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A Duodecimal Scale

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The mode of reckoning by *twelves* or *dozens*, may be supposed to have had its origin in the observation of the celestial phenomena, there being twelve months or lunations commonly reckoned in a solar year. The Romans likewise adopted the same number to mark the subdivisions of their unit of measure or of weight. The scale appears also in our subdivisions of weights and measures, as twelve ounces to a pound, twelve inches to a foot; and is still very generally employed in wholesale business, extending to the second and even to the third term of the progression. Thus, *twelve* dozen, or 144, make the *long hundred* of the northern nations, or the *gross* of traders; and twelve times this again, or 1728, make the *double* or *great gross*.

A Duodecimal Scale

As already explained, any number may be made the basis of a system of numeration and notation. The decimal basis is a mere accident, and in some respects an unfortunate one, both for science and art. The duodecimal basis would have been greatly superior, giving greater simplicity to the science, and facilitating its various applications. In this chapter it will be explained how arithmetic might have been developed upon a duodecimal basis.

In order to make the matter clear, I call attention to two or three principles of numeration and notation. First, the bases of numeration and notation should be the same; that is, if we write numbers in a duodecimal system, we should also name numbers by a duodecimal system. Second, in naming numbers by any system, we give independent names up to the base, and then reckon by groups, using the simple names to number the groups. Bearing these principles in mind, we are ready to understand Numeration, Notation, and the Fundamental Rules in Duodecimal Arithmetic.

Numeration.—In naming numbers by the duodecimal system, we would first name the simple numbers from one to eleven, and then, adding one more unit, form a group, and name this group twelve. We would then, as in the decimal system, use these first names to number the groups. Naming numbers in this way, we would have the simple names, *one*, *two*, *three*, etc., up to *twelve*. Continuing from *twelve*, we would have *one and twelve*,

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two and twelve, *three and twelve*, etc., up to *twelve and twelve*, which we would call *two twelves*. Passing on from this we would have *two twelves and one*, *two twelves and two*, etc., to *three twelves*, and so on until we reach *twelve twelves*, when we would form a new group containing *twelve twelves*, and give this new group a new name, as *gross*, and then employ the first simple names again to number the *gross*. In this way we would continue grouping by twelves, and giving a new name to each group, as in the decimal scale by tens, as far as is necessary.

These names, in the duodecimal system, would naturally become abbreviated by use, as the corresponding names in the decimal system. Thus, as in the decimal system *ten* was changed to *teen*, we may suppose twelve to be changed to *teel*, and omitting the "and" as in the common system, we would count *one-teel*, *two-teel*, *thir-teel*, *four-teel*, *fif-teel*, *six-teel*, etc., up to *eleven-teel*. *Two-twelves* might be changed into *two-tel*, or *twen-tel*, corresponding to *two-ty* or *twenty*, and we would continue to count *twentel-one*, *twentel-two*, etc. *Three-twelves* might be contracted into *three-tel* or *thirtel*, corresponding to *three-ty* or *thirty* of the decimal system; *four-twelves* to *fourtel*, *five twelves* to *fiftel*, etc., up to a *gross*. Proceeding in the same manner, a collection of twelve gross would need a new name, and thus on to the higher groups of the scale.

In this manner, the names of numbers according to a duodecimal system could be easily applied. Were we actually forming such a system, the simplest method would be to introduce only a few new names for the smaller groups, and then take the names of the higher groups of the decimal system, with perhaps a slight modification in their orthography and pronunciation, to name the higher groups of the new scale. Thus, *million*, *billion*, etc., could be used to name the new groups without any confusion, as they do not indicate any definite number of units to the mind, but merely so many collections of smaller collections. Indeed, even the word thousand, with a modification of its orthography, say *thousun*, might be used to represent a collection of twelve groups, each containing a *gross*, without any confusion of ideas. Their etymological formation would not be an objection of any particular force, as no one in using them thinks of their primary signification. These terms are not suggested as the best, but as the simplest in making the transition from the old to the new system. It will also be noticed that our departure in the decimal scale from the principle of the system, by using the terms *eleven* and *twelve*, would facilitate the adoption of a duodecimal system.

