Manual of the Dozenal System

compiled by the

Dozenal Society of America

Numeration Throughout is Dozenal (Base Twelve)

0 1 2 3 4 5 6 7 8 9 ↊ ↋ 10

where ↊ is ten, ↋ is eleven, and 10 is a dozen

Dozenal numeration is a system of thinking of numbers in twelves, rather than tens. Twelve is a much more versatile number, having four even divisors—2, 3, 4, and 6—as opposed to only two for ten. This means that such hatefulness as "0.333. . . " for $1/3$ and "0.1666. . . " for $1/6$ are things of the past, replaced by easy "0;4" (four twelfths) and "0;2" (two twelfths).

In dozenal, counting goes "one, two, three, four, five, six, seven, eight, nine, ten, elv, dozen; dozen one, dozen two, dozen three, dozen four, dozen five, dozen six, dozen seven, dozen eight, dozen nine, dozen ten, dozen elv, two dozen, two dozen one. . . " It's written as such: 1, 2, 3, 4, 5, 6, 7, 8, 9, ↊, ↋, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1↊, 1↋, 20, 21. . .

Dozenal counting is at once much more efficient and much easier than decimal counting, and takes only a little bit of time to get used to. Further information can be had from the dozenal societies, as well as in many other places on the Internet.

The Dozenal Society of America
`http://www.dozenal.org`

The Dozenal Society of Great Britain
`http://www.dozenalsociety.org.uk`

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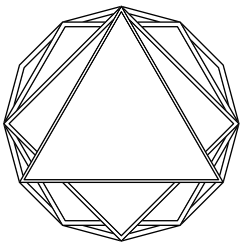


A Quick FAQ

1. **What is "dozenal"?**
"Dozenal" is the name given to counting numbers by *twelves*, rather than by *tens*. Another name for doing this is **duodecimal**; other, older, and rather less common names are **duodenal**, **duodenary**, and **uncial**. But **dozenal** is favored by most of those who actually count by twelves. In other words, dozenal is counting in base twelve.

2. **How do most people count now?**
Most of the world counts using the *decimal* system; that is, it counts by tens.

3. **Why should we bother considering anything other than the decimal (counting by ten) system?**
We should consider bases other than decimal because, despite being quite widespread, ten is a suboptimal base. It is difficult to use even fractions in base ten; only fifths and halves come out to one digit (0.2 and 0.5), and thirds are infinitely repeating (0.3333…). No base can have only even fractions; but we can do better than ten. In other words, ten is literally *un-satis-factory* because it has *not-enough-factors*.

4. **Why is twelve a better base than ten?**
Twelve is a better base than ten because it has more factors; more precisely, it has twice as many factors. While ten has only 2 and 5, twelve has 2, 3, 4, and 6. This means that halves, thirds, quarters, and sixths all come out to even, single-digit fractions (0;6, 0;4, 0;3, and 0;2). Furthermore, eighths are only two digits (0;16), as are sixteenths (0;09) and ninths (0;14). Finally, this abundance of factors makes the multiplication table for dozenal much easier to learn and use than that in decimal.

5. What about other numbers, like computer bases eight and sixteen?

Eight and sixteen are fine bases for computers because computers use *binary* (base two), and both of these numbers are powers of two. However, they are poor bases for the human mind. Compared to twelve, they are factor-poor; in fact, twelve has more factors than any number less than twenty-four, making it the most promising candidate for a human base.

6. Is it really possible to change to base twelve?

It *is* really possible to change to base twelve; many people have done so, and there is no reason that a whole society could not, as well. Societies have undergone much greater changes — for example, changes from one alphabet to another, or from Roman numerals to Hindu-Arabic numerals — without great upheaval, and reaped immense benefits from these changes.

7. What is the chief difficulty in changing to dozenal?

The chief difficulty in changing to dozenal is remembering to read numbers in twelves, rather than tens. For example, seeing "10" and not thinking "ten and zero," but thinking "twelve and zero" instead. This can be done in a matter of a few hours to a few days; after this, a little practice with the dozenal multiplication tables will suffice to have one quite conversant in dozenals, to the point that one will be quite frustrated when forced to use decimal again.

8. So how do you write "ten" and "eleven"?

"Ten" and "eleven" can be written in many different ways. For a time the Dozenal Society of America used "*" and "#", but has since returned to a stylized version of "X" and "E". In plain text, "X" and "E" are most common, though "A" and "B" are also common. In this booklet we use "Ɛ" and "Ɛ", which were devised for this purpose by Sir Isaac Pitman.

9. How does one talk about numbers when one is no longer naming numbers by tens?

One can name numbers when counting in twelves in many different ways, as well. Traditional English can do so up to 10^3, where twelve is "dozen," a dozen dozen is a "gross," and a dozen gross is a "great-gross". The Dozenal Society of America has often used the do-gro-mo system, which is described later in this booklet. A system of growing popularity is Systematic Dozenal Nomenclature (SDN), which is also described later in this booklet.

Ɛ. What about zeroes and just moving the decimal point?

Adding and taking away zeroes, and otherwise just moving the decimal point, works identically in dozenal as in decimal. One just needs to remember that 10 means *twelve*, not ten. So 0;6 × 10 is 6, simply moving that dozenal point one place; but we're multiplying six twelfths by twelve, rather than six tenths by ten. This benefit exists regardless of the base being used.

Ɛ. Is counting by twelves (dozenal) a new idea?

Dozenal (counting by twelves) is *not* a new idea, by any means. The first serious discussion of beginning to count by dozens seems to have been around the time of the French Revolution, when the committee which ended up developing the awkward metric system considered switching to the dozenal base instead. Leclerc, the famous Comte de Buffon, wrote to this effect in 1041 (decimal, 1777); this was echoed by Laplace in 109Ɛ (decimal, 1846). English authors like John Playfair (1067, or 1807) and Thomas Leech (10Ɛ6, or 1866) also wrote movingly in favor of dozenal. In more modern times, the writings of F. Emerson Andrews gave rise to the Dozenal Society of America, which along with the Dozenal Society of Great Britain has been carrying the torch ever since.

10. How can we tell when a number is dozenal?

We can tell that a figure is dozenal in several ways. First, if the number contains either Ɛ or Ɛ, then it must be dozenal. Second, we have already seen that dozenal often uses " ; " as its "decimal point" (more properly called a *fraction point*); if a number uses " ; " rather than the decimal " . ", then it must be dozenal. Sometimes we do this even with integers; for example, "10;" is dozenal (twelve), while "10." is decimal (ten). Finally, sometimes we simply state that a number is dozenal or decimal. In this booklet, for example, numbers should be assumed to be dozenal unless stated otherwise.

11. What are some examples of dozenal being better?

Some prominent examples of dozenal superiority are:

a. Packing. Dozenalists have noted that there are more ways to pack a dozen items than ten; and in fact, all of these ways are more efficient in terms of packing material and volume than any way of packing ten.

b. Time. Since an hour is divided into twelve equal parts of five minutes each, dozenal is naturally suited to it. Telling the time is a simple matter of reading the number the hands are pointed to; for example, twenty after four is simply "4;4", twenty minutes being the four on the clock. Parts of an hour are simply the fractional value: 3 is a quarter of twelve, and 4;3 is a quarter after four. And while half-past being when the hand points to 6 is odd in decimal, it makes perfect sense in dozenal.

c. Parts of the whole. The dozenal analog of percentage is much simpler and more useful than percentage is, being 100 dozenal (144.) is much more versatile than 84 (100.) is.

Dozenal makes nearly everything easier; but these are some of the most prominent examples.

A Little about Numbers

Numbers? For nerds. Certainly non-mathematicians don't have to think about them in any depth. Most people try to think about numbers as little as possible. Run the budget, balance the checkbook, make sure the paycheck is the right amount, and do the taxes in April, then forget about them. And there's certainly nothing wrong with that; nobody *has* to deal with numbers any more than he needs to.

Still, everybody needs to deal with them *sometimes.* And for that reason, everybody has an interest in how numbers work; specifically, everybody has an interest in making sure that our numbers are as easy to use and as logically organized as possible. Nobody would want to use Roman numerals when balancing their checkbook, for example; it would make an already unpleasant task into a nightmare.

Our numbers are written using a system of *place notation.* Place notation simply means that the location of a digit in a given number dictates what that digit really means. For example, the "4" in "94" and the "4" in "49" don't mean the same thing. Why not? Because they're in different places.

To use place notation, we first select a *base*; that is, a number on which the meaning of all digits will depend. In our current system, we use ten as a base. We also need to have a number of digits equal to that base; since we use ten, we need to have ten digits. The digits we all use are well-known; we'll list them here for illustration:

$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$$

Considered all by themselves, the meaning of each of these digits is simple. "4," by itself, simply means "4 units of one," or four. Each digit means something similar as we go up the line; "6" means six, "8" means eight, "9" means nine. But then we run out of digits; so how do we write ten without using letters?

We use *two* digits, and write "10."

