Is the decimal system appropriate for modern mathematics?

Extended Project Qualification

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Contents

Abstract

Despite its extreme relevance in all places of the world, the concept of a number base is unaccustomed to many confronted with the question of this work. First of all, the use of numbers and their origin is looked at, to gain greater perspective of what the purpose of number systems are. This information can then be applied to form arguments for and against the use of base ten, going onto analysis of another seemingly useful base: 12. The question therefore leaves no stone unturned when addressing mathematical application; the 'modern mathematics' mentioned refers to mathematical operations in science and furthering research, which the decimal system is accused of hindering. This dissertation explores the reason for counting at all; the logic behind choosing the decimal system; and the base claimed to be an appropriate alternative.

This document will avoid in all possible cases to use number digits, in light of the misrepresentation of the same digits under a separate base.

Words in bold can be found in the glossary on page 26.

Introduction

"Numbers do not come from things themselves, but from the mind that studies things" - Georges Ifrah

For thousands of years, philosophers and intellectuals alike have attempted to fathom what we are now fed and taught from the age of three. Counting in a number other than 10 is a difficult concept for the average mind to grasp, given we only use digits from 0-9 in everyday life. Since the only number system we encounter now, apart from computer science students and over-enthusiastic mathematicians, is the **decimal** system, the whole idea of number bases is somewhat lost on the common person. A **base** is defined very simply by Carol Vorderman as the number of values that can be shown using only one digit. As mentioned, our most used base – the decimal – is also known as base 10. We all use numbers and patterns constantly, sometimes without being aware, hence maths is a core subject in the national curriculum; all ages and variations will be affected by a different counting system.

To provide context, the QI 'elves' (BBC) and J J O'Connor at the University of St Andrews both agree: the Babylonians are the people around circa 4000 BC, in the ancient tribe of Babylonia, whom can be held responsible for the birth of modern mathematics as we know it. It is from these that we find our angles in a circle, seconds in a minute and minutes in an hour. The ancient people hence used base 60 to count with – called **sexagesimal**. This example is the birth of many other number systems that varied from continent to continent and were revolved around the culture and needs of the people at that time.

Arguments against the decimal system are assuredly not a new event for 21st century scientists. In fact, they can be traced back to when the **metric system** was first conjured. Since then, many mathematicians have voiced their opinion on the subject, many of whose work will be included in the sources and analysed over the course of the dissertation. In particular, the **duodecimal** system has been favoured as an alternative to the current decimal, with societies in Great Britain, America and Australia being established especially for the cause. The connection between all their minds and this work comes down to a simple question, which – whilst being in plain sight – goes unconsidered by the majority: should we be counting in tens?

1: The relevance of the number

"*Counting is one of the very few things which modern man takes for granted*" –
F. Emerson Andrews, 1934

An essential ground to cover before analysing and evaluating the use of numbers is the understanding of counting itself. Frankly, a background understanding of where this tool originated will provide essential insight when drawing a conclusion of the appropriateness of particular systems.

1:1 Numbers as a sixth sense

It could be suggested that counting itself is a natural instinct with which humans are born. Ifrah (1994) gives good examples in his writing to propose a concept of size and number naturally occurring in beings by looking at animals less intelligent than humans, such as birds and some land mammals. The Goldfinch bird, for example, when presented with two piles of seed of different quantity, is able to distinguish between three and one seed, three and two seeds, and six and three seeds, and so forth. In numbers larger than this, Goldfinches would confuse the two piles. This demonstrates a cognitive sense of numbers and quantity present in animals and humans alike. Sensual judgement decreases as magnitude increases; that is, it is more difficult to judge a larger quantity (Ananstthwamy 2017), a philosophy perfectly explained by the Goldfinch experiment. Included in this article is a reference to Stanislas Dehaene's book 'The Number Sense' (1997), in which Deheane proposes the idea that we are born with a cognitive sense of numbers, just as we are colours. From the evidence given, it could be concluded that beings have a natural concept of quantity, therefore begging the creation of a numeral system.

1:2 Numbers as a counting tool

Numbers began when food and trade came into the matter, as mentioned by both Ifrah and Ball in their respective books. Tradesmen began to use their fingers to monitor how many goods they were giving and receiving. This, essentially, could be marked the start of using base ten. Many mathematicians, including Ball, have named it an 'accident of biology' that we have ten fingers and therefore use ten digits. Ball goes further to suggest that if there is other intelligent life with eight fingers, they will very likely have based their number system around base eight. Being a successful mathematician and popular presenter of children's television, this can be deemed a reliable source of information, though possibly modified to be made understandable for a younger age group.

Number systems are typically limited to the needs of the people; this is true for people as numbers were created and in modern day. In a study carried out on nearly two hundred Australian aboriginal languages, three quarters were found to not have a translation for numbers greater than four, with a further twenty-one not using numbers above five (Anansthwamy 2017). For all dealing in trade and larger numbers, however, five was not a sufficient number of digits to be used in counting. In fact, many ancient philosophers actually studied geometry and astronomy because the numbers of the time were not yet capable of the complicated **calculus** their theorems would go on to show (Andrews 1934). And so the Romans, arguably the greatest empire of all time, devised a method to handle counting beyond five.