To illustrate the subject more fully, let us adopt the names suggested, and apply them to the scale. Naming numbers according to the method explained, we would have the names as indicated in the following series:

one	oneteel	twentel-one	one gross and one
two	twoteel	twentel-two	one gross and two
three	thlrteel	twentel-eight	two gross and five
four	fourteel	twentel-eleven	six gross and seven
five	fifteel	thirtel-one	ten gross and eight
six	sixteel	fortel-two	eleven gross am nine
seven	seventeel	fiftel-six	one thousun
eight	eighteel	sixtel-eight	one thousun and five
nine	nineteel	seventel-nine	one thousun four gross and seven
ten	tenteel	tentel-ten	
eleven	eleventeel	eleventel-eleven	two thousun seven gross and fortel-one
twelve	twentel	one gross	

Notation.—The writing of numbers by the duodecimal system would be an immediate outgrowth of the method of naming numbers in this system. As in the decimal system of notation, it would be necessary to employ a number of characters one less than the number of units in the base, besides the character for nothing. Since the group contains *twelve* units, the number of significant characters would be *eleven*—two more than in the decimal system. For these characters we should use the nine digits of the decimal system, and then introduce new characters for the numbers *ten* and *eleven*. To illustrate, we will represent *ten* by the character Φ and *eleven* by Π.

These characters, with the zero, would be combined to represent numbers in the duodecimal scale in the same manner as the nine digits represent numbers in the decimal scale. Thus, *twelve* would be represented by 10, signifying one of the groups containing *twelve*; 11 would represent *one and twelve*, or *oneteel*; 12 would represent *two and twelve*, or *twoteel*; 13 would represent *thirteel*; 14, *fourteel*; 15, *fifteel*, etc. Continuing thus, 20 would represent *two twelves*, or *twentel*; 21, *twentel-one*; 23, *twentel-three*, etc. The notation of numbers up to a *thousun* may be indicated as follows:—

one, 1	twelve, 10	thirtel, 30
two, 2	oneteel, 11	thirtel-two, 32
three, 3	twoteel, 12	thirtel-five, 35
etc., etc.	twentel, 20	thirtel-ten, 3Φ
nine, 9	twentel-one, 21	thirtel-eleven, 3Π
ten, Φ	twentel-ten, 2Φ	one gross, 100
eleven, Π	twentel-eleven, 2Π	one thousun, 1000

Extending the series as explained above, we should have the following notation table:—

Trillyuns.	Gross of Billyuns.	Twelves of Billyuns.	Billyuns.	Gross of Millyuns.	Twelves of Millyuns.	Millyuns.	Gross of Thousuns.	Twelves of Thousuns.	Thousuns.	Gross.	Twelves.	Units.
8	5	Π	4	6	Π	8	5	7	Φ	3	6	5

From the explanation given it is clearly seen that a system of duodecimal arithmetic might be easily developed, and readily learned and reduced to practice. Employing the names which I have indicated, or others similar to them, the change from the decimal to the duodecimal system would be much less difficult than has usually been supposed. It would be necessary to learn the method of naming and writing numbers, which we have seen is very simple, and a new addition and multiplication table, from which we could readily derive the elementary differences and quotients. The rest of the science would be readily acquired, as all of its methods and principles would remain unchanged. Indeed, so readily could the change be made, that in view of the great advantages of the system, one is almost ready to believe that the time will come when scientific men will turn their attention seriously to the matter and endeavor to effect the change.

Fundamental Operations.—In order to show how readily the transition could be made, I will present the method of operation in the fundamental rules. We would proceed first to form an addition table containing the elementary sums, which, as in the decimal system, we would commit to memory. From this we could readily derive the elementary differences used in subtraction. Such a table is given on page 6.