It seems so elementary, but when one sits back to think about it, pretending that one hasn't always thought about numbers in this way, its elegance is really striking. Here we have two digits, a "1," which means one, and a "0," which means zero. But if we put them together, they mean neither one nor zero, but ten. And only if we put them together in a particular way; if we wrote "01," we wouldn't be talking about ten anymore. Only "10" means ten, nothing else (as far as our current system is concerned).

It means this because each successive digit to the left means its normal value *multiplied by the appropriate multiple of ten, our base.* The first digit, farthest to the right, means only itself, its normal meaning. So in "10," the "0" means exactly that, zero. The "1," however, is more complicated. Since it's the digit *second* farthest to the right, it means itself, one, *multiplied by ten.* So it means not one, but *ten times one*, or ten. Add the ten that the "1" means and the zero that the "0" means and we've got ten.

This system easily lends itself to writing numbers of completely arbitrary complexity. Take another string of digits; say, "4,657." We have four digits; only one means itself. Each digit further to the left means the digit itself multiplied by the next multiple of ten. Visually, here's is what we're doing with the numbers:

$$\begin{array}{cccc} 4 & 6 & 5 & 7 \\ \times & \times & \times & \times \\ 10^3 & 10^2 & 10^1 & 10^0 \\ = & = & = & = \\ 4000 & 600 & 50 & 7 \end{array}$$

And finally sum the bottom row to get 4,657.

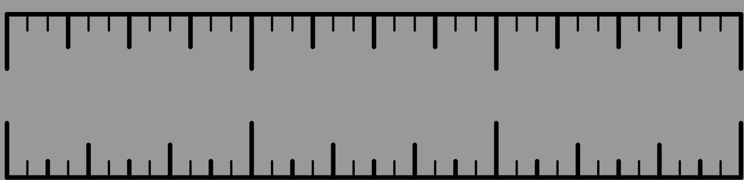
This is exactly what we're doing in our heads every time we read a number; we've just done it so frequently for so long that we don't even notice ourselves doing it anymore. But that's how the system works, and it's a beautiful system, complex and yet easy to use.

Surely, with this incredibly beautiful system, we've reached the peak of perfection in our number system. How could anything be easier or more logical than this?

We're used to our number system, and many people have a difficult time even imagining that there could possibly be anything else. But there is a better system. Not a better system than place notation; everyone agrees that place notation is the best way to write numbers, at least when one intends to do any calculations with them. But our system is not just the place notation system, but the place notation system *with a base of ten.* The fact is that *any* whole number greater than one could be chosen as a base. What makes ten so special?

Absolutely nothing. Other numbers would be better than ten, and would make our calculations quicker and easier, even in these days of pocket calculators and computers. The best base of all, in fact, would be the number twelve; adopting a number system with the base of twelve would make everything we do, from the complex calculations of mathematicians to the basic number-juggling of harried tax preparers, easier to do and quicker to figure.

Why would a base of twelve be better? Let's take a look.



Dozenal's flexibility allows us to have many different easy divisions; as in this ruler, with thirds divided into quarters on the bottom but quarters divided into thirds on the top.

How to Count by Twelves

Before we do that, though, we should probably look a little at how to count by twelves; knowing how to do it will make it much easier to talk about why we should bother actually doing it.

Just as before, the position of the digit in a number tells us what it means; the "4" in "94" and the "4" in "49" are very different numbers. However, rather than multiplying again by *ten* as we move to the left, we multiply again by *twelve*. This means that we need single digits for the numbers ten and eleven; while many have been proposed, let's just use "↊" for ten and "↋" for eleven, at least for now. These are the digits chosen by Sir Isaac Pitman long ago, and still are commonly used by dozenalists today.

When we get to eleven, then, we're out of digits; we have to move one place to the left. So we write a "1" in the next place to the left, and a "0" in the ones place, like so: "10." It is hard, at first, to realize that "10" doesn't necessarily mean "ten." What it really means is "one group of some number, and zero units." In decimal, this means "one ten and zero ones"; in dozenal, it means "one dozen and zero ones." And so it goes:

1	2	3	4	5	6	7	8	9	↊	↋	10
11	12	13	14	15	16	17	18	19	1↊	1↋	20
21	22	23	24	25	26	27	28	29	2↊	2↋	30
						…					
↊1	↊2	↊3	↊4	↊5	↊6	↊7	↊8	↊9	↊↊	↊↋	↋0
↋1	↋2	↋3	↋4	↋5	↋6	↋7	↋8	↋9	↋↊	↋↋	

Then, just as we do in decimal when we reach 99, we proceed to three digits, like so: ↋↊, ↋↋, 100, 101, 102,…109, 10↊, 10↋, 110, and so on, just so long as we care to continue the exercise.

That's how we count with digits; we can count with words similarly: one (1), two (2), three (3), four (4), five (5), six (6), seven (7), eight (8), nine (9), ten (↊), eleven (↋), dozen (10); dozen-one (11), dozen-two (12), dozen-three (13), dozen-four (14), dozen-five (15), dozen-six (16), dozen-seven (17), dozen-eight (18), dozen-nine (19), dozen-ten (1↊), dozen-eleven (1↋), two dozen (20); two dozen one (21), two dozen two (22),... eleven dozen eight (↋8), eleven dozen nine (↋9), eleven dozen ten (↋↊), eleven dozen eleven (↋↋), one gross (100); one gross one (101), one gross two (102), one gross three (103), one gross four (104), one gross five (105),... one gross nine (109), one gross ten (10↊), one gross eleven (10↋), one gross one dozen (110); one gross one dozen one (111), one gross one dozen two (112), and so on, as long as we wish.

Some people don't like the symbols "↊" and "↋" for ten and eleven; and some people don't even like the names "ten" and "eleven." For many dozens of years, the Dozenal Society of America has used cursive variants of "X" (the Roman numeral ten) and "E" (the initial letter of "eleven") for ten and eleven, and called them "dek" and "el." This is a very common convention, as is "↊" and "↋." Eleven has also been called "elf," "elv," "len," "lef," and "lev." Whichever one you decide you like best will, of course, work just fine; people can use the symbols and names that they like, and over time, as dozenal gains more acceptance, conventions will arise and eventually be used by all.

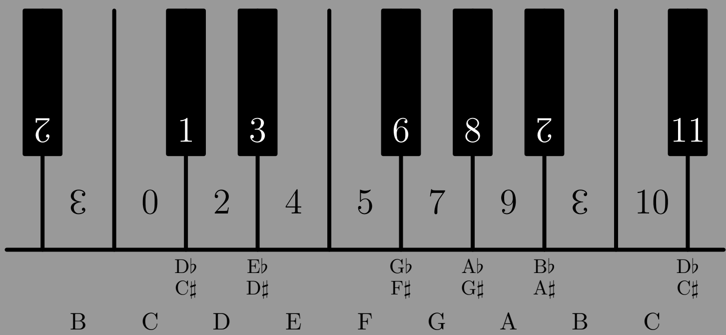
In plain text, though, it should be noted that "X" and "E," or "A" and "B," are by far the most common conventions. Sometimes one even sees "*" and "#"!

For fractional parts, we use the same process we use in decimal. Some people simply use a period (the "decimal point") for this, but most dozenalists use a semicolon, which we call the "dozenal point" or "Humphrey point" (after H. K. Humphrey, who pioneered its use). While in decimal the period is pronounced "point," in dozenal the point is usually pronounced "dit," no matter how it's written.

$$0;6 = \text{“zero dit six”} = 1/2$$

Remember here that instead of doing parts of ten, we’re doing parts of twelve; so “0;6” is “zero ones and six twelfths,” not “zero ones and six tenths” as “0.6” would be in decimal. Six twelfths is, of course, a half.

Finally, many people find it cumbersome to talk in dozens and grosses. No doubt; but there are systems which make it not only just as easy to talk in dozens as it is to talk in tens, but even easier. We’ll discuss these in time[1]; for now, we’ll stick with plain old cumbersome basic English, which gives us a foundation upon which we can approach our next subject.



Many dozenalists have noted that in music, the so-called “octave” actually has twelve, *not eight and certainly not ten, notes. This has produced several complete systems for writing music which are vastly easier than our current one, but which are out of the reach of decimal.*

[1]See page 29.

Why Twelve?

So why does twelve make such a better base? Simply put, twelve is better because it has more factors; that is to say, twelve can be divided evenly by more numbers than ten can. In fact, twelve can be divided evenly by *twice as many* numbers as ten can. Ten can be divided evenly only by 2 and 5; twelve can be divided evenly by 2, 3, 4, and 6.

So what, you ask? First of all, it makes the multiplication tables *much easier*. Let's take a look at that now:

1	2	3	4	5	6	7	8	9	↊	↋	10
2	4	6	8	↊	10	12	14	16	18	1↊	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	↊	13	18	21	26	2↋	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2↋	36	41	48	53	5↊	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
↊	18	26	34	42	50	5↊	68	76	84	92	↊0
↋	1↊	29	38	47	56	65	74	83	92	↊1	↋0
10	20	30	40	50	60	70	80	90	↊0	↋0	100

Notice the cycles in almost every single one of the rows:

Twos Cycles through the even numbers, then increments the next digit to the left and starts again.