Roman numerals follow the decimal rules yet have many issues when used in mathematical settings. Beyond four, drawing vertical lines to represent fingers, supposedly, began to get cumbersome. According to Andrews (1934), the scribe then notated a V when five was reached, to represent a whole hand. Upon reaching two full hands, two Vs would be drawn crossing over – in the shape of X. Larger powers of ten were given their own letters: C represents one hundred (from the Latin *centum*), M represents one thousand (*mille*) and L for fifty. Since this was early A.D., not many digits were required to transcribe the year, making the method suitable for their time, which reinforces the idea of limitations to a number system due to cultural demands. When it comes to writing current years, such as 1999, and carrying out complicated mathematics as we do today, however, the Roman numeral system serves very little purpose. Andrews gives the simple example of multiplying CCCLXV by III, requiring a long method to calculate the product DCCCCLLXXXVV. As is evident, a new way of writing numbers would have to surpass the Roman numeral system in order for mathematics to advance. This journalist has opinionated views in favour of the dozenal argument (see Chapter 4), so the information included may be biased. As well as this, the articles were published in the mid twentieth century, so new intelligence found since may deem the articles invalid. Despite this, other sources such as Ball agree with Andrews, making the material credible.

1:3 Numbers as they are today

Today, regardless of base, most of the world uses the Indo-Arabic numerals: 0 1 2 3 4 5 6 7 8 and 9. These arrived on the English shore in 1551, due to a commercial traveller Leonardo Pisano (Aitken 2010). Pisano witnessed the notation's superiority whilst on business and spread the idea; whether this means the advantages of using base ten were overwhelming or not cannot be known.

Other than the regular educational mathematics class, numbers appear in the most unlikely places and are integrated in our language and society. 'As Easy As Pi' (Buchan 2009), illustrates well where we use numbers every day; the simple example of 'sixes and sevens' is as follows.

This expression is used to describe someone lost or confused in England, but the saying actually originated in France in the eighteenth century. Gambling games were popular then, one in particular being to bet on the outcome of a die roll. Players of the game considered betting on higher numbers to be more risky, though the maths of probability contrasts this. Thus, someone betting on fives or sixes (cinq et six, in French) was assumed to be confused. This phrase made its way to England by the nineteenth century but upon arrival was translated incorrectly into 'sixes and sevens'. Strangely enough, this saying seems to make more sense than the original 'fives and sixes', since there is no seven on a die and their sum is the unlucky thirteen.

The perplexing anecdote demonstrates the effect of communication and correct translation. Whilst this was a harmless example, included in the decimal chapter are further examples of mistakes that had more of an impact than the humiliation of an Englishman visiting France.

2: The Decimal argument

"That is why, with these nine numerals, and with this sign 0 [...] one writes all the numbers one wishes" - Leonardo of Pisa, also known as Fibonacci, 1202

Ten is a common place in the life of man and any alternative is a difficult life to picture. While it may seem logical, the original idea to use the tenth base came from our own anatomy. A story told originally by L.Weyl-Kailey in 1985 about the importance of number association with the body is as followers: simply enough, in a teacher's attempt to encourage mental maths, he forbade the use of counting on fingers. By the end of the lesson, the students were relieved to be able to use their finger to do maths again (Ifrah 1994). This short anecdote suggests that, especially at a young age, we are very dependent of the visual aspect of counting.

But does that mean we should use base ten? As said in the previous section, Ball (2005) reckons that if we had any other number of fingers, we would use a different number system. So if the number ten is so 'accidental', why have we chosen the decimal base as the foundations for all our current mathematics in favour of the rest? Should such a prominent mathematician such as Fibonacci think it wise, surely - one must assume - the flaws are minimal. To gain an insight on why the metric system and all other decimal implications were a positive change, the period before must be studied.

2:1 The Awaiting of Base Ten

The TV programme Horrible Histories (2011) includes a scene depicting a traveller arriving on the English shore needing a currency exchange. The two men partake in a puzzling conversation, in which the Englishman attempts to explain the division of their currency to a bewildered Frenchman, who cannot make sense of it (see Appendix 2). When using this source, it has to be considered that it is a programme aimed at children – aired on CBBC – so fast paced talking and an introduction to the sketch saying the system was "very complicated" exaggerates the effect. The information extracted is still useful, nonetheless; the way it is presented demonstrates the point that the **imperial currency** in this time period was incomprehensible to people not used to the methods. By putting us in this position, relating most to the befuddled traveller, the scene suggests our modern society would not easily accept a system other than the decimal.

The underlying message in the sketch can be shown with quantitative, modern data. In a survey, nearly 40% of the respondents rated this previous system of sovereigns, crowns and shillings to be 'difficult' to calculate with, compared with the 3% who rated the same for the decimal system. Interestingly enough, when asked why they thought the system changed, the responses suggest an average view that the decimal system was transferred to as an easier alternative to the imperial (see Appendix 3).

