; $\pi(78) = 20$; $\Pi d|78 = 31\,6768$.

79. 3 × 27; C_{58}; $SF_{4↊}$; $D_{5↋}$; PSK_{26}; $\tau(79) = 4$; $\phi(79) = 50$; $\sigma(79) = ↊8$; $\pi(79) = 20$; $\Pi d|79 = 5009$.

7↊. 2 × 3↋; C_{59}; $SF_{4↋}$; D_{60}; PSK_{27}; $\tau(7↊) = 4$; $\phi(7↊) = 3↊$; $\sigma(7↊) = 100$; $\pi(7↊) = 20$; $\Pi d|7↊ = 5144$.

7↋. 5 × 17; $C_{5↊}$; SF_{50}; D_{61}; PSK_{28}; $\tau(7↋) = 4$; $\phi(7↋) = 60$; $\sigma(7↋) = ↊0$; $\pi(7↋) = 20$; $\Pi d|7↋ = 5281$.

80. $2^5 \times 3$; $C_{5↋}$; A_{19}; CO_{20}; $\tau(80) = 10$; $\phi(80) = 28$; $\sigma(80) = 190$; $\pi(80) = 20$; $\Pi d|80 = 1078\,5400\,0000$.

81. p_{21}; $4^2 + 9^2$; $2^4 + 3^4$; SF_{51}; D_{62}; $\tau(81) = 2$; $\phi(81) = 80$; $\sigma(81) = 82$; $\pi(81) = 21$; $\Pi d|81 = 81$.

82. 2×7^2; $7^2 + 7^2$; C_{60}; D_{63}; $1^4 + 2^4 + 3^4$; $\tau(82) = 6$; $\phi(82) = 36$; $\sigma(82) = 123$; $\pi(82) = 21$; $\Pi d|82 = 39\,4808$.

83. $3^2 \times$ ↋; C_{61}; D_{64}; $2^3 + 3^3 + 4^3$; $\tau(83) = 6$; $\phi(83) = 50$; $\sigma(83) = 110$; $\pi(83) = 21$; $\Pi d|83 = 3↊\,9623$.

84. $2^2\times5^2$; C_{62}; $A_{1↊}$; 6^2+8^2; $↊^2$; $S_{↊}$; $1^3+2^3+3^3+4^3$; $\tau(84)=9$; $\phi(84)=34$; $\sigma(84)=161$; $\pi(84)=21$; $\Pi d|84=2\,3↊↊9\,3854$.

85. p_{22}; SF_{52}; D_{65}; $1^2+↊^2$; $\tau(85)=2$; $\phi(85)=84$; $\sigma(85)=86$; $\pi(85)=22$; $\Pi d|85=85$.

86. $2\times3\times15$; C_{63}; SF_{53}; $A_{1↋}$; CO_{21}; $\tau(86)=8$; $\phi(86)=28$; $\sigma(86)=160$; $\pi(86)=22$; $\Pi d|86=3030\,0900$.

87. p_{23}; SF_{54}; D_{66}; $\tau(87)=2$; $\phi(87)=86$; $\sigma(87)=88$; $\pi(87)=23$; $\Pi d|87=87$.

88. $2^3\times11$; C_{64}; A_{20}; $2^2+↊^2$; $\tau(88)=8$; $\phi(88)=40$; $\sigma(88)=156$; $\pi(88)=23$; $\Pi d|88=3321\,8194$.

89. $3\times5\times7$; C_{65}; SF_{55}; D_{67}; T_{12}; $\tau(89)=8$; $\phi(89)=40$; $\sigma(89)=140$; $\pi(89)=23$; $\Pi d|89=3485\,9969$.

8↊. 2×45; C_{66}; SF_{56}; D_{68}; PSK_{29}; 5^2+9^2; $\tau(8↊)=4$; $\phi(8↊)=44$; $\sigma(8↊)=116$; $\pi(8↊)=23$; $\Pi d|8↊=6604$.