Threes A four-item cycle: 3, 6, 9, 10; 13, 16, 19, 20; 23, 26, 29, 30.

Fours A three-item cycle: 4, 8, 10; 14, 18, 20; 24, 28, 30; and so forth.

Sixes A two-item cycle: 6, 10; 16, 20; 26, 30; and so forth.

Eights A three-item cycle: 8, 4, 0; 8, 4, 0.

Nines A four-item cycle: 9, 6, 3, 0; 9, 6, 3, 0.

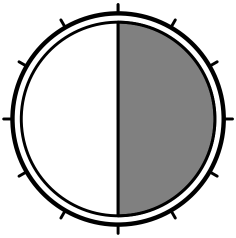
Elevens The rightmost digit counts down from ↋, with the leftmost digit increased by one each time.

By comparison, the decimal table is bereft of patterns. The twos and the fives are quite easy; but the threes and the nines, for example, are *ten-item cycles*! Every third number is a multiple of three; and yet decimal handles threes horribly. The fours, sixes, and eights are *five-item cycles*. Elevens do come out fairly neatly; but technically these are not really part of the decimal tables, being one greater than the base, and furthermore 11 (one dozen one) has exactly the same pattern in dozenal.

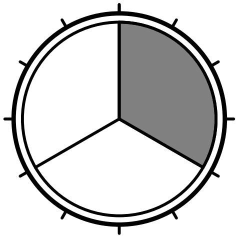
Dozenal really shows its superiority, however, in the *fractions*. Because it has so many more factors than ten, twelve has a more even fractions than ten.

The most important fractions are *halves*, *thirds*, and the halves and thirds of each (*quarters* and *sixths*), with the halves of quarters (*eighths*), the thirds of thirds (*ninths*), and the thirds of quarters (*twelfths*) being less vital but still important. These are the most important fractions because two and three are the first *prime numbers* (that is, the first numbers besides one which are divisible by only themselves and by one). They are also important because they are the most common numbers; every even number is divisible by two, and every third number is divisible by three, so having a base which works well with these numbers is convenient. Finally, we can easily divide physical things into roughly equal halves and thirds visually, and into fractions of these by halving the quarters or thirding the halves or what have you, but larger divisions (say, into fifths) are much harder to do without equipment, as well as much more rarely required. We're much more likely to need to divide by three than we are by five.

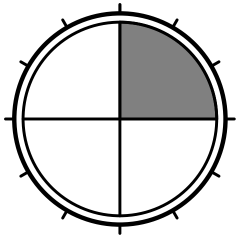
On the next page, you will find some visual representations of the most important fractions, including even the halves of eighths, together with their decimal and dozenal expansions. Note that dozenal handles all of them extremely well; and even when decimal handles them decently, dozenal still handles them better.



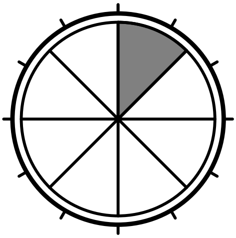
$^1\!/_2 = 0;6$
(0.5)



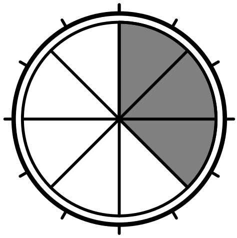
$^1\!/_3 = 0;4$
(0.3333…)



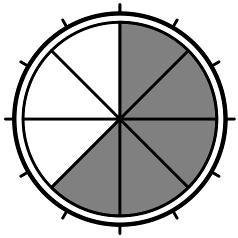
$^1\!/_4 = 0;3$
(0.25)



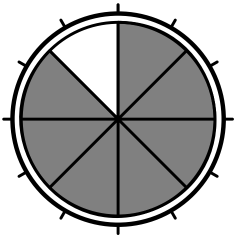
$^1\!/_8 = 0;16$
(0.125)



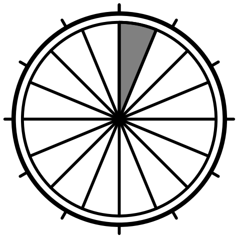
$^3\!/_8 = 0;46$
(0.375)



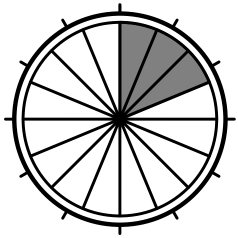
$^5\!/_8 = 0;76$
(0.625)



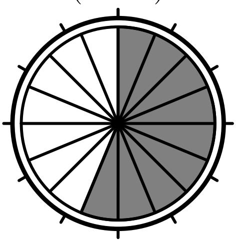
$^7\!/_8 = 0;$↊6
(0.875)



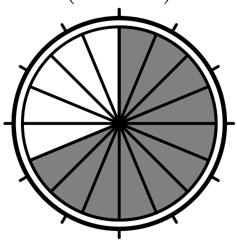
$^1\!/_{14} = 0;09$
(0.0625)



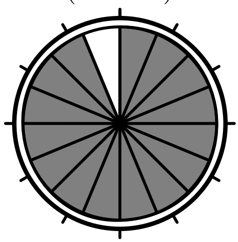
$^3\!/_{14} = 0;23$
(0.1875)



$^9\!/_{14} = 0;69$
(0.5625)



↋$/_{14} = 0;83$
(0.6875)



$^{13}\!/_{14} = 0;$↋3
(0.9375)

"14," of course, here is "dozen-four," or in decimal 16 (sixteen, or ten and six).

These easier fractions make nearly everything else in mathematics easier, as well. Since fractions are really nothing other than division, *division* is made much easier. Everyday tasks like splitting checks, calculating supplies, dividing up supplies, making change, and all manner of other common calculations are made much easier by counting and thinking in twelves. Telling time becomes perfectly natural; no need to multiply the number the big hand is pointing at by five to get minutes, because five minutes is a twelfth of an hour. Yearly interest can be converted to monthly just by moving the dozenal point. The applications are literally endless.

Indeed, dozenal has many more even fractions for its size than decimal has:

Fraction	Dozenal	Decimal
1/2	0;6	0.5
1/3	0;4	0.333...
1/4	0;3	0.25
1/5	0;2497...	0.2
1/6	0;2	0.1666...
1/7	0;186↊35...	0.142857...
1/8	0;16	0.125
1/9	0;14	0.111...
1/↊	0;124972...	0.1
1/↋	0;111...	0.0909...
1/10	0;1	0.083...

Only four nonterminating fractions on the dozenal side, but six on the decimal side; and one of the decimal ones is the vital and frequent 1/3. Those that are regular on the decimal side are often also regular on the dozenal side, but shorter; e.g., 0;3 as opposed to 0.25, 0;16 as opposed to 0.125. This is because dozenal is an even multiple of two, three, four, and six; but three eights is also two dozen, and two nines is a dozen and a half. Having so many even fractions, and so many of them important fractions, is a strong advantage.

Moving the Fraction Point

Oftentimes the most important question to people hearing about dozenal is, "What about just moving the decimal point?" To multiply or divide by ten, or by a hundred, or by a thousand, we can just move the decimal point (properly the "fraction point") to the right or the left the appropriate number of places. Easy, right? Wouldn't we lose that if we switched to dozenal?

No. The beauty of the place notation system we discussed earlier is that *any* base has this same property. It's not multiplying by *ten* that has this property; it's multiplying by *the base.* So in dozenal, we can simply move the dozenal point exactly as we do in decimal, but when we're multiplying or dividing by a dozen, or by a gross, and so on.

$$5;782 \times 100 = 578;2$$
$$578;928 \div 100 = 5;78928$$

Just as simple as in decimal; and more immediately useful, too. If your monthly interest on a loan is 0;5↊8 perbiqua (a dozenal form of percent: one part per gross), then your yearly interest on that loan is 5;↊8 perbiqua. Try *that* in decimal!

Conversion Rules

Numbers can be converted from decimal to dozenal and back again by pretty simple procedures. While there are, of course, many tools to allow computers to do this work, doing it by hand is also relatively easy.

Integers, Decimal to Dozenal: To convert an integer from decimal to dozenal, divide the number repeatedly by twelve, saving the remainder; the remainders in reverse order is the answer. The following is in decimal:

$$\begin{array}{rl} 124 & \mathrm{R}\ 6 \\ 12\,\overline{)\,1494} & \end{array}$$

$$\begin{array}{rl} 10 & \mathrm{R}\ 4 \\ 12\,\overline{)\,124} & \end{array}$$

Then, since decimal 10 is less than decimal 12, 10 is itself the remainder. Reverse the order of the remainders, convert 10 or 11 into ↊ or ↋ as necessary, and one has the answer: ↊46.