Numbers aren't just apparent in counting money; the systems of measurement have, as will be seen in the following section, transformed greatly over time due to the differing bases. Ball (2009) tells of the complicated units that could be found in the same era as the TV sketch aforementioned, including the disconcerting chains, furlongs, rods, and ells. The ell especially had a misleading representation. It was first used in medieval England, and is said to be obtained from the length of a man's arm: 57 centimetres. Whilst this seems a sensible and rather practical measuring unit, time did the ell no favours, as many other ways were found to interpret it. In Germany, the ell was a 40 centimetre unit; if in Scotland, a 95 centimetre length; even the English Parliament couldn't keep the length consistent and doubled what it originally was a few years later. Eventually, Switzerland referred to sixty-eight different lengths as being an 'ell'.

From evidence, it seems the imperial system was not the most suitable for everyday needs; an alternative, however, did not arrive until the eighteenth century.

2:2 The Arrival of Base Ten

In the late eighteenth century, the French revolution had overcome the leaders in power and the new students saw a need to change what used to be. According to the **Dozenal Society of America (DSA)**, 1790 saw the National Assembly of France form the Commission of the French Academy of Sciences. The students involved in this organisation saw the decimal system as the most accessible to the public, and therefore the best choice for a new measuring system. Nate Barksdale (2014), tells of the French scientists who wanted to standardise measurements across the globe and remove the four-hundred different methods of measurements as it stood at the time. They did this by dividing the distance between the North Pole and the Equator by ten million, an appropriate and immutable standard for the metre. The millimetre (one-thousandth of a metre) conveniently provided the basis for one centimetre-cubed of water - the millilitre - which in turn provided the foundation of the gram. This easy conversion means chemists can deduce the mass of water based on its volume.

Though this wasn't the first use of base ten, the metric system can mark the beginning of a new kind of numerical revolution. As expected, there was much retaliation to the change, as Barksdale continues; the infamous leader Napoleon abolished the metric measurements in

1812, only for it to return due to popular demand twenty-eight years later. The Dozenal Society of America describes, in the Manual of the Dozen System, Napoleon's opinion on this matter, quoting: "I can understand the twelfth part of an inch, but not the thousandth part of a metre."

Whilst the introduction of metres and kilograms can be considered pivotal to the promotion of base ten, another major point to consider is the decimalisation of the coin. As illustrated in the above subsection, money and different monetary values caused much confusion in history. This all seemed to be resolved when the coin was changed to the same intervals as measurements were now kept in, in 1971. A BBC news article , released to mark forty years since this decimalisation, informs the reader of Britain's late arrival into this new trend; the USA and France had already made the transition in the late eighteenth century (Freeman 2011). This change, though unapparent to many, was very gradual and went through many debates leading up to the final decision. Right back in the 1820s, Britain saw the change happening in other countries and were tempted to do the same. The most the currency would change in that century was the introduction of the florin (Royal Mint 2017); this coin, introduced in 1849, was worth one tenth of a pound - a degree of which is common to us now but would have been outlandish to the then population who were used to the factors of four, six and twelve. The information page extends to acknowledge Sir Christopher Wren's opinion on the matter, saying that in 1696 he agreed there should be five farthings to a penny, showing the architect's preference of the decimal system even before it had become the norm.

The article continues until the 1960s, in which Australia, New Zealand, and South Africa all converted their currency to decimal. By this point, Britain were far behind the norm and currency exchanges were proving a serious issue. Dr Kevin Clancy is mentioned and quoted to say that the government did not want this decimalisation to occur because it would cause too many difficulties and changes. British obstinance eventually proved unproductive as the overwhelming need to change the base of currency finally took its toll in 1966. Freeman simply describes the decision as a simple one, to which the Chancellor of Exchequer said "Why not?" A lengthy transfer of old coins to new coins meant Britain was officially declared to have decimalised their currency in 1971. Sixpences were not easily let go, and remained until the 1980s - mainly due to the work of public campaigners claiming it was 'part of their heritage'.

2:3 The Appreciation of Base Ten

At last, the metric system and pound sterling currency had rooted itself in British and global history and the people were ready to use their ten fingers for all the practical applications of maths in the real world.

This system is, today, the most common and has since been renamed the International System of Units (Ball 2009). The seven main units are the metre, kilogram, second, ampere, kelvin, candela and mole; these correspond to length, weight, time, current (electricity), temperature, luminosity and concentration respectively.

There are overwhelming advantages to the universal system of ten as we know it. One of these can be explained looking at, as illustrated before, the alternatives and what could be without one system used overall. In 1983, a Boeing 767 refuelled with 22,600 pounds (lb) of fuel, where the actual instruction was to fill up with 22,600 kilograms (Ball 2009). The amount of fuel then in the jet was less than half the required amount in kilograms. This meant that the aircraft ran out of fuel - much a surprise to the pilot - and an emergency landing was needed. Whilst none were killed in this incident, it was accident that should be avoided in all cases and could be avoided by the universal use of the metric kilogram.