8↋. p_{24}; SF_{57}; D_{69}; $\tau(8↋)=2$; $\phi(8↋)=8↊$; $\sigma(8↋)=90$; $\pi(8↋)=24$; $\Pi d|8↋=8↋$.

90. $2^2\times3^3$; C_{67}; A_{21}; $\tau(90)=10$; $\phi(90)=30$; $\sigma(90)=1↋4$; $\pi(90)=24$; $\Pi d|90=2176\,6900\,0000$.

91. p_{25}; SF_{58}; $D_{6↊}$; $3^2+↊^2$; $\tau(91)=2$; $\phi(91)=90$; $\sigma(91)=92$; $\pi(91)=25$; $\Pi d|91=91$.

92. $2\times5\times↋$; C_{68}; SF_{59}; $D_{6↋}$; $5^2+6^2+7^2$; $\tau(92)=8$; $\phi(92)=34$; $\sigma(92)=160$; $\pi(92)=25$; $\Pi d|92=4104\,8014$.

93. 3×31; C_{69}; $SF_{5↊}$; D_{70}; $PSK_{2↊}$; $\tau(93)=4$; $\phi(93)=60$; $\sigma(93)=108$; $\pi(93)=25$; $\Pi d|93=7169$.

94. $2^4\times7$; $C_{6↊}$; A_{22}; $\tau(94)=↊$; $\phi(94)=40$; $\sigma(94)=188$; $\pi(94)=25$; $\Pi d|94=34\,↋↊06\,8714$.

95. p_{26}; $SF_{5↋}$; D_{71}; 7^2+8^2; $\tau(95)=2$; $\phi(95)=94$; $\sigma(95)=96$; $\pi(95)=26$; $\Pi d|95=95$.

96. $2\times3\times17$; $C_{6↋}$; SF_{60}; A_{23}; $\tau(96)=8$; $\phi(96)=30$; $\sigma(96)=180$; $\pi(96)=26$; $\Pi d|96=4869\,0900$.

97. $5\times1↋$; C_{70}; SF_{61}; D_{72}; $PSK_{2↋}$; $\tau(97)=4$; $\phi(97)=74$; $\sigma(97)=100$; $\pi(97)=26$; $\Pi d|97=77↊1$.

98. $2^2\times25$; C_{71}; D_{73}; $4^2+↊^2$; $\tau(98)=6$; $\phi(98)=48$; $\sigma(98)=156$; $\pi(98)=26$; $\Pi d|98=63\,3368$.

99. $3^2\times11$; C_{72}; D_{74}; 6^2+9^2; $\tau(99)=6$; $\phi(99)=60$; $\sigma(99)=132$; $\pi(99)=26$; $\Pi d|99=65\,2↊39$.

9↊. $2\times4↋$; C_{73}; SF_{62}; D_{75}; PSK_{30}; $\tau(9↊)=4$; $\phi(9↊)=4↊$; $\sigma(9↊)=130$; $\pi(9↊)=26$; $\Pi d|9↊=8084$.

9↋. 7×17; C_{74}; SF_{63}; D_{76}; PSK_{31}; $\tau(9↋)=4$; $\phi(9↋)=80$; $\sigma(9↋)=100$; $\pi(9↋)=26$; $\Pi d|9↋=8241$.

↊0. $2^3\times3\times5$; $5!$; C_{75}; A_{24}; MP_1 (3-Perfect); T_{13}; CO_{22}; TH_8; $L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8$; $\tau(↊0)=14$; $\phi(↊0)=28$; $\sigma(↊0)=260$; $\pi(↊0)=26$; $\Pi d|↊0=295↊\,6454\,0000\,0000$.

↊1. $↋^2$; C_{76}; D_{77}; $S_{↋}$; $L_1+L_3+L_5+L_7+L_9$; $\tau(↊1)=3$; $\phi(↊1)=92$; $\sigma(↊1)=↋1$; $\pi(↊1)=26$; $\Pi d|↊1=92↋$.