Integers, Dozenal to Decimal: To convert it back to decimal, do essentially the same thing, but divide repeatedly by ↊ instead, saving and then reversing the remainders in the same way:

$$\begin{array}{rl} 105 & \mathrm{R}\ 4 \\ ↊\,\overline{)\,↊46} & \end{array}$$

$$\begin{array}{rl} 12 & \mathrm{R}\ 9 \\ ↊\,\overline{)\,105} & \end{array}$$

$$\begin{array}{rl} 1 & \mathrm{R}\ 4 \\ ↊\,\overline{)\,12} & \end{array}$$

And the 1 will itself be the next remainder, of course; so we gather our remainders, 4941, and reverse them to get our answer, 1494.

Fractionals, Decimal to Dozenal: Multiply repeatedly by twelve; the successive final carries are the dozenal number. In other words, the integer part of the products is the answer. The following is in decimal:

$$
\begin{array}{rr}
 & 0\,.\,0\,2\,4 \\
\times & 1\,2 \\
\hline
 & 0\,.\,2\,8\,8 \\
\times & 1\,2 \\
\hline
 & 3\,.\,4\,5\,6 \\
\times & 1\,2 \\
\hline
 & 5\,.\,4\,7\,2
\end{array}
$$

Take the the integer parts and string them together, without reversing, and you get the answer, 0.024 = 0;035.

Fractionals, Dozenal to Decimal: Do the same thing, but multiply repeatedly by ↊ rather than by (decimal) 12.

$$
\begin{array}{rr}
 & 0;0\,3\,5 \\
\times & ↊ \\
\hline
 & 0;2\,↊\,2 \\
\times & ↊ \\
\hline
 & 2;4\,5\,8 \\
\times & ↊ \\
\hline
 & 3;8\,8\,8 \\
\times & ↊ \\
\hline
 & 7;3\,2\,8
\end{array}
$$

The integer parts of these successive products gives us 0.0237, which can of course round to our original number, 0.024. Fractionals will often not convert exactly, so rounding will sometimes be necessary.

Basic Operations

In dozenal, as in any other base, we need to know the algorithms (the rules) for doing the basic four functions of arithmetic. Fortunately, all algorithms for these four functions are the same in any base, as long as we are still using place notation. We must simply remember to “carry” or “borrow” at *twelve* rather than at *ten*.

Fortunately, this mostly winds down to remembering to think of “10” as meaning a dozen rather than ten. Since we have to jump this hurdle anyway, learning these functions is still a pretty easy task.

Addition

Addition is characteristically easy, even in dozenal. We simply line the numbers up (based on their dozenal points; if they have none, line them up based on their rightmost digit) and add each column together, *carrying* to the next column to the left whatever that column’s dozens digit, if any, might be.

$$
\begin{array}{rrrrrrr}
 & {\scriptstyle 1} & & {\scriptstyle 1} & {\scriptstyle 1} & & \\
 & & 4 & 5; & 7 & 8 & \\
+ & 3 & \text{↊} & 3; & \text{↋} & 9 & 3 \\
\hline
 & 4 & 2 & 9; & 7 & 5 & 3
\end{array}
$$

This should be clear enough. No digit is the same as 0, so $0 + 3 = 3$. $8 + 9 = 15$ (one dozen five, “seventeen” in decimal parlance), so we place the 5 below and *carry* the 1 to the next column to the left. $7 + \text{↋} = 16$; add the 1 that we carried and get 17. Place the 7 below, carry the 1 to the next column to the left. $5 + 3 = 8$, plus the one that we carried make 9; place the 9 below, and no need to carry anything (since this total is less than 10, twelve), and move on.

$4 + \text{↊} = 12$ (one dozen two, "fourteen" in decimal parlance); place the 2 below, carry the 1 to the next column to the left. Finally, no digit is the same as 0, so we have $0 + 3 = 3$, plus the one that we carried make 4. Our final answer, then, is 429;753.

We can clearly see that this is identical to our method of adding in decimal, except that we carry at twelve, not at ten. That is, in both cases we carry when the total is 10 or greater; but in dozenal, "10" means *twelve*, one dozen and zero ones. This remembered, we have no difficulty.

Subtraction

In subtraction, of course, we don't carry anything; rather, we *borrow*. When we stack our numbers for adding, we line them up on the dozenal point, or if there is none on the rightmost digit, just as we do with addition; unlike in addition, though, we must be sure that our higher number is on the top. We then subtract each column, one by one, until we reach the end.

If, in a given column, the higher digit is smaller than the lower digit, we must *borrow* 10 from the next column to the left. This means that we reduce the top digit in the next column by one, and *add* 10 to the digit in the column we're dealing with. We may have to do this several times before we can solve.

$$
\begin{array}{rrrrrrr}
 & & & & 11 & 10 & \\
 & \text{↊} & 14 & 4 & \cancel{12} & \cancel{11} & 10 \\
 & \text{↋} & 4 & 5; & 2 & 1 & \\
- & 3 & \text{↊} & 3; & \text{↋} & 9 & 3 \\
\hline
 & 7 & 6 & 1; & 2 & 3 & 9
\end{array}
$$

No digit is the same as 0; but $0 < 3$, so we have to *borrow*. However, in the next column to the left, we have the same problem; that is, the upper digit is smaller than the lower. So we keep moving left until we find a column with a larger upper digit; we find that when we find $5 > 3$. So we *borrow* 10 from the 5 by crossing it out and making it 4 (shown here as a superscript); we then give the 10 to the 2 to the right. $2 + 10 = 12$ (one dozen two), so we cross out the 2 and put 12 above it. But we must borrow from that, too; so we cross that out and put 11 still above it, taking the 10 over to the right. This makes our 1 into 11 because $1 + 10 = 11$ (one dozen one); but we must borrow again to fill out zero; so we take the 10 from the 11, make it 10, and add 10 to the last column, 0. $0 + 10 = 10$, of course. So now we can actually begin to subtract. One column at a time, we find $10 - 3 = 9$, $10 - 9 = 3$, $11 - ↋ = 2$, and $4 - 3 = 1$. Our next column, though, again has an upper digit smaller than its lower ($4 < ↊$); so again we must borrow. We borrow 10 from the ↋, making it ↊, and add it to the 4, making it 14 (one dozen four). $14 - ↊ = 6$, so we place that below; then we move on, noting that $↊ - 3 = 7$, and place that below. And there is our answer: 761;239.

There's no doubt that subtraction, especially in a problem like this with repeated borrowing, is more difficult than addition. But we do it in dozenal precisely the same way that we do it in decimal: by borrowing 10. 10 simply means "one dozen and zero ones," rather than "one ten and zero ones." Quite simple.

Multiplication

Multiplication is very common, probably among our more common operations, such that we nearly all learn our "times tables" in grade school. To begin using dozenal, we must learn new times tables; however, as we've already seen,[2] the dozenal multiplication table is so much more regular and simple than the decimal that this is a very easy thing to do indeed.

[2] See the multiplication tables discussed above at page 12.

For long multiplication, though, our algorithm is precisely the same as it in decimal, keeping in mind as always that 10 means a *dozen*, not ten.

$$
\begin{array}{cccccccc}
 & & & {\scriptstyle 8} & {\scriptstyle 7} & {\scriptstyle 3} & {\scriptstyle 1} & \\
 & & & {\scriptstyle 2} & {\scriptstyle 2} & {\scriptstyle 1} & & \\
 & & & 6 & ↊ & 9; & 4 & 2 \\
\times & & & & & & ↊ & 3 \\
\hline
 & & 1 & 8 & 8 & 4; & 0 & 6 \\
+ & 5 & 8 & 8 & ↋ & 9 & 5;8 & 0 \\
\hline
 & 5 & ↊ & 8 & 5 & 9; & 8 & 6
\end{array}
$$

Then, since we started with a total of two dozenal places (;42), we "point off" the same number in our answer, giving us 5↊859;86.

Multiplication can often be a lengthy process, so there is no need to go step-by-step through what we have just done here. In any case, by now the concept is clear: remember that 10 means a *dozen*, not ten, and then do precisely what you'd do in decimal.

Division

Division is, like multipication, extremely common but often rather cumbersome. But we all grew up practicing our long division, and we do our long division in dozenal the same way we do it in decimal: very carefully. Particularly because many numbers come to infinitely repeating expansions, oftentimes we could go on dividing for the rest of our lives if we cared to (as in the example we are about to see). However, division is very frequently necessary and must be handled.

$$
\begin{array}{r}
71;8\,\text{↋}\,77 \\
83\,\big)\,\overline{4\,\text{↊}\,\text{↋}\,5;0000} \\
499 \\
125 \\
83 \\
620 \\
560 \\
800 \\
769 \\
530 \\
499 \\
530
\end{array}
$$

At this point we can be pretty confident that we've gotten close enough, and we'll just keep getting sevens; and besides, we've already got more digits than we started with, so likely what we get is meaningless anyway.