Another measurement issue occurred for the Nasa team in control of an orbit droid around Mars; one team had measured the distance using the metric system, and one using the imperial. Their mistake meant the orbiter crashed into the planet, compromising years of research, volumes of fuel and an abundance of funds (ibid).

As can be concluded, the conversion to base ten was an advantageous one. With regard to ease of use, people today find the decimal system far more appropriate when compared to the previous system, where a consistent division of ten was not seen.

3: Alternative bases

It is suggested by Ball (2005) that should we have evolved with a different number of fingers, we would not automatically revert to base 10 but would favour another base. The following have been used by other civilisations both in the past and the present, showing the potential for use in the future.

3:1 Base 2

Binary will most commonly be seen in computing, and is transcribed using the digits 1 and 0. Computers will comprehend only "on" and "off" signals, where 1 digits represent an on signal and 0 an off signal (Vorderman 2014). These single binary digits are referred to as 'bits', with four bits being a '**nibble**', and eight reaching a '**byte**'. Where, in base ten, the column headings of tens, hundreds and thousands are used, the binary system refer to successive powers of two: 1, 2, 4, 8, 16, 32 etc. (decimal notation). For example, the number 1101 refers to one 8, one 4, no 2 and one 1, therefore notating 8 + 4 + 1 = 13.

In practical and everyday use, however, binary is cumbersome. To give an example, base two takes all of eight digits to express the number 132, which only takes three in decimal; even further, the duodecimal system can notate the same value with only two digits. Whilst the coding and digits can represent simple 'on' and 'off' values, highly useful in computer coding, as a widespread use, this system is not deemed appropriate.

3:2 Base 20

This base, known more formally as **vigesimal**, can be traced back to the Mayan era of 250 - 900 AD (Ball 2005). As with many civilisations, their counting system originated from objects that surrounded them on a day-to-day basis: sticks and beans. Unlike other societies, however, their numbers were written in vertical columns.

A probable source and aid of base twenty is something the majority of people have and can use: twenty digits - ten fingers and ten toes (ibid). Analysing the suitability for worldwide use, the Mayans may have built many famous monuments but their number base would not prove very beneficial to the average person. The vertical orientation of their scriptures does not match the horizontal order in which we write our words and letters; thus, if literacy and numeracy differ in the way they are written, use of either will become tangled to the extent of being counter-productive.

3:3 Base 60

The sexagesimal base is associated with the Babylonians and Sumerians, aforementioned in the introduction (Ifrah 1994, Ball 2005). In their time period - about 5,000 BC, according to Ball - the Sumerian people successfully built cities, roads and developed architecture before any other people. From tablets and scriptures, historians can also conclude they had a concept of infinity, or at least everlasting time. Their expertise in mathematical education can therefore be respected, so their choice of base would appear to have logical reason.

Sixty can be seen in many parts of simple mathematics and our order of time; for example, there are sixty seconds in a minute, sixty minutes in an hour, 360 degrees in a circle, and the measure of car speed from 0-60mph.

When examining the way that they count, one could argue the Babylonians actually used the decimal system; it could be concluded that they made the connection, as would anyone, between the ease of counting and their ten fingers. Analysis of the previous use of the sexagesimal system slowly turns into an argument supporting base ten.

Arguably, the figures of the sexagesimal system were clumsy to note down (see Appendix 1) much like the roman numerals. Introducing base sixty to the modern age would have some advantages though; one and a half minutes would be written accordingly, as 1.30, rather than the confusing conversion of 1.5 from the decimal base. The sexagesimal system is very rarely seen in use today, though, and the only alternative to scribing the difficult ‘59’ using Sumerian digits would be to create sixty different, individual digits - an almost impossible feat.

4: The Dozenal argument

But the man said to Ten, "Little Ten,

you just don't have the grace or the ken;

You're just five and a two,

And what good can five do?

Only Twelve is my sweetheart, my zen.

- DSA Newscast (2015)

The dozenal or 'zen' seems to be the most reasonable contender to oppose the decimal use. In this system, twelve is the base and the digits are from 0 to E (11, in base ten). A main advantage of twelve over ten is its superior number of factors; twelve has two, three, four and six, where ten only has five and two. So far, this system has been found to be the favoured by many other mathematicians who consider the same topical question.

4:1 The Toils of Ten

Criticism of the decimal system to which we are so accustomed is difficult to approach. Even in reading '10', one cannot refrain from thinking 'ten'. While this integer has rooted itself in our history and is unavoidable in everyday life, in reality it is impractical; when considering ideas such as music and packaging goods, the multiples of five or ten are decisively inconvenient.

The original Roman calendar was ten months, until two more were added (Ifrah 1998). Ten months in a year and twenty hours in a day was conclusively rejected, evidence that nature cannot work by multiples of ten. Andrews (1934) brings attention to the splitting of North, East, South and West - **cardinal directions** that cannot be split into ten.

