The Duodecimal Measure of Angle

A key consideration in the establishment of any number base is measurement. What will be the basis of a new system of measurement? This is a large topic, because of this, let’s focus on a relatively simple part of it. In this article, decimal units are preceded by decimal figures, dozenal units by dozenal figures.

In general, there seem to be two modes of measuring the world. The first method sets up a base unit (the king’s foot, the cubit, the number of wavelengths of light emitted by an unstable isotope) then concatenates this unit across the object to be measured until the number of units can be counted. The base unit is usually more or less arbitrary. In the first case, the civilization number base will simply be applied, bundling the quantity according to that base. If the boat is the square of a dozen cubits long, and you’re in a decimal culture, you’ve got 144 cubits; if a sexagesimal culture, you have 2 sixty and two dozen units. The challenge rests mostly in selecting a stable and reproducible base unit.

A second method takes into account that there is a more or less undeniable master unit (the day, the circle), then divides the master unit into aliquot parts which will serve as secondary units. Because we are dividing a master unit into aliquot parts, this method tends to use a highly factorable number to arrive at an arrangement friendly to the commonest fractions. In a later article we’ll explore this more deeply.

For now, let’s examine the easiest case of the second method. How should we divide the circle, given a dozenal number base? The Founders of the DSA explored this question, as have many others. In the Manual of the Dozen System on page 28; and 29; we are presented with “The Duodecimal Circle”, which then serves as the basis for that document’s exploration of trigonometry. The unit circle is divided into a dozen “duors” (two hours of astronomical right ascension, 30°, or π/6 radians). The duor is divided into a dozen “temins” (ten minutes of astronomical right ascension, 2½°, or π/72 radians).

Being new to dozenals, I encountered the Duodecimal Circle on my own in 1983 and later in 2003, and felt skeptical about a purely dozenal division of a circle. In our decimal civilization, we use a three hundred sixty degree circle. Trigonometry classes and certain calculations use radians. Why use a dozenal measurement, when no one in history aside from us doodekaplies seems to have arrived at a circle divided dozenally? On the internet a few years back, I read an assertion by a hexadecimal proponent that claimed dozenal proponents often construct studies that artificially uplift the dozen over ten or sixteen to further their case for dozenal. In the case of the division of a circle, we’ll start from the beginning to try to avoid this bias. We’ll see the most useful angles are geometrically determined. Their geometric utility demands human acknowledgement.

Dozenalists and proponents of hexadecimal and octal will agree that multiplication and division by factors of two are of utmost importance. We depart from accord when proponents of binary-power bases assert that two is the only important factor.

We begin by examining the simple fact that a circle can be produced by fully rotating a line segment in a plane about one of its points (see Figure 2A). Such a unit circle is the basis for all measurements of angle.

The first regular (two-dimensional) polygon many people will think of is the square, which can be made by propagating a line of a given length ℓ in a direction perpendicular to its length for the same length ℓ (See Figure 2C). The resultant figure has four equal sides with four equal, right angles. This figure can be rotated so that exactly 4 such figures contain the circle in Figure 2A—such a circle possesses a radius of ℓ. The square can be copied and tiled to fill up an infinite (Euclidean) two-dimensional plane.

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**The Duodecimal Circle**

In the measurement of time and angle, the greatest simplicity is attained by using the circle and the day as the fundamental units, and the lesser division as duodecimals of these. In this way no conversion is necessary between minutes of time and minutes of angle. Time and longitude are expressed by the same number. The superscript (c) can replace the degree (°) symbol.

<table>
<thead>
<tr>
<th>(dozenal)</th>
<th>(decimal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1c</td>
<td>2 hours or 30°</td>
</tr>
<tr>
<td>0.01c</td>
<td>10 minutes 30°</td>
</tr>
<tr>
<td>0.001c</td>
<td>50 seconds 12° 30′</td>
</tr>
<tr>
<td>0.0001c</td>
<td>4.166⅔ seconds 1° 2⅔′</td>
</tr>
</tbody>
</table>

**Key Trigonometric Functions**

\[
\begin{align*}
\sin \theta & = \frac{1}{2} \\
\cos \theta & = \frac{\sqrt{3}}{2} \\
\tan \theta & = \frac{1}{\sqrt{3}}
\end{align*}
\]

→ Figure 1, Remastered from material in the Manual of the Dozen System, pages 28;–29;

Volume 50; Number 1; Whole Number 10; one dozen two 12;
Graph paper demonstrates that the tiling of squares can be very handy. The Cartesian coordinate system with its familiar $x$ and $y$ axes, the street grids of cities like Chicago and Phoenix, the arrangement of columns in a big box store, all employ orthogonal arrangements of elements. Most of the built environment is based on the right angle. Our homes, offices, factories, streets, and cities commonly employ orthogonal geometry. Thus the right angle, a division of the circle into quarters, an angle of $90°$ ($\pi/2$ radians), is perhaps the most important division of the circle in everyday life.

The simplest regular polygon is a triangle, shown by Figure 2B. We can construct a triangle having three sides of equal length and three corners with the same angles, thus an equilateral triangle, simply by placing a compass at one end of a line, drawing a circle as in Figure 2A, then doing the same at the other end. We can draw straight lines from the intersections to both ends of the first lines to obtain a triangle. An equilateral triangle, if we were to copy it and cut it out, can be used to fill the circle in Figure 2A: exactly six equilateral triangles with a common vertex can fill the circle. In fact, we can tile the equilateral triangle to fill up an infinite plane just like the square. We can make equilateral triangle “graph paper”. In trigonometry, the cosine of $60°$ ($\pi/3$ radians) is exactly $\frac{1}{2}$. Because the equilateral triangle is the simplest regular polygon, because it can fill two-dimensional planes, and because precisely 6 equilateral triangles can fill a circle, it follows that such a 3-sided figure is important. Its geometry is thus important. The angles we’ve generated are all $60°$ ($\pi/3$ radians), $1/6$ of a full circle. So dividing a circle into six equal angles is an important tool.