↊2. 2×51; C_{77}; SF_{64}; D_{78}; PSK_{32}; $1^2+↋^2$; $\tau(↊2)=4$; $\phi(↊2)=50$; $\sigma(↊2)=136$; $\pi(↊2)=26$; $\Pi d|↊2=8744$.

↊3. 3×35; C_{78}; SF_{65}; D_{79}; PSK_{33}; $L_{↊}$; $\tau(↊3)=4$; $\phi(↊3)=68$; $\sigma(↊3)=120$; $\pi(↊3)=26$; $\Pi d|↊3=8909$.

↊4. $2^2\times27$; C_{79}; $D_{7↊}$; $\tau(↊4)=6$; $\phi(↊4)=50$; $\sigma(↊4)=168$; $\pi(↊4)=26$; $\Pi d|↊4=77\,↋454$.

↊5. 5^3; $C_{7↊}$; $D_{7↋}$; PSK_{34}; $5^2+↊^2$; $2^2+↋^2$; $\tau(↊5)=3$; $\phi(↊5)=84$; $\sigma(↊5)=110$; $\pi(↊5)=26$; $\Pi d|↊5=9061$.

↊6. $2\times3^2\times7$; $C_{7↋}$; A_{25}; 1^3+5^3; $4^2+5^2+6^2+7^2$; $\tau(↊6)=10$; $\phi(↊6)=30$; $\sigma(↊6)=220$; $\pi(↊6)=26$; $\Pi d|↊6=5476\,2778\,3000$.

↊7. p_{27}; M_7; SF_{66}; D_{80}; $\tau(↊7)=2$; $\phi(↊7)=↊6$; $\sigma(↊7)=↊8$; $\pi(↊7)=27$; $\Pi d|↊7=↊7$.

↊8. 2^7; 8^2+8^2; 4^3+4^3; 2^6+2^6; C_{80}; D_{81}; CO_{23}; $\tau(↊8)=8$; $\phi(↊8)=54$; $\sigma(↊8)=193$; $\pi(↊8)=27$; $\Pi d|↊8=75↊9\,4714$.

↊9. 3×37; C_{81}; SF_{67}; D_{82}; PSK_{35}; $\tau(↊9)=4$; $\phi(↊9)=70$; $\sigma(↊9)=128$; $\pi(↊9)=27$; $\Pi d|↊9=9769$.

↊↊. $2\times5\times11$; C_{82}; SF_{68}; D_{83}; 7^2+9^2; $3^2+↋^2$; $\tau(↊↊)=8$; $\phi(↊↊)=40$; $\sigma(↊↊)=190$; $\pi(↊↊)=27$; $\Pi d|↊↊=7↋79\,7694$.

↊↋. p_{28}; SF_{69}; $\tau(↊↋)=2$; $\phi(↊↋)=↊↊$; $\sigma(↊↋)=↋0$; $\pi(↊↋)=28$; $\Pi d|↊↋=↊↋$.

↋0. $2^2 \times 3 \times$ ↋; C_{83}; A_{26}; τ(↋0) = 10; ϕ(↋0) = 34; σ(↋0) = 240; π(↋0) = 28; $\Pi d|$↋0 = 715261000000.$2^2 \times 3 \times$ ↋; C_{83}; A_{26}; τ(↋0) = 10; ϕ(↋0) = 34; σ(↋0) = 240; π(↋0) = 28; $\Pi d|$↋0 = 7152 6100 0000.

↋1. 7×17; C_{84}; $SF_{6↊}$; D_{85}; PSK_{36}; 2^3+5^3; τ(↋1) = 4; ϕ(↋1) = 90; σ(↋1) = 114; π(↋1) = 28; $\Pi d|$↋1 = ↊2↊1.

↋2. 2×57; C_{85}; $SF_{6↋}$; D_{86}; PSK_{37}; τ(↋2) = 4; ϕ(↋2) = 56; σ(↋2) = 150; π(↋2) = 28; $\Pi d|$↋2 = ↊484.