By means of this table we can readily find the sum or difference of numbers expressed in the duodecimal system. To illustrate, required the sum of 487Π, 5Φ38, 63Π7, Φ856. The solution of this would be as follows: Adding the column of units, 6 units and 7 units are 11 units, and 8 units are 19 units, and Π units are 28 units, or 2 twelves and 8 units; writing the units, and carrying 2 to the column of twelves, we have 2 twelves and 5 twelves are 7 twelves, and Π twelves are 16 twelves, and 3 twelves are 19 twelves, and 7 twelves are 24 twelves, or 2 gross and 4 twelves; writing the twelves, and carrying 2 to the third column, we have 2 gross and 8 gross are Φ gross, and 3 gross are 11 gross, and Φ gross are 1Π gross, and 8 gross are 27 gross, or 2 thousuns and 7 gross; 2 thousuns and Φ thousuns are 10 thousuns, and 6 thousuns are 16 thousuns, and 5 thousuns are 1Π thousuns, and 4 thousuns are 23 thousuns; hence the amount is 23748.

Operation
487Π
5Φ38
63Π7
Φ856
23748

To illustrate subtraction let it be required to find the difference between 6428 and 2564. We would solve this as follows: Subtracting 4 units from 8 units we have 4 units remaining; we cannot take 6 twelves from 2 twelves, so we add 10 twelves and have 12 twelves; 6 twelves from 12 twelves leaves 8 twelves; carrying 1 to 5 we have 6 gross; we cannot take 6 gross from 4 gross; adding 10 as before we have 6 gross from 14 gross leaves Φ gross; adding 1 thousun to 2 thousuns, we have 3 thousuns from 6 thousuns leaves 3 thousuns; hence the remainder is 3Φ84.

Operation
6428
2564
3Φ84

In order to multiply and divide, we first form a multiplication table similar to that now used in the decimal system, and commit it to memory. This table need not extend beyond "twelve times," as in our present system there is no need of extending beyond "ten times." From this table of elementary products, we can readily derive the table of elementary quotients as we do in the decimal system. Such a table will be found on page 6.

It will be interesting to notice several peculiarities of this table, similar to those of the decimal system. As the column of "five times" ends alternately in 5 and 0, making it so easily learned by children, so the column of "six times" in the duodecimal table will end alternately in 6 and 0. In our present table the sum of the two terms of each product in the column of "nine times" equals nine, so in the duodecimal table, the sum of the two terms of each product in the column of "eleven times" equals eleven. We also notice that each product in the column of "twelve times" ends in 0, as does each product in the column of "ten times" of our present table.

By means of the multiplication table we can readily find the product or quotient of numbers expressed in the duodecimal scale. To illustrate multiplication, let it be required to find the product of 54Φ8 by 3Π7. We would solve this as follows: Using the first term of the multiplier, 7 times 8 are 48, 7 times Φ are 5Φ, and 4 are 62, 7 times 4 are 24 and 6 are 2Φ, 7 times 5 are 2Π and 2 are 31, making the first partial product 31Φ28; multiplying by Π we have Π times 8 are 74, Π times Φ are 92 and 7 are 99, Π times 4 are 38 and 9 are 45, Π times 5 are 47 and 4 are 4Π; 3 times 8 are 20, 3 times Φ are 26 and 2 are 28, 3 times 4 are 10 and 2 are 12, 3 times 5 are 13 and 1 are 14. Adding up the partial products, we have as the complete product, 1953768.

Operation
34Φ8
3Π7
31Φ28
4Π594
14280
1953768

To illustrate division, let it be required to find the quotient of 1953768 divided by 3Π7. We would solve this as follows: We find that the divisor is contained in the first four terms of the dividend 5 times, and multiplying 3Π7 by 5 we have 179Π; subtracting this from the dividend we have a remainder, 174; bringing down the next figure of the dividend and proceeding as before, we have for the quotient 54Φ8.