As with multiplication, there is no need for us to go over all the esoterica of long division. We drilled in it plenty in our school days, and we'll have to do it here and there for the rest of our lives. But so long as we remember that 10 equals *twelve*, we will have no more difficulty than we otherwise would.

Of course, if we have to do many calculations in decimal, we would use a calculator. Fortunately, there are calculators which work in dozenal; we will discuss some of these later, starting on page 3↊.

Divisibility Tests

We all know about divisibility tests in decimal, even if not by that name. Most elementarily, we all can recognize even numbers at a glance, and we know that they are even because they are divisible by 2. (Rather, that "divisible by two" is what "even" means.) We can also find clear patterns based on the factors of ten; e.g., a number with a 5 at the end is a *half* of something. A number with a 25 at the end is a *quarter* of something. And so on. In dozenal, there are *many* more such patterns to latch onto.

In decimal, we can immediately identify numbers divisible by two and by five; with some additional trouble, we can identify numbers divisible by four; only with still more additional work can we identify numbers divisible by three or by nine.

In dozenal, we can easily identify numbers divisible by 2, 3, 4, and 6 at a glance. With a slightly closer look, we can identify numbers divisible by 8 and 9. With some additional work, we can also identify numbers divisible by ↋; and by learning a trick, we can identify numbers divisible by 5 and ↊. The field for this sort of thing in dozenal is vastly wider than it is in decimal.

At a Glance

At a glance, we can identify numbers divisible by 2, 3, 4, and 6. To do this, we need only look at the last digit of the number, so matter how large it is.

Divisible by 2 If it's an *even* number; that is, if its last digit is 0, 2, 4, 6, 8, or ↊.
Divisible by 3 A number is *threefold* if its last digit is 0, 3, 6, or 9.
Divisible by 4 A number is *fourfold* if its last digit is 0, 4, or 8.
Divisible by 6 A number is *sixfold* if its last digit is 0 or 6.

We're quite used to thinking, in decimal, about the number 5 as meaning "half." In dozenal, the number 6 means half. Furthermore, 6 being the product of 2 and 3, and sixths being both thirds of halves and halves of thirds, it's much more important to be able to identify things divisible into sixths than those divisible into fifths. So this is an extremely useful test.

Decimal has a tolerably easy test for determining fourfoldness; it's more difficult than dozenal's, however. Fourfoldness is quite common, so this is a great advantage for dozenal, as well.

Threefoldness is where dozenal can really shine. After halving, thirding is the most common division we must make. (Quartering, of course, is really just halving twice.) So while in decimal determining divisibility by three is trickier, and dealing with thirds is hatefully difficult due to their infinitely repeating decimal expansions, in dozenal dealing with thirds is a positive joy.

Of course, in addition to this, any number which ends in 0 is divisible by twelve; but that is obvious, and hardly worth mentioning.

So much for the "at a glance" tests; let's look at the slightly more involved ones.

Closer Look

Divisibility by 2, 3, 4, and 6 by doing nothing more than looking at the final digit is pretty convenient; but say we need divisibility by a less obvious number. Dozenal has easy tests for some other numbers, too, which take very little more work than our four simple friends above.

Divisible by 8 A number is *eightfold* if:

- its penultimate (second-to-last) digit is *even* and its ultimate (last) is 0 or 8; *or*
- its penultimate digit is *odd* and its ultimate is 4.

Divisible by 9 A number is *ninefold* if:

- its penultimate digit is 0, 3, 6, or 9, and its ultimate is 0 or 9; *or*
- its penultimate digit is 1, 4, 7, or Ɛ̃, and its ultimate is 6; *or*
- its penultimate digit is 2, 5, 8, or Ɛ, and its ultimate is 3.

Divisible by Ɛ A number is *elevenfold* if, when all its digits are summed, the sum is divisible by Ɛ. If you still can't tell, sum the digits of the sum; this can be repeated as many times as necessary.

Note that the test for 9 looks quite a bit more complex than it really is. Start at zero and count up by threes, and remember those numbers followed by 0 or 9; then start at one and count up by threes, and remember those numbers followed by 6; then start at two and count up by threes, and remember those numbers followed by 3. These simple rules will suffice.

Split, Promote, Discard

We can also determine divisibility by 5 (and by extension ↊) if we are willing to do some calculation. While more involved, this can still be done mentally, and is easier than simply manually dividing to find out.

First, all the two-digit multiples of 5 must be memorized. This goes beyond the multiples of five in the multiplication table. They are:

5	↊	13	18	21	26	2Ɛ	34	39	42	47	50
55	5↊	53	58	71	76	7Ɛ	84	89	92	97	↊0
↊5	↊↊	Ɛ3	Ɛ8								

Note that once you know the first 10 (twelve) multiples, you really know all of them; it's just a matter of adding them to 50, and then to ↊0.

This done, we then *split*, *promote*, and *discard* our way to finding out the answer. For example, let's take 23↊↊93854. Dividing this by five to determine divisibility would take considerable time and effort; this test (called SPD for short) makes it much simpler.

First, we *split* off the last two digits, 54. Then we *promote* them to the nearest multiple of 5; as we can see from the list above, this is 55. We had to add 1 to 54 to get there, so we also add one to the remainder of the number, 23↊↊938, which gives us 23↊↊939.

Then we repeat the process, splitting the number into 23↊↊9 and 39. 39 is already a multiple of 5; so we discard it and move on. We split the number again, 23↊ and ↊9. We then *promote* the second number to the nearest multiple of twelve; this is ↊↊, to get which we must add 1. So we add 1 to the other number, too, giving us 23↋. Split the last two digits off: 2 and 3↋. Promote the second to the nearest multiple of 5, which is 42. To get there, we added 3. So we add 3 to the other number, 2, giving us 5. The number can't be split anymore, and the last part is divisible by 5; therefore, the whole number is divisible by 5.

Divisibility by ↊ is done the same way; if the number is divisible by both 5 and 2, then it is divisible by ↊.

As said, this clearly requires some effort; however, it's much simpler than the process of simply dividing to determine divisibility; and further, it means that dozenal has workable mental tests for *every single number less than and including itself except for 7.* That's truly hard to beat.

Other Properties

Dozenal provides many other interesting and helpful properties, all of which can be extremely convenient when working with numbers. Not all of these need to be memorized by most people (though all of them can be interesting), but the fact that dozenal provides them speaks volumes about its worth.

Squares and Other Powers All squares and even powers in dozenal end in 0, 1, 4, or 9. Due to that, all powers of 0, 1, 4, and 9 always end in the very same figure; for example, 4^n ends in 4. All powers of 6 end in 0, and all powers of Ƨ end in 4. All odd powers of 3, 5, 7, 8, and Ɛ also end in the same figure; e.g., 3^{n-1} ends in 3. Finally, *no* power of any number ends in 2, 6, or Ƨ.

Prime numbers Prime numbers have long been a fascination of mathematicians, amateur and professional. These are numbers, besides 1, which are divisible only by 1 and themselves. In dozenal, all prime numbers (other than 2 and 3) end in 1, 5, 7, or Ɛ; any number which does not end in one of these four digits cannot be prime. The minimal set of all primes can be expressed as $\{2, 3, (6n \pm 1)\}$. Not coincidentally, 2, 3, and 6 are all factors of the dozen.

Approximations of transcendental numbers Transcendental numbers are extremely important in mathematics; numbers like π, Euler's number (e), and the Golden Ratio (φ, pronounced "phi," like "fie") show up constantly in geometry, statistics, trigonometry, and many other studies. There are also non-transcendental numbers which have infinite expansions, such as certain reciprocals (in dozenal, for example, $\frac{1}{5}$ or $\frac{1}{7}$) or $\sqrt{2}$ (the square root of two). Since these numbers continue infinitely, we are always forced to deal with approximations; we *must* round them to some convenient value. In dozenal, though, these approximations are usually much closer to true than in decimal; this trait can often be extremely useful.

A few of the more interesting of these transcendentals and irrational fractions are charted below, with their errors charted in parts per biqua (per gross).

	Dozenal			Decimal		
π	3;184809	3;185	0;03↋%	3.141592	3.142	0;085%
		3;1848	0;0009%		3.1416	0;001↊%
e	2;875236	2;875	0;024%	2.718281	2.718	0;05↊%
		2;8752	0;0036%		2.7183	0;0046%
φ	1;74↋↋67	1;750	0;0054%	1.618033	1.618	0;0085%
		1;7500	0;0054%		1.6180	0;0085%
$\sqrt{2}$	1;4↋7917	1;4↋8	0;02↊%	1.414213	1.414	0;045%
		1;4↋79	0;0017%		1.4142	0;0034%
$\frac{1}{5}$	$0;\overline{2497}$	0;24↊	0;04↊%	$\frac{1}{3}$ $(0.\overline{3})$	0.333	0;06↋%
		0;2497	0;0025%		0.3333	0;0083%
$\frac{1}{7}$	$0;\overline{\text{186↊35}}$	0;187	0;024%	$0.\overline{142857}$	0.143	0;030%
		0;186↊	0;0036%		0.1429	0;00↊8%

Notice that, because 1⁄3 is simple in dozenal and 1⁄5 is simple in decimal, we've compared the two to each other as analogous.