George Orwell's famous novel, nineteen eighty four (2000), even includes hints towards a progressively dominating decimal system; chapter eight of the dystopian, futuristic story describes an old man arguing with the bartender, who refuses to give him a pint of beer - because he is unaware of the imperial measurement. In this projected future, the beer is served only in litres and half-litres. Clinging onto memories of the past, the old man's reactions could mirror the general response if every measurement was converted to base ten in the same way. This cannot be taken as fact, since the book is written as fiction. As with the TV sketch mentioned in Section 2, though, the author conveys his projection of society's reaction to the change - an idea that could not be caused by mere speculation.

French revolutionaries attempted to, in the 1800s - a rapidly progressive time for scientific discovery - introduce one hundred minutes in an hour, degrees around a point and ten months in the year. Aitken (2010) claims that this did not catch on due to the indivisibility and unfamiliarity of ten. If natural progression of time and rules of geometry were not susceptible to the decimal system, surely it cannot be appropriate for use in simple nor complex mathematics.

Simon Stevin, a prominent mathematician in the sixteenth century, invented the decimal point; this was revolutionary for his time because for the first time people could express fractions - or specifically tenths - of a whole number without much complication. However, despite his invention, he discovered the whole number would best be divided into twelfths rather than tenths. As quoted, Stevin could well be credited with creating bases as a fundamental, but himself declared base twelve would be more appropriate (The Duodecimal Society of America 1960). Stevin is quoted: "the decimal base is unsatisfactory because it has not enough factors" (Beard 1948). Considering he was disputably the most influential mathematician to study the concept of bases and counting, Stevin's opinion can in most ways be taken as a valid contribution to the cause. Despite the strong arguments, Beard comes to the conclusion that no change should be made globally, but that intellectuals studying and practising the mathematical sciences would find it more helpful to use base twelve, phrasing: "Duodecimal should be man's second mathematical language". Considering the time period this was written in, Beard had hopes that by 2000, this would be a more popular base to use. Evidently, there were more hurdles to this adoption than he foresaw.

4:2 The Transcendence of Twelve

The Duodecimal Society of America, though not widely known, have spread their ideas and support for base twelve across the globe. Their organisation meets regularly, produces a newscast, and has its own logo (see Appendix 6). Andrews (1960) criticises the blatant acceptance of the decimal system, as follows.

Although the Arabic system provided a way for calculus to be carried forward with ease, the drawbacks came because there were only ten digits. There are other examples of where multiples of twelve were used in everyday life such as the measuring system, now known to us as the Imperial system. Using this, the foot – consisting of twelve inches – could be divided into halves, thirds, quarters, or sixths; all of the results gave easy integers. This advantage is also evident in another imperial measure: the yard. A yard can be split into two, three, four, six, nine, twelve or eighteen parts, a wide range that proves very useful to anyone using it. If a yard were thirty-five inches, as would seem appropriate under the decimal system, it could only be split into fifths or sevenths. As well as this, groceries are always packaged in multiples or factors of twelve. An interesting observation is that the word 'grocer' comes from the same roots as 'gross' – meaning twelve lots of twelve, 144. Thus a definition is produced: a grocer is a labourer who deals by the gross; the organisation of groceries into multiples of twelve makes divisions easier, since there are more appropriate factors.

The evidence suggests that using base twelve is much more practical in daily life than base ten. If this is indeed the case, it must be questioned whether the duodecimal system could be adopted in modern and future mathematics.

4:3 A Fortifiable Future

Goodman, the President of the DSA, was asked about his reasons for believing so strongly in the future of a duodecimal system; he claims the only difference between using the duodecimal and decimal systems is the notation. Fundamentally and physically, there is no change. A box of eggs twelve spaces by two will still contain the same number of eggs; whichever way we choose to express this number does not affect how many eggs there truly are. Goodman also mentions that a main advantage of the dozenal system is human interaction with mathematics; judgement of measurements is said to be far easier with counting in twelves. A useful base for human interaction would have the factors 2 (halves), 3 (thirds), 4 (halves of halves), 6 (halves of thirds), 8 (halves of halves of halves), and 9 (thirds of thirds). Twelve is the first number to possess most of these as factors, and 8 and 9 are easy to express under base twelve.

Bellos (2012) writes about Goodman's ideas and about the dozenal movement, which has been active since the 1600s; Isaac Pitman, inventor of shorthand, is said to have suggested using upside down versions of 2 and 3 as the two additional digits needed, called ‘dek’ and ‘el’. Since this is a newspaper source, though, the information given is liable to have been exaggerated for effect. The system that has been devised by proud dozenalists of such societies contains ‘;’ as its “duodecimal point”, where the decimal system uses a full stop to separate unit columns (Manual for the Dozenal System 1960). In other places, these digits are notated using an X and E respectively (Appendix 5).

Because of our fast progressing society and knowledge, it is argued that the benefits are worth the change (ibid). Like the French decimal system, science has arguably reached the point where a new method is required to overcome further boundaries. The main change that needs to take place in order to adopt the dozenal system is our humane habits; a short period of translation between decimal and dozenal numbers would most likely be necessary, but it is claimed that the dozenal multiplication tables are much easier to compute than the decimal (Goodman 2017). Andrews (1934) demonstrates how more numbers in the dozenal multiplication table come out “in round numbers” than in the decimal. Due to its bigger base, the base-12 system is also able to express all its tables in two digit numbers, with the exception of the final product, ‘gro’ (100). It is recognised that it may be difficult to transfer from the decimal multiplication tables that today’s population so know and love; in doing this, Andrews insists it is a minor flaw that can easily be overcome, since all it requires is the training of memory. Some other advantages are that there are not as many prime numbers, factoring is easier and percentages are more applicable.