Operation

```
3Π7)1953768)54Π8
     179Π
    ------
      1747
      13Φ4
    ------
       3636
       337Φ
    ------
        2788
        2788
    ------
```

The method of finding the square or cube root of a number expressed in the duodecimal scale is similar to that used in the decimal scale, as may be shown by an example. Thus, find the square root of Π5301. The greatest square in Π is 9; subtracting and bringing down a period, and dividing by 2 times 3 or 6, we find the second term of the root to be 4; completing the divisor and multiplying 64 by 4, we have 214; subtracting and bringing down, we have 3Π01, and dividing by 2 times 34, or 68, we have 7 for the last figure of the root; completing the divisor and multiplying it by 7, we have 3Π01, which leaves no remainder.

Operation

```
   Π·53·01(347
   9
64)253
   214
687)3Π01
    3Π01
```

The above tables and calculations seem awkward to one who is familiar with the decimal system; but it should be remembered that a beginner would learn the addition and multiplication tables and the calculations based on them, just as readily as he now learns them in the decimal system. The practical value of such a system, in addition to what has already been said, may be seen in the calculation of interest, the rules for which would be greatly simplified on account of the relation of the number of months in a year (12) to the base, and also of the relation of the rate to the same, which would be some 8% or 9% ; that is, 8 or 9 per gross. I hope to be able in a few years to publish a small work in which the whole science of arithmetic shall be developed on the duodecimal basis.

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Addition Table

$2+1=3$	$3+1=4$	$4+1=5$	$5+1=6$	$6+1=7$	$7+1=8$	$8+1=9$	$9+1=\Phi$	$\Phi+1=\Pi$	$\Pi+1=10$	$10+1=11$
$2+2=4$	$3+2=5$	$4+2=6$	$5+2=7$	$6+2=8$	$7+2=9$	$8+2=\Phi$	$9+2=\Pi$	$\Phi+2=10$	$\Pi+2=11$	$10+2=12$
$2+3=5$	$3+3=6$	$4+3=7$	$5+3=8$	$6+3=9$	$7+3=\Phi$	$8+3=\Pi$	$9+3=10$	$\Phi+3=11$	$\Pi+3=12$	$10+3=13$
$2+4=6$	$3+4=7$	$4+4=8$	$5+4=9$	$6+4=\Phi$	$7+4=\Pi$	$8+4=10$	$9+4=11$	$\Phi+4=12$	$\Pi+4=13$	$10+4=14$
$2+5=7$	$3+5=8$	$4+5=9$	$5+5=\Phi$	$6+5=\Pi$	$7+5=10$	$8+5=11$	$9+5=12$	$\Phi+5=13$	$\Pi+5=14$	$10+5=15$
$2+6=8$	$3+6=9$	$4+6=\Phi$	$5+6=\Pi$	$6+6=10$	$7+6=11$	$8+6=12$	$9+6=13$	$\Phi+6=14$	$\Pi+6=15$	$10+6=16$
$2+7=9$	$3+7=\Phi$	$4+7=\Pi$	$5+7=10$	$6+7=11$	$7+7=12$	$8+7=13$	$9+7=14$	$\Phi+7=15$	$\Pi+7=16$	$10+7=17$
$2+8=\Phi$	$3+8=\Pi$	$4+8=10$	$5+8=11$	$6+8=12$	$7+8=13$	$8+8=14$	$9+8=15$	$\Phi+8=16$	$\Pi+8=17$	$10+8=18$
$2+9=\Pi$	$3+9=10$	$4+9=11$	$5+9=12$	$6+9=13$	$7+9=14$	$8+9=15$	$9+9=16$	$\Phi+9=17$	$\Pi+9=18$	$10+9=19$
$2+\Phi=10$	$3+\Phi=11$	$4+\Phi=12$	$5+\Phi=13$	$6+\Phi=14$	$7+\Phi=15$	$8+\Phi=16$	$9+\Phi=17$	$\Phi+\Phi=18$	$\Pi+\Phi=19$	$10+\Phi=1\Phi$
$2+\Pi=11$	$3+\Pi=12$	$4+\Pi=13$	$5+\Pi=14$	$6+\Pi=15$	$7+\Pi=16$	$8+\Pi=17$	$9+\Pi=18$	$\Phi+\Pi=19$	$\Pi+\Pi=1\Phi$	$10+\Pi=1\Pi$
$2+10=12$	$3+10=13$	$4+10=14$	$5+10=15$	$6+10=16$	$7+10=17$	$8+10=18$	$9+10=19$	$\Phi+10=1\Phi$	$\Pi+10=1\Pi$	$10+10=20$