Every single time the dozenal abbreviation is more precise than the decimal. Even where dozenal does not perform particularly well, as in 1⁄5, it still performs better than decimal does in analogous situations (e.g., 1⁄3).

Sometimes, this improvement of precision is only marginal, as in the three-digit rounding of 1⁄7; 0;019% and 0;021% are quite close to each other, though even there dozenal comes out ahead. However, we see the four-digit rounding of 1⁄7 is nearly twice as precise in dozenal as in decimal.

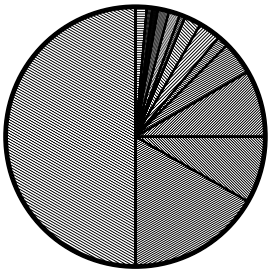
Others are even more remarkable. π, for example, which is ubiquitous in mathematics, is *more than twice* as precise when rounded to three digits or four digits than decimal is. e is nearly three times as precise when rounded to three digits. $\sqrt{2}$ is more than twice as precise when rounded to four digits. It's true that, in general, dozenal numbers are more precise than decimal to the same number of digits, because the denominator is larger (a multiple of twelve, rather than that of the smaller ten); but dozenal's greater precision in these important roundings is remarkable, and a powerful tool.

Percentage and Perbiqua Percentage (parts per hundred) and perbiqua (parts per gross) show another notable advantage for dozenal. While a hundred does improve on ten in dividing at least into even quarters, it cannot divide into even eighths or sixteenths; and most damningly, it cannot divide into even *twelfths*, which considering our twelve-month year and two-dozen-hour day makes it a particularly bad choice.

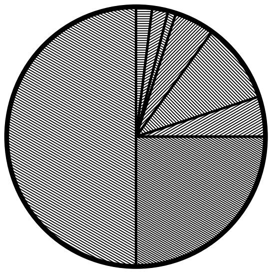
	Divisors	Total
Perbiqua	1, 2, 3, 4, 6, 8, 9, 10, 14, 16, 20, 30, 40, 60, 100	13
Percent	1, 2, 4, 5, 10, 20, 25, 50, 100	9

Perbiqua here are dozenal, percent decimal.

Pictorially, the greater number of even parts per gross than per hundred is even more striking:



Perbiquas



Percents

These figures make it clear that dozenal has six more even divisors than decimal has ($13 - 9 = 6$, in dozenal), which makes all manner of interesting things, particularly things like yearly or monthly interest rates (which often go in eighths and sixteenths, two numbers that percentages miss entirely), much easier to deal with.

Talking in Dozens

Ever since dozenals were first discussed, there has been discussion over how we should talk about them. On a basic level, of course, we have our normal English words that we've been using so far in this Manual; that is, "dozen" and "gross." Traditionally, a dozen gross was known as a "great-gross." And while these terms do work, they are a bit clumsy, and they fall apart when we start talking about larger numbers; also, they make no provision for fractional parts.

Really, any system would work; just as with symbols for ten and eleven, we all simply use what we want, and as dozenals gain more acceptance, one or another system will win out. So here, we will simply review the two most common and popular systems; this will give us a taste of the possibilities.

The earliest and for a long time the most widespread system is what we will call here the *do-gro-mo* system, which is shown as a table below:

10	Do	0.1	Edo
100	Gro	0.01	Egro
1000	Mo	0.001	Emo
1 0000	Do-mo	0.0001	Edo-mo
10 0000	Gro-mo	0.0000 1	Egro-mo
100 0000	Bi-mo	0.0000 01	Ebi-mo
1000 0000	Tri-mo	0.0000 001	Etri-mo

This is an admirably simple system which has been around for over six dozen years; for many years it was printed on the back of every issue of *The Duodecimal Bulletin.* This system can fairly easily name any number, no matter how large or complex. For example:

58↊9 233↋

This cames to "five tri-mo eight ten nine two three three eleven." The down side is that we must remember that the eighth digit marks the number of tri-mo, though there's nothing about it that makes one think either "tri" (normally associated with "three") or "mo" (which otherwise means "1000").

These shortcomings led to the creation of another system in recent years, known as Systematic Dozenal Nomenclature (SDN). This system uses endings already extremely widely and internationally known, and takes advantage of the fact that *uncia* is an ancient word meaning a twelfth, and builds a very straightforward and easy system from it.

Num.	Root	Pos.	Neg.
0	Nil	Nilqua	Nilcia
1	Un	Unqua	Uncia
2	Bi	Biqua	Bicia
3	Tri	Triqua	Tricia
4	Quad	Quadqua	Quadcia
5	Pent	Pentqua	Pentcia
6	Hex	Hexqua	Hexcia
7	Sept	Septqua	Septcia
8	Oct	Octqua	Octcia
9	Enn	Ennqua	Enncia
↊	Dec	Decqua	Deccia
↋	Lev	Levqua	Levcia
10	Unnil	Unnilqua	Unnilcia
11	Unun	Ununqua	Ununcia

This system can be extremely deep and flexible, but at its root it's quite simple. Count the digits, starting with zero; use the root for the number of digits; add *qua* to the end. And that's it. So our example from above:

58↊9 233↋

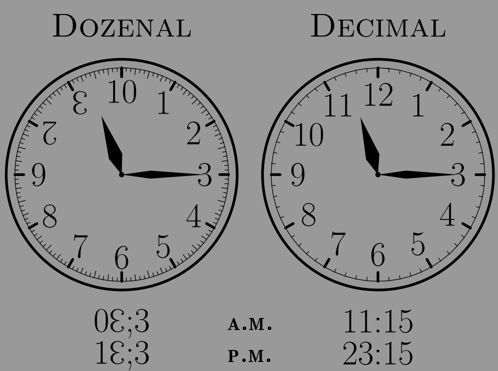
winds up, very simply, as “five septqua eight ten nine two three three eleven.” This is not only straightforward, but it is considerably simpler even than the traditional system we use for decimal numbers.

For fractions, we can use *cia* rather than *qua*; for example, 0;003 is “three tricia” (count the digits, starting with zero, including the ones place). Oftentimes, of course, we’ll simply refer to these the way we do now, and say “zero dit zero zero three.”

This system should explain why we called our dozenal version of percentages “parts per biqua”; because 100 is *biqua* in SDN. “Perbiqua” is a reasonable shorthand for this.

For two-digit numbers, another common shorthand is to drop the “un” syllable; e.g., “38” can be either “three unqua eight” or “three-qua eight.”

This system also allows creation of words based on very simple and straightfoward rules. For example, a 200th anniversary in SDN could be called a *binabiquennium*, and a 80th anniversary a *octaunquennium*. Some references on using SDN in this way can be found in the bibliography.



Even something as simple as telling time becomes still simpler with dozenal.

Measurement

There exists no really satisfactory system of measurement in the world today. Essentially, there are two major systems, one of which is nearly extinct. Decimal metric, often called SI for short (though what is commonly used is almost never strictly SI) is used in most of the world, including Anglophone countries, with the exception of the United States and, to a certain extent, the United Kingdom. This system purports to be rational (though it isn't), based on Earth-centered standards (though it isn't), and consistently decimal (though it isn't). It offers little of interest once we recognize that dozenal is a better base than decimal.

The other is probably best called *customary-imperial*, though the "imperial" part is nearly gone. This is the system that grew up in the United Kingdom and was spread therefrom around the world to the various colonies of the Empire. Perhaps ironically, the UK has officially abandoned the system, while the first colonies to break away from the old British Empire, the United States, have retained it rather jealously to this day. The UK does still use the old imperial system in certain cases (e.g., beer is still sold in imperial pints, and roadways are still marked in imperial miles), but it is clearly an officially disfavored system. The United States, on the other hand, uses customary (an older version of imperial) nearly exclusively; though those in medical and scientific fields do frequently use metric, they convert these measurements to customary for their patients and public. This is a mongrel system, containing elements of dozenal, but also elements of base sixteen, base eight, binary, ternary, quarternary, and even base fourteen ("stones," still used somewhat in the UK, which are fourteen pounds).

Both of these systems suffer from the same two principal faults: they are not dozenal, and they are not consistent. Since they are decimal, they suffer all the faults that a suboptimal base like decimal must suffer: difficult thirds, few regular fractions, and so forth.

Since they are not consistent, they cannot be easily understood and recreated. In metric, for example, the liter is supposedly a cubic *deci*meter, although the full meter is supposedly the standard unit of length; but the liter is in fact slightly smaller than a cubic decimeter anyway, and the SI version of metric does away with it entirely. Furthermore, the *kilo*gram is the basic unit of mass, though it carries a prefix which means "thousand"; and despite its status, the basic unit of amount of substance, the mole, is based on a quantity of *twelve* (!) *grams*, not ten kilograms. The customary system remedies this problem somewhat, allowing even thirds in certain cases (e.g., in the dozenal foot and troy pound), but still relies on decimal, and can only allow even thirds at the cost of complete chaos in the base system.

