The Dozenal Society of America

A Brief Introduction to Dozenal Counting

by Professor Gene Zirkel

Most of the world evolved *a counting system based on ten*, but *a system of weights and measures based on twelve.* **Why?**

Origins

For the most part, our ancestors counted on their fingers. In a world where communication was limited, most societies independently developed a ten-based counting system. Of course there were exceptions. A few barefoot tribes counted in twenties, the Babylonians used sixty, and one tribe in South America counted in threes. Can you guess upon what parts of their bodies they counted? *(Answer given below.)*

At the same time, practical people measured in dozens. Once again, people throughout the world independently arrived at the same conclusion. Thus:

- the baker sold donuts in collections of twelve
- the carpenter divided the ruler into twelve subdivisions
- the grocer dealt in dozens and in dozens of dozens or grosses
- the pharmacist and the jeweler still use the twelve ounce pound
- the minters divided the shilling into twelve pence, etc.

Why? Counting in tens is *a biological accident*. If only we had been born with twelve fingers, how much simpler all this would be. But measuring was not accidental. It was devised by practical people who used the fractions: ½, ⅓, and ¼. That is why merchants and tradespeople chose to divide their units of weights and measures into twelve parts. Simply put, by choosing twelve subdivisions, they could have their cake and eat it too. They could use the three most common fractions without having to actually employ fractional notation. For ½, ⅓, and ¼ of a foot are 6, 4, and 3 inches respectively—whole numbers, not fractions!

Thus using the period (.) for the fraction point in base ten and the semicolon (;) in base twelve, we obtain the following:

	DECIMAL			DOZENAL		
½	5 tenths	0.5	1 digit	6 twelfths	0;6	1 digit
⅓	± 3 tenths	± 0.333…	∞ digits	4 twelfths	0;4	1 digit
¼	2½ tenths	0.25	2 digits	3 twelfths	0;3	1 digit

Solutions

Over the course of time, many suggested that we try to align our counting and our measurements. Proposals were made advocating various bases. For example base eight was offered as a solution to this dilemma, since in base eight halves, quarters and eighths are simplified. Computer scientists use a similar idea when they switch between bases two and sixteen.

The desirability of aligning our counting—which is based on a biological accident—with our measuring—which was devised by pragmatic people—was well understood at the time of the French Revolution. It was evident that either counting should be changed to base twelve or that measuring should be changed to base ten so as to be in agreement with one another. The French blundered into changing the wrong one. Maladroitly, they decided to keep the accidental and to change the practical. It is analogous to cutting off one's toes instead of obtaining a larger shoe. (Or as G. K. Chesterton said, "Cutting heads to fit hats" [1].)

Human Progress

Good ideas are often resisted when they are first presented. For example, some localities passed laws that a person holding a lantern was required to walk in front of an automobile lest these new-fangled, frivolous toys frighten horses that were needed for commerce and industry. Of course, eventually good ideas do win out. But not once—*never*—in the course of history has any society, anywhere, ever voluntarily adopted the unfortunate decimal metric system. Why is it that in every country where it is required today, it had to be forced upon an unwilling populace by law with the threat of fines and/or imprisonment? Are all of us everywhere so ignorant of what is good for us that a few Big Brothers in government must tell us how we must sell butter and rugs to one another? I don't think so. I think that common people have resisted and rejected this accident in favour of simple ordinary fractions because they know which is really more convenient.

In the United States, every pupil in science class is taught the so-called advantages of the abominable decimal metric system. Metric measuring devices are available. Yet when given a chance to measure something for their own use, a chance to use whatever measure they prefer, they use dozenal measures because fractional parts of units are easier to handle.

A Misconception

Some people wrongly believe that the ability to multiply and divide by powers of the base by simply moving the fraction point is an advantage special to base ten. But such is *not* the case. It is not "ten-ness" that gives this property (after all it wouldn't work with ten-based Roman Numerals). No, *this advantage exists in every base*, for it is a property of the *place value notation* we use for expressing numbers along with *a symbol for zero*. Thus we see that

$$(110.11) / 10^2 = 1.1011$$

is always true, *no matter what base one is using.*

Counting

In base ten counting we use ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The numeral 342 represents $(3 \times \text{ten}^2) + (4 \times \text{ten}) + 2$. In dozenal counting we use twelve symbols, adding two digits to represent ten and eleven since 10 still represents the base. The numeral 342 represents $(3 \times \text{twelve}^2) + (4 \times \text{twelve}) + 2$. Thus 342; in dozenals represents 482. in the familiar awkward decimal base.

Many people use ↊ and ↋ to represent the digits for ten and eleven[2]. They are pronounced *ten* and *elv*. Counting proceeds as in the accompanying base twelve multiplication table.

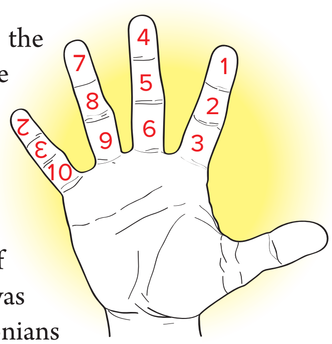
The Base Twelve Multiplication Table

1	2	3	4	5	6	7	8	9	↊	↋	10
2	4	6	8	↊	10	12	14	16	18	1↊	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	↊	13	18	21	26	2↋	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2↋	36	41	48	53	5↊	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
↊	18	26	34	42	50	5↊	68	76	84	92	↊0
↋	1↊	29	38	47	56	65	74	83	92	↊1	↋0
10	20	30	40	50	60	70	80	90	↊0	↋0	100

Conclusion

The above are some of the reasons why thinking people advocate a gradual change to dozenal counting. Because of the prevalence of computers, many students at present are being taught about base two and base sixteen counting. It would be simple to teach children both a dozenal metric system and the ill- advised decimal metric system, and then allow them to freely use the one they prefer. In one generation awkward systems would go the same way ancient Roman Numerals have gone—relegated to clocks, cornerstones and other curiosities. Remember, until the Crusaders brought what are called the Hindu-Arabic numerals to the West, all of European commerce was dependent upon Roman Numerals, and many people were convinced that they would never be changed.

In answer to the earlier question: the South Americans mentioned above counted on the segments (phalanges) of the fingers. If one uses the thumb as a pointer, one can easily count to twelve on one hand.



Incidentally, whereas the basis of almost every system of counting was the result of biology, the Babylonians were the one civilization that intelligently developed a number base—base sixty. If twelve has the advantage of the factors 2, 3, 4, and 6, sixty has 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. Note that sixty is the least common multiple of [twelve] and ten.

Notes

1 "One very common form of the blunder is to make modern conditions an absolute end, and then try to fit human necessities to that end, as if they were only a means. Thus people say, 'Home life is not suited to the business life of today.' Which is as if they said, 'Heads are not suited to the sort of hats now in fashion'. Then they might go round cutting off people's heads, and calling it The Hat Problem." From *The Collected Works of G.K. Chesterton: The Illustrated London News, Vol. 34, 1926-1928.* Entry "December 11, 1926". 1991. San Francisco: Ignatius Press. ISBN 9780898702941.

2 Prof. Zirkel's original article used the "Bell" transdecimal numerals (numerals symbolizing digits greater than digit-nine), where ⅹ = digit-ten and ♮ = digit eleven. This has been converted to the DSGB's standard, Issac Pitman's transdecimal numerals, where ↊ = digit-ten and ↋ = digit eleven.

This document was remastered 18 January 2011 by Michael Thomas De Vlieger. The dozenal numerals ten and elv were changed, all tables and figures were updated, and notes were added.