Multiplication Table

$2\times1=2$	$3\times1=3$	$4\times1=4$	$5\times1=5$	$6\times1=6$	$7\times1=7$	$8\times1=8$	$9\times1=9$	$\Phi\times1=\Phi$	$\Pi\times1=\Pi$	$10\times1=10$
$2\times2=4$	$3\times2=6$	$4\times2=8$	$5\times2=\Phi$	$6\times2=10$	$7\times2=12$	$8\times2=14$	$9\times2=16$	$\Phi\times2=18$	$\Pi\times2=1\Phi$	$10\times2=20$
$2\times3=6$	$3\times3=9$	$4\times3=10$	$5\times3=13$	$6\times3=16$	$7\times3=19$	$8\times3=20$	$9\times3=23$	$\Phi\times3=26$	$\Pi\times3=29$	$10\times3=30$
$2\times4=8$	$3\times4=10$	$4\times4=14$	$5\times4=18$	$6\times4=20$	$7\times4=24$	$8\times4=28$	$9\times4=30$	$\Phi\times4=34$	$\Pi\times4=38$	$10\times4=40$
$2\times5=\Phi$	$3\times5=13$	$4\times5=18$	$5\times5=21$	$6\times5=26$	$7\times5=2\Pi$	$8\times5=34$	$9\times5=39$	$\Phi\times5=42$	$\Pi\times5=47$	$10\times5=50$
$2\times6=10$	$3\times6=16$	$4\times6=20$	$5\times6=26$	$6\times6=30$	$7\times6=36$	$8\times6=40$	$9\times6=46$	$\Phi\times6=50$	$\Pi\times6=56$	$10\times6=60$
$2\times7=12$	$3\times7=19$	$4\times7=24$	$5\times7=2\Pi$	$6\times7=36$	$7\times7=41$	$8\times7=48$	$9\times7=53$	$\Phi\times7=5\Phi$	$\Pi\times7=65$	$10\times7=70$
$2\times8=14$	$3\times8=20$	$4\times8=28$	$5\times8=34$	$6\times8=40$	$7\times8=48$	$8\times8=54$	$9\times8=60$	$\Phi\times8=68$	$\Pi\times8=74$	$10\times8=80$
$2\times9=16$	$3\times9=23$	$4\times9=30$	$5\times9=39$	$6\times9=46$	$7\times9=53$	$8\times9=60$	$9\times9=69$	$\Phi\times9=76$	$\Pi\times9=83$	$10\times9=90$
$2\times\Phi=18$	$3\times\Phi=26$	$4\times\Phi=34$	$5\times\Phi=42$	$6\times\Phi=50$	$7\times\Phi=5\Phi$	$8\times\Phi=68$	$9\times\Phi=76$	$\Phi\times\Phi=84$	$\Pi\times\Phi=92$	$10\times\Phi=\Phi0$
$2\times\Pi=1\Phi$	$3\times\Pi=29$	$4\times\Pi=38$	$5\times\Pi=47$	$6\times\Pi=56$	$7\times\Pi=65$	$8\times\Pi=74$	$9\times\Pi=83$	$\Phi\times\Pi=92$	$\Pi\times\Pi=\Phi1$	$10\times\Pi=\Pi0$
$2\times10=20$	$3\times10=30$	$4\times10=40$	$5\times10=50$	$6\times10=60$	$7\times10=70$	$8\times10=80$	$9\times10=90$	$\Phi\times10=\Phi0$	$\Pi\times10=\Pi0$	$10\times10=100$