Dozenal offers a number of solutions to this problem, none of which have really gained prominence. As dozenal gains traction, some systems will be used more than others, and at some point one or another one will become the standard. For now, though, there are plenty of systems which showcase the power of dozenal arithmetic as applied to metric systems. We will briefly examine the two most popular here: Takashi Suga's Universal Unit System, and Tom Pendlebury's TGM.

Universal Unit System

Takashi Suga's Universal Unit System (UUS) attempts to formulate a system of measure based on what it considers to be fundamental, universal constants. It takes advantage of the fact that many of these constants, when raised to some more or less arbitrary power of twelve, come to suprisingly convenient unit sizes.

UUS is therefore based on the following constants: the speed of light in vacuum, the quantum of action, and the Boltzmann constant. In a less direct way, the Rydberg constant, the atomic mass unit, the Bohr radius, and half of the Planck length are also used. These quantities are then manipulated so as to produce useful units for all fields of endeavor.

Furthermore, UUS doesn't use the standard seven SI units, forming all other units from these. Rather, it relies on units of impedance, plane angle, logarithmic quantity, amount of sustance, length, time, energy, and thermodynamic temperature. In other words, energy is used as a base unit rather than mass, impedance is used rather than current, and luminous intensity is entirety done away with (due to being biologically based rather than physically).

This yields twenty-four (dozenal two dozen, 20) primary units, some of which are shown in the table on the facing page. This table is culled from the Summary table of the paper outlining UUS, *Proposal for the Universal Unit System*, and the parts we present here are reproduced from that table verbatim. For this reason, all the numbers are decimal.

Some criticisms of the system are that it relies on "fundamental" physical constants, the accuracy of our measurements of which is continuously refined, which thus necessitates revising the system to conform to these. Also, UUS offers no real unit names; the unit of length, for example, appears to be simply "meter," despite this being precisely the same as the unit of length in SI metric. Sometimes "harmonic" is appended to distinguish between the two; e.g., the "harmonic gram" as opposed to simply the gram. However, this does make the system more difficult to use.

Notably, Suga himself asks his readers to "please understand that the author has no intention to promote the use of the 'Universal Unit System' in the real world." This perhaps explains the lack of common names for the units.

Those interested in further information on UUS are encouraged to go directly to the source, Suga's own paper proposing the system: `http://www.dozenal.com`.

Universal Unit System Summary								
Category	Dimen. / Item	Sym.	Value		Comment			
Base units	length	m_u	27.21028842	cm	12^8	$\times$	1	D
that are not	time	s_u	390.2675219	ms	12^{16}	$\times$ 1	D/c_0	
natural units	energy	J_u	64.1433465	mJ	12^{16}	$\times$ 12^{-2}	$\hbar c_0/D$	
	thermodyn. temp.	K_u	1.211831	K	12^{-4}	$\times$	12^{-2}	$\hbar c_0/Dk_B$
Derived units	mass	g_u	131.950228	g	$12^3 2$	$\times$	12^{-2}	$\hbar/c_0/D$
of dynamical	work	W_u	164.357378	mW	1	$\times$	12^{-2}	$\hbar c^2/D^2$
quantities	force	N_u	235.731961	mN	12^8	$\times$	12^{-2}	$\hbar c_0/D^2$
	pressure	P_u	3.18384692	Pa	1	$\times$	12^{-2}	$\hbar c_0/D^3$
Derived units	charge	C_u	28.8965942	mC	12^{16}	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_n}$
of electro-	elec. current	A_u	74.0430416	mA	1	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_n}c_0/D$
magnetic	field strength	O_u	272.114137	mA/m	12^{-8}	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_n}c_0/D^2$
quantities	flux density	G_u	390.283662	mC/m^2	1	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_n}/D^2$

Tim, Grafut, Maz (TGM)

Tom Pendlebury, a long-time member and stalwart of the Dozenal Society of Great Britain, spent unquennia ("unqua" + "ennia," dozens of years) developing a system of measure called Tim, Grafut, Maz (TGM), so called for its three primary units. The system got quite a bit of notice in dozenal circles when published in the early 1190s (decimal, 1980s; the current year is 1200), and is still popular today.

Rather than build a system up from a unit of length, TGM builds one up from a unit of time. Called the Tim, this unit is exactly 0;21 seconds, 0;0001 of an hour. SDN works well with TGM, so we can call a dozen Tims an unquaTim (equal to 2;1 seconds, or a "unctic" for short), a dozen of those a biquaTim (21 seconds, a "bictic"), a dozen of those a triquaTim (210 seconds, or five minutes, a "block"), and a quadquaTim an hour. TGM thus fits in with our current dozenal division of a half-day into twelve hours, and we can keep our current analog clocks unchanged (except, of course, for replacing "10," "11," and "12" with the proper "Ɫ," "Ɛ," and "10").

TGM then assumes that the mean acceleration of gravity on Earth equals 1, and derives a length unit from that. This length unit, the Gravity Foot or Grafut, is about 0;Ɛ783 of an English foot, and about 0;366Ɛ of a meter (very nearly exactly between one quarter and one third). Units of speed (the Vlos) and acceleration (the Gee, as mentioned equal to the mean pull of gravity on Earth) come from these.

Mass and weight are identical quantities (assuming that one is on the planet Earth), while in both customary and in metric they are often confused, causing great problems. The Maz is the mass of one cubic Grafut of pure water at maximum density; this is about 49;0154 pounds, or 21;Ɫ254 kilograms. The whole system derives from these three units.

TGM produces a great many interesting coincidences, with both customary and SI metric. For example, three triquaGrafut is only a little less than a mile, and two triquaGrafut is almost two-thirds of a kilometer. A Surf (a square Grafut) is almost exactly a twelfth of a square meter and only a little bit less than a square foot. A Volm (a cubic Grafut) is almost 7 gallons and just under 22 liters. Four quadciaVolm is just a little more than a teaspoon and very nearly exactly 5 milliliters.

One triquaTim is exactly five minutes; one quadquaTim is exactly one hour. 15 Vlos is nearly 55 miles per hour (in decimal, 65) and just a touch more than 88 kilometers per hour (in decimal, 104), a very convenient number for highway speed limits. A Freq (one per Tim, the unit of frequency) is almost six hertz, and five triciaFreq is exactly one revolution per minute (RPM).

A biquaMaz is a bit more than 4 imperial tons and a bit under 4 metric tons. A standard atmosphere is almost exactly 2Ɛ Prem. A Pov is just over six horsepower and just under 300 watts. A Tregree is only a little more than two degrees Fahrenheit and 1 ⅕ degrees Celsius. A Kur is just under half an ampere, and 6 hexciaKur is so close to one microampere as to make very little difference. And so on.

TGM strongly emphasizes having a 1 : 1 correspondence between basic units as much as possible. It is also very open to *auxiliary* units; that is, units which aren't necessarily part of the system itself, but which are nevertheless useful in important ways. An example is the standard atmosphere, the Atmoz; this is 2Ɛ Prem, not 1 : 1 by any means, but nevertheless an extremely useful quantity. Users of TGM can fill in any gaps they find with such units as they like; in this way TGM can be useful in any field of endeavor.

Furthermore, TGM encourages the use of auxiliary *names* for such units, and even for its core units. TGM users have coined names such as "tick" for the Tim, "block" for a triquaTim (a period of five minutes), "Gravmile" for 3 triquaGrafut, "Pintvol" for three

Length, Area, Volume			
Grafut		0;↋783 ft	0;366↋ m
Gravinch	$_{1}$Gf	0;↋783 in	2;569↊ cm
Gravyard	3 Gf	0;↋783 yd	0;↊789 m
Gravmile	3 ^{3}Gf	0;↋51↊ mi	1;6488 km
Gravklick	2 ^{3}Gf	0;7752 mi	1;0319 km
Surf		0;↋362 ft^2	0;1070 m^2
	^{4}Sf	0;5461 acres	0;2213 ha
Volm		6;9↋47 gal	21;↊254 L
Pintvol	3 $_{2}$Vm	1;17↊9 pt	0;6567 L
Cupvol	1;6 $_{2}$Vm	1;17↊9 cp	0;3293 L
Supvol	$_{3}$Vm	1;0182 tbs	12;↋624 mL
Sipvol	4 $_{4}$Vm	1;0182 tsp	4;↋↊09 mL

Time, Motion, and Frequency			
Tim			0;21 s
Tick	Tm		0;21 s
Unctic	^{1}Tm		2;1 s
Bictic	^{2}Tm		21 s
Block	^{3}Tm	5 min	210 s
Hour	^{4}Tm	1 hr	50 min
	$_{3}$Tm		0;1257 ms
Vlos		3;9874 mph	6;16↊↊ kph
SpL̇im.	15 Vl	54;9248 mph	88;2946 kph
St. Grav.	1 Gee	28;2280 ft/s^2	9;9879 m/s^2
Freq	$^{1}/_{\text{Tm}}$		5;9153 Hz
	5 $_{3}$Fq		1 RPM