↋3. $3^3 \times 5$; C_{86}; D_{87}; $3^2+4^2+5^2+6^2+7^2$; τ(↋3) = 8; ϕ(↋3) = 60; σ(↋3) = 180; π(↋3) = 28; $\Pi d|$↋3 = 932↊ 0969.

↋4. $2^3 \times 15$; C_{87}; D_{88}; CO_{24}; 6^2+↊^{2}; T_{14}; τ(↋4) = 8; ϕ(↋4) = 54; σ(↋4) = 180; π(↋4) = 28; $\Pi d|$↋4 = 9669 ↋854.

↋5. p_{29}; SF_{70}; D_{89}; $4^2 +$ ↋^{2}; τ(↋5) = 2; ϕ(↋5) = ↋4; σ(↋5) = ↋6; π(↋5) = 29; $\Pi d|$↋5 = ↋5.

↋6. $2 \times 3 \times 1$↋; C_{88}; SF_{71}; A_{27}; τ(↋6) = 8; ϕ(↋6) = 38; σ(↋6) = 200; π(↋6) = 29; $\Pi d|$↋6 = ↊156 0900.

↋7. $p_{2↊}$; SF_{72}; $D_{8↊}$; $2^2+3^2+4^2+5^2+6^2+7^2$; τ(↋7) = 2; ϕ(↋7) = ↋6; σ(↋7) = ↋8; π(↋7) = 2↊; $\Pi d|$↋7 = ↋7.

↋8. $2^2 \times 5 \times 7$; C_{89}; A_{28}; $1^2+2^2+3^2+4^2+5^2+6^2+7^2$; τ(↋8) = 10; ϕ(↋8) = 40; σ(↋8) = 240; π(↋8) = 2↊; $\Pi d|$↋8 = ↊173 3645 6454.

↋9. 3×3↋; $C_{8↊}$; SF_{73}; $D_{8↋}$; PSK_{38}; τ(↋9) = 4; ϕ(↋9) = 78; σ(↋9) = 140; π(↋9) = 2↊; $\Pi d|$↋9 = ↋609.

↋↊. 2×5↋; $C_{8↋}$; SF_{74}; D_{90}; PSK_{39}; τ(↋↊) = 4; ϕ(↋↊) = 5↊; σ(↋↊) = 160; π(↋↊) = 2↊; $\Pi d|$↋↊ = ↋804.

↋↋. ↋ $\times 11$; C_{90}; SF_{75}; D_{91}; $PSK_{3↊}$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9 + F_{↊}$; τ(↋↋) = 4; ϕ(↋↋) = ↊0; σ(↋↋) = 120; π(↋↋) = 2↊; $\Pi d|$↋↋ = ↋↊01.

100. $2^4 \times 3^2$; One Gross; C_{91}; A_{29}; F_{10}; 10^2; $4! + 5!$; $3! + 4!$; $F_1 + F_3 + F_5 + F_7 + F_9 + F_{↋}$; S_{10}; $\tau(100) = 13$; $\phi(100) = 40$; $\sigma(100) = 297$; $\pi(100) =$ 2↊; $\Pi d|100 = 1000\,0000\,0000, 0000$.

This document was originally published in *The Duodecimal Bulletin* 38:3 (WN 74), pp. 9–1↋. The original printing in the *Bulletin* contained a printing error, which combined the paragraphs for 9↊ and 9↋ into one, labelled 9↊, and did not have a separate paragraph for 9↋. This has been fixed; this repair required a few additions. Specifically, ϕ(9↊), σ(9↊), π(9↊), $\Pi d|$9↊, τ(9↋), and ϕ(9↋) had to be determined by consulting the OEIS, since the original printing did not include them. Otherwise, however, all the calculations were done by Prof. Jay Schiffman, DSA. The Dozenal Society of America is proud to present this entirely newly typeset and corrected version to the world, August 1201.