Graham (2010) argues that there is a desperate need for the change to duodecimal; again, the writer appreciates that a full change of number system would be difficult to implement. As Beard (1948) mentioned before, Graham persuades the blog readers that progression into further scientific discoveries depends on the abandonment of our current use of the decimal system. Calling the metric system “tragic”, it is suggested that changing the currently decimal SI units to measurements divisible into twelfths could be a primary step forward. This could be a plausible and logical step forward, should it be decided that the decimal system is inefficient.

Conclusion

The question to be answered, then, remains: is the base we are most familiar with appropriate for future use? Regarding modern mathematics, there are many requirements the number base has to fulfil to allow suitable progression and open doors for scientific investigation. This dissertation has explored the functionality of the current decimal system, as well as the convenience of bases other than ten.

The journalist Andrews (1970) mentions a comment he received in response to his original article, in which a man says the decimal system is too integrated in society to make the change. If a man in that period of time thought the decimal system was already too involved, in today's society the idea is hardly feasible. Buchan (2009) comments on the duodecimal society's effort to promote their base choice: "unsurprisingly, they were entirely unsuccessful". The writer takes a view that any effort to make alterations to the current number system would undoubtedly result in failure. As well as this, the strong arguments against using decimal coinage in 1918 have not stood; ten must have seemed much more advantageous that it should be adopted into currency. This is not, however, definitive proof of its practicality for modern mathematics, more its applicability to the average user.

Ultimately, our choice of decimal is due to the number of fingers on two hands. Whilst this makes association of the body with maths much simpler, the fundamental notation of numerals is easier, and thus the performing of maths is made more efficient. Under the duodecimal system, a price sale reduce of a third can be advertised as '40% off!' rather than '33.3333% off!', considerably more ambiguous than the former. As A.C.Aitken (2017) said, "It is truly extraordinary that arithmetic should have been hindered so long by a vestigial remnant from anatomy".

As seen from the survey in Appendix 3, modern society would not be ready to make the change from base ten, since they find ten's indices far easier than the multiples of twelve and six as seen in the imperial system. As Beard (1948) suggested, a more likely approach would be to change the SI units, or at least introduce a duodecimal alternate system for use by mathematicians. The gradual acceptance of this new system would mean it can eventually be integrated into society.

If this work were to be continued, an assessment of the public's opinion on accepting the duodecimal system could be carried out, not unlike the survey mentioned in the decimal argument. A society as specified as that of today's may not become easily accustomed to the idea of numbers representing different values.

Unintentionally, and most likely negligible to a reader, this work has been divided into twelve subsections: four groups of three. Consider how this easy arrangement could not have been achieved with ten subsections, limiting the dissertation to five groups of two or vice versa. Although it is sufficient for daily humane demands, the decimal system, in my own opinion and that of some established mathematicians, seems not to be appropriate for modern mathematics. In the words of French Mathematician Jean Essig, "Douze, notre dix futur" - twelve, our future ten.

References

Books

Ball, J., 2005. *Think of a Number: Everything You Need to Know about Maths*. London: Susan Leonard. Dorling Kindersley Ltd

Ball, J., 2009. *Mathmagicians*. London: Mary Ling

Ifrah, G., *The Universal History of Numbers*. Harvill Press, 1998.

Orwell, G., 2000. *Nineteen Eighty-Four,* London: Penguin Classics.

Vorderman, C., *Computer Coding for Kids*. Dorling Kindersley, 2014.

Internet

Aitken, A.C., "Twelves and Tens." *Twelves and Tens | Hexnet*, August 3 2017. Available at: hexnet.org/content/twelves-tens.

Barksdale, N., "Who Invented the Metric System?" *History.com*, A&E Television Networks, August 27 2017. Available at: www.history.com/news/ask-history/who-invented-the-metric-system.

Beard, R., "Why Change?" *Why Change? | The Dozenal Society of America*, August 2 2017. Available at: www.dozenal.org/drupal/content/why-change.html.

Manual of the Dozen System. Duodecimal Society of America Inc., 1960. 7 October 2017. Available at: http://www.dozenal.org/archive/ManualOfTheDozenSystem1174-web.pdf

O'Connor, J.J., "An Overview of Babylonian Mathematics." *Babylonian Mathematics*, July 10 2017. Available at: www-history.mcs.st-and.ac.uk/HistTopics/Babylonian_mathematics.html.

'Graham'., "An Argument For Dozenalism." *An Argument For Dozenalism | Hexnet*, August 3 2017. Available at: hexnet.org/content/argument-dozenalism

"Making the Change to Decimal Coinage." *The Royal Mint*, August 6 2017. Available at: www.royalmint.com/discover/decimalisation/the-change-to-decimal-coinage.