Mass, Force, and Density			
Maz		48;↋↊72 lb	21;↊254 kg
	^{2}Mz	4;130↋ ton	3;8804 t
Oumz	2 $_{3}$Mz	1;07↋8 oz	25;↋048 g
Poundz	3 $_{2}$Mz	16;↋↋64 oz	0;6567 kg
Denz		52;5146 lb/ft^3	6↋3;↋7↋7 kg/m^3
Mag		108↊;2↋62 pdl	191;7151 N
		49;0154 lbf	21;↊38↊ kgf
Werg		47;37↊7 lbf·ft	62;↋96↋N·m
Prem		0;506↋ lbf/in^2	1818;6↋↋0 Pa
Atmoz	2↋ Pm	12;8836 lbf/in^2	4↊900;4916 Pa
		25;↋89 inHg	535;56↋ mmHg
Pov		0;6↋45 hp	2↋↋;↊↊08 W

Temp., Elec., and Chemistry			
Calg		0;0021 °F	0;0012 K
Decigree	^{2}Cg	0;21 °F	0.1 K
Tregree	^{3}Cg	2;1 °F	1;2497 K
Kur		Current	0;5↋47 A
		6 $_{6}$Kr	0;↋↋53 μA
Pel		Elec. Pot.	607;3167 V
		$_{3}$Pl	0;6073 V
		2 $_{2}$Pl	10;1263 V
Og		Resistance	1025;6860 Ω
Quel		Elec. Quant.	0;1048 C
		^{1}Ql	1;0487 C
Molz			21;↊254 kmol

biciaVolm, and so forth. These add to the system, and make it personalizable for any language while still maintaining an official and international character.

Some criticisms of TGM have been that its units of volume (the Volm) and of mass (the Maz) are rather larger than their comparable units in the two dominant systems; that the Tim corresponds with an hour rather than with a two-hour period derived from dozenally dividing a full solar day; and that its names are not aesthetic. TGM supporters argue that "convenient size" is an arbitrary metric, and that in any case there are plenty of auxiliary units to fill in the gap if the default units are not suitable for a particular purpose, and more can be created if needed. Further, they argue that aesthetic names can be developed very easily if those already provided do not appeal. Finally, they argue that the Tim corresponding with the hour comports with the true nature of angle, and that it eases transition to a dozenal system.

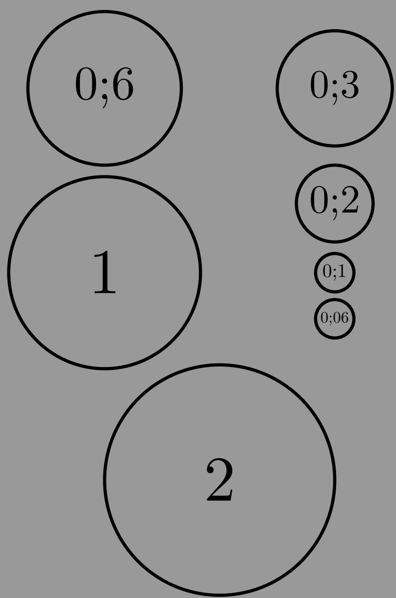
Other Systems

Dozenal arithmetic is so rich that these are far from the only systems of measure that have been proposed. For many years the Do-Metric system was quite popular. This was an interesting system based on customary and imperial measures, devised in the days when the British Empire had not yet abandoned them, which is notable in that it has *two* scales of length, a "mechanic's scale" based on subdivisions of the foot and a "basic scale" based on the yard. It also involved an interesting system of volumes and weights, mixing traditional terms like "carat" and "ounce" with metric ones like "grams," and even throwing in some made-up ones like "dribs" for good measure.

Jean Essig in 116Ɛ (1955.) wrote the first modern non-English defense of dozenalism, entitled *Douze notre dix futur* ("Twelve, our future ten"). M. Essig proposed a dozenalized metric system which took the meter more or less as it was and built a system up around it, simply prefixing the metric terms with "duodecimal." This system was never fully developed; for example, there were no electrical units.

Other projects, such as Primel (a version of TGM with a Tim based on a two-hour period), and other systems like UUS but based on different fundamental constants or taking different dozenal multiples of those constants, have also been devised.

As noted above, a sort of natural selection will inevitably produce a dominant system. The real takeaway here is that dozenal offers ways to fix all that is wrong with our current methods of measuring. Measurement, so thoroughly mixed with our counting and our arithmetic, stands to gain just as much from the transition to a more sensible base as those two fields do.



Superior divisibility gives superior flexibility in arrangement. For weights, for example, we can weigh all intermediate weights up to 4 with only one of each size with only seven weights; decimal requires nine weights, requiring two duplicates, and even then can't match dozenal's range.

Tools for Dozenals

We often wish to do our calculations by means of mechanical aids. Formerly this was typically by means of abaci or slide rules; these days it tends to be with electric calculators and computers. There are plenty of such tools for using dozenal; we will list only a few of the more prominent ones here.

Conversion

While we've seen that conversion of bases is a relatively simple process, it is often long and tedious. For this reason, a good number of tools for automating this process have been developed.

doz and **dec** are command-line utilities which are part of the dozenal suite of programs. They are capable of doing conversions of any number, exponential notation, fractions, integers, and with a variety of symbols for Ƨ and Ɛ.

Dozenal / Decimal Converter Calculator is an online resource which converts numbers in real time. It uses the relatively unusual "Bell" numerals ("*" for Ƨ and "#" for Ɛ), but is very quick and easy.

Calculators

dozdc is a complete Reverse Polish Notation (RPN) command-line calculator, performing the normal four functions as well as arbitrary roots, logarithms to arbitrary bases, trigonometric functions and their inverses, hyperbolic functions, and simplistic programming capabilities. It is, like doz and dec, part of the dozenal suite; there is also a graphical interface for those with Perl/Tk.

ZCalculator is a full-featured calculator with an effective graphical interface designed by Michael Punter of the Dozenal Society of Great Britain. It runs primarily in Windows, but does work effectively in `wine` on other systems.

Display

`dozenal` is a LaTeX 2ε package which makes using dozenal in LaTeX easy and automated. It provides fonts for the so-called Pitman characters ("Ɛ" and "Ɛ"), but makes it easy to use different characters from the same or different fonts if one so desires. It also redefines all the standard LaTeX counters, so that (for example) page numbers are automatically output in dozenal. This package was used for typesetting this booklet.

`Treisaran SVG Fonts` provide Pitman characters in SVG (Scalable Vector Graphics) format. They are useful when working in word processors such as LibreOffice or Microsoft Word, or when putting these characters into pictures or diagrams.

Metric Converters

`tgmconv`, another part of the `dozenal` suite, is a complete metric converter which will convert units arbitrarily to and from any of TGM, metric, or customary-imperial measures. It is a command-line only utility; however, there is an online interface, as well.

`A Converter`, by Takashi Suga, is an online converter which allows conversion between a number of different systems in different bases, including SI, customary-imperial, TGM, and Suga's own UUS.

There are many other tools which will assist in utilizing the dozenal base; these and others may be accessed on the Dozenal Society of America's website, `http://www.dozenal.org`, particularly on the Resources page.

Further Reading

The following is arranged very roughly by topic, and more precisely by suitability for those unfamiliar with dozenal; those closer to the top are more suitable for neophytes than those farther.

Malone, James. *Eggsactly a Dozen*. http://www.dozenal.org/drupal/content/eggsactly-dozen.

Zirkel, Gene. *A Brief Introduction to Dozenal Counting*. http://www.dozenal.org/drupal/content/brief-introduction-dozenal-counting.

Schiffman, Jay. *Fundamental Operations in the Duodecimal System*. http://www.dozenal.org/drupal/content/fundamental-operations-duodecimal-system.

Zirkel, Gene. *Decimal-Dozenal Conversion Rules*. http://www.dozenal.org/drupal/content/decimal-dozenal-conversion-rules.

DeVlieger, Michael. *Featured Figures: Basic Operations*. http://www.dozenal.org/drupal/content/featured-figures-basic-operations.

Treisaran. *Dozenal Divisibility Tests Quick Guide*. http://www.dozenal.org/drupal/content/dozenal-divisibility-tests-quick-guide.

None. *The Aspirant's Tests*. http://www.dozenal.org/drupal/content/aspirants-tests.

Pendlebury, Tom and Donald Goodman. *TGM: A Coherent Dozenal Metrology*. http://www.dozenal.org/drupal/content/tgm-coherent-dozenal-metrology.

Suga, Takashi. *Proposal for the Universal Unit System*. http://www.dozenal.com.

DeVlieger, Michael. *Dozenal Frequently Asked Questions*. http://www.dozenal.org/drupal/content/dozenal-frequently-asked-questions.