We can observe the importance of triangles in general and equilateral triangles in particular in our everyday society. Structural engineers design trusses, bar joists, and space frames with an equilateral arrangement, because the equilateral triangle is the sturdiest two-dimensional figure. Because its sides are equal, it can be mass-manufactured. The equilateral triangle is perhaps not as apparent as arrangements made with right angles (orthogonality), but it is important in the building of our everyday structures.

For the sake of this article, we’ll call the equilateral triangle and the square “cardinal shapes”. Figures 2D and 2E show these cardinal shapes bisected (cut in half). There are three ways to bisect an equilateral triangle using one of its points, which are congruent if we rotate the triangle $120°$ ($2\pi/3$ radians). There are two ways to bisect a square using one of its points, which are congruent if we rotate the square $90°$ ($\pi/2$ radians). When we bisect an equilateral triangle, we obtain a right triangle with angles that measure $30°$, $60°$, and $90°$ ($\pi/6$, $\pi/3$, $\pi/2$ radians); see Figure 2F. When we bisect a square, we obtain a right triangle with angles measuring $45°$ and $90°$ ($\pi/4$ and $\pi/2$ radians); see Figure 2G. These bisections are important because they relate a corner of an equilateral triangle with the midpoint of its opposite side, or the diametrically-opposed corners of a square to one another. In trigonometry, the sine of $30°$ ($\pi/6$ radians) is exactly $\frac{1}{2}$. Thus, $30°$ and $45°$, one dozenth and one eighth of a circle, respectively, are of secondary importance. We’ll call the bisected equilateral triangle and the diagonally bisected square the “bisected cardinal shapes” for the sake of this article.

Figure 2H shows that the difference between the bisecting angles of the cardinal shapes is $15°$ ($\pi/12$ radians), one two-dozenth of a circle. Using $15°$ or one two-dozenth of a circle as a snap-point, one can construct any incidence of the bisecting angles of a square or equilateral triangle. In fact, before the advent of computer-aided design and drafting (CADD), draftsmen commonly used a pair of “45°” and “30°–60°” drafting triangles (see Figure 2I), along with a T-square or parallel bar precisely to obtain the common angles which are two dozenth of a circle. Thus, it is not by dozenalist design but sheer utility that the two-dozenth of a circle, or $15°$ ($\pi/12$ radian) angle is deemed important.
To be sure, other regular polygons can be drawn. The pentagon appears in regular three dimensional polyhedra, in the dodecahedron and the icosahedron, and in their symmetries. Geometrically, it is an important figure, however it cannot tile two dimensional space, and is not in common use in everyday life. The fact that most Americans can name precisely one building that is shaped like a pentagon contributes to the case that pentagonal arrangements are a curiosity because they are rare. The hexagon is important: we can see in Figure 2C that the outline of the group of triangles inscribed in the left circle is a regular hexagon. Thus the hexagon can tile two dimensional planes. Role playing games in the eighties employed the hexagonal grid system. There are some curious places (the Nassau Community College campus, the Price Tower in Bartlesville, Oklahoma, and other Frank Lloyd Wright buildings) which are arranged in a triangular-hexagonal geometry. The geometry of the hexagon is corollary to that of the equilateral triangle. There certainly are many regular polygons in use in human civilization and apparent in nature, but the commonest and most important geometries appear to be linked to the equilateral triangle and the square.

Other angles are important. On a map, we commonly hear “north northwest” or “east southeast”, the sixteen partitions of the circle. The 22½° (π/8 radian) angle, one sixteenth of a circle, is important, perhaps more significant than 15° divisions in cartography. The 22½° (π/8 radian) angle, one sixteenth of a circle, shouldn’t be ignored for this reason.

The decimal division of the circle into thirty dozen degrees is a quite handy tool (See Figure 4A and 4B). Each of the two dozen related to the equilateral triangle and the square (let’s call these two dozen angles “basic angles”) are resolved in the system of degrees without fractions. The system neatly accommodates fifths and tenths of a circle, although these are comparatively rarely used.

Under a dozenal system, we may discover that using a strictly dozenal division of the circle, perhaps using “temins” (perhaps abbreviated t) for convenience, neatly accommodates all the basic angles as well as the sixteenths of a circle without fractions (See Figure 5A and 5B). The basic angles are simply multiples of 6°. The sixteenths of a circle are multiples of 9t. If we desire to “unify” the basic and the sixteenth-circles into one system, we might regard 3t (7½°, π/24 radian) as the dozenal “basic angle”. We surrender the fifths (pentagonal symmetry) and tenths to repeating fractions, but maintain a fairly simple measure of the commonest angles. A strictly dozenal division of a circle, as presented by our Founders, thus appears sound and sufficient for everyday use.

A further exploration of division-based measurement will appear in a coming issue.

Notes for Figures 4 & 5: The unit circles shown here are drawn so that the black tickmarks are the length of the reciprocal of their denominator. Thus the tickmarks at ½ circle is ½ unit long, those at ¼ and ¾ are ¼ unit long, and those at ⅛ and ⅜ are ⅛ unit long. Red lines are used to indicate fractions of a circle which are multiples of the reciprocals of powers of two such as quarters and eighths, with the heaviest line indicating ⅛. Gold lines indicate multiples of the reciprocals of multiples of three, with the thirds receiving the heaviest lines. Blue lines indicate multiples of the reciprocals of the multiples of five, with fifths receiving the heaviest line. In the enlarged quadrants at right, the dashed line reminds us that the 60° angle is one which generates an equilateral triangle.