Magazine

Ananthaswamy, A., "The Origin of Mathematics." *New Scientist*, vol. 235, no. 3141, 2017, pp. 30–35., doi:10.1016/s0262-4079(17)31721-9.

Andrews, F.E., "My Love Affair With Dozens." *Michigan Quarterly Review: Vol. 11, No. 2*, MPublishing, University of Michigan, 1 Jan. 1970, Available at: quod.lib.umich.edu/m/mqrarchive/act2080.0011.002?view=pdf&seq=37&size=100.

TV

QI, series 13 episode 5, 'Maths' [2015]. Dave. 3 Jul. 2017.

Horrible Histories, [2009] CBBC. 27 Aug. 2017

Newspaper

Bellos, A., "Dozenalists of the World Unite! Rise up against the Tyranny of Ten! | Alex Bellos." *The Guardian*, Guardian News and Media, 12 Dec. 2012, Available at: www.theguardian.com/science/alexs-adventures-in-numberland/2012/dec/12/dozenalists-world-unite-tyranny-ten.

Freeman, L., "How Britain Converted to Decimal Currency." *BBC News*, BBC, 5 Feb. 2011, Available at: www.bbc.co.uk/news/business-12346083.

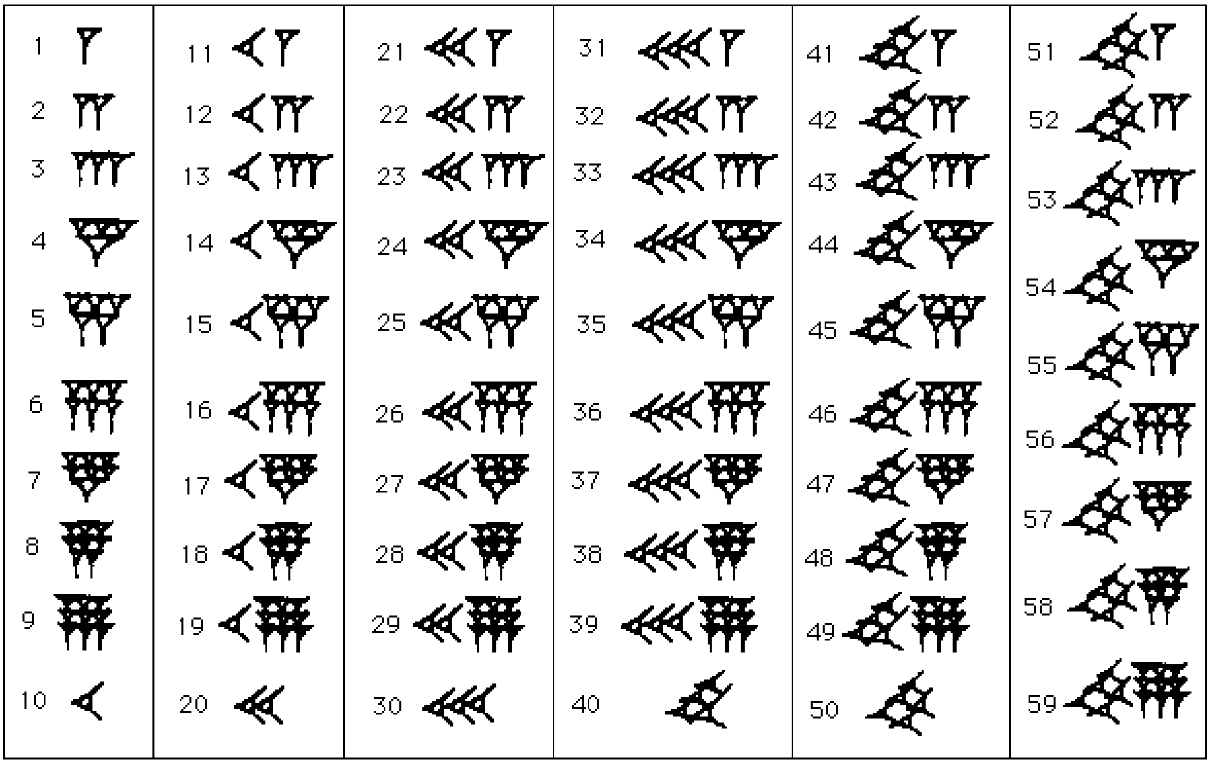
Glossary

- **Base:** The number that is raised to various powers to generate the principal counting units for a system
- **Binary:** Of or relating to a system having two as its base
- **Byte:** A unit of data equal to eight bits
- **Calculus:** The branch of mathematics that deals with limits and the differentiation and integration of functions
- **Cardinal directions:** Four principal points on a compass: North, East, South or West
- **Decimal:** A number written using base ten
- **DSA:** The Dozenal Society of America
- **Duodecimal:** Of, relating to, or based on the number 12
- **Imperial currency:** See below
- **Imperial system:** Units conforming to standards or definitions legally established in Britain
- **Metric System:** Of, or relating to, the metre
- **Nibble:** A unit of data equal to four bits
- **Vigesimal:** Of, or relating to, or based on the number 20
- **Sexagesimal:** Of, or relating to, or based on the number 60
- **SI units:** An international system of scientific units

as defined by The Free Dictionary by Farlex, https://www.thefreedictionary.com

Appendices

Appendix 1:



Source: O'Connor, J.J., "An Overview of Babylonian Mathematics." *Babylonian Mathematics*, July 10 2017. Available at: www-history.mcs.st-and.ac.uk/HistTopics/Babylonian_mathematics.html.

Appendix 2:

Name	Number of pennies
Sovereign	360
Pound	240
Angel	120
Crown	60
Shilling	12
Sixpence	6
Groat	4
Farthing	1/4

Source: *Horrible Histories*, [2009] CBBC. 27 Aug. 2017

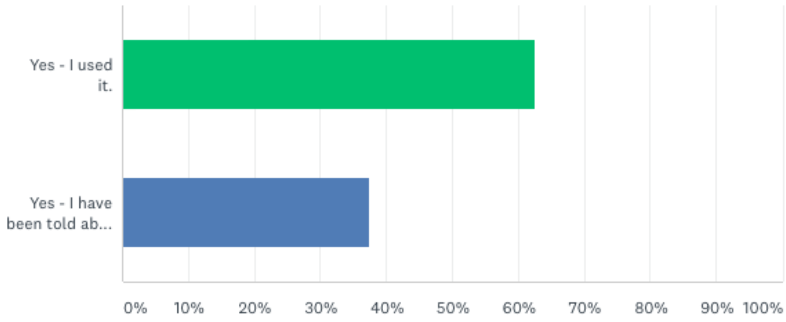
Appendix 3:

Q1

Customize Export

Do you remember the imperial coinage system before the change in 1971? (crowns, shillings, etc.)

Answered: 40 Skipped: 0



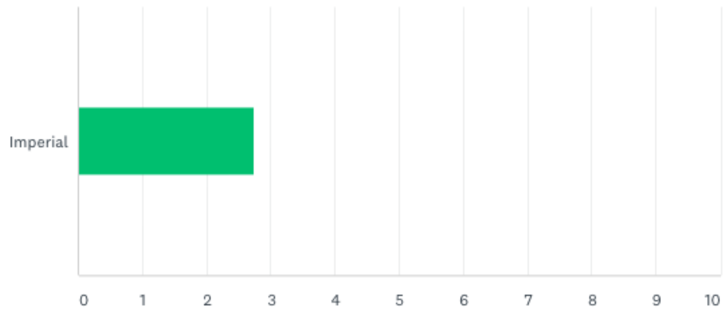
ANSWER CHOICES	RESPONSES	
Yes - I used it.	62.50%	25
Yes - I have been told about it.	37.50%	15
TOTAL		40

Q2

Customize Export

Rate how easy you think the imperial system was to use and calculate.

Answered: 39 Skipped: 1



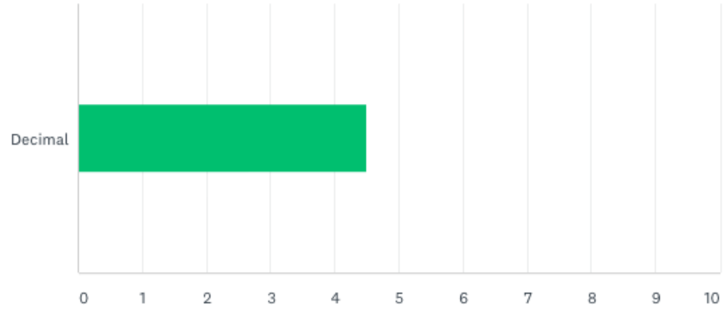
	VERY DIFFICULT	DIFFICULT	REASONABLE	EASY	VERY EASY	TOTAL	WEIGHTED AVERAGE
Imperial	7.69% 3	38.46% 15	33.33% 13	12.82% 5	7.69% 3	39	2.74

Q3

Customize Export

Rate how easy you think the decimal system is to use and calculate.

Answered: 39 Skipped: 1



	VERY DIFFICULT	DIFFICULT	REASONABLE	EASY	VERY EASY	TOTAL	WEIGHTED AVERAGE
Decimal	0.00% 0	2.56% 1	2.56% 1	38.46% 15	56.41% 22	39	4.49

Q4

Export

Since the 1600s, individuals have argued to change the way we count money. Briefly explain why you think the coin system changed from imperial to decimal in 1971.

Answered: 39 Skipped: 1

RESPONSES (39) TEXT ANALYSIS MY CATEGORIES

Categorize as... Filter by Category Search responses

Showing **39** responses

to make it easier

9/10/2017 2:59 PM View respondent's answers

It was easier

8/28/2017 12:37 PM View respondent's answers

To bring us into line with most other countries, as nearly every other country was using a decimal unit of currency and we were lagging behind.

8/28/2017 10:28 AM View respondent's answers

It was easier to calculate, for Brits, for tourists, and for use as a reserve currency.

8/27/2017 4:20 PM View respondent's answers

Easier to use and compatible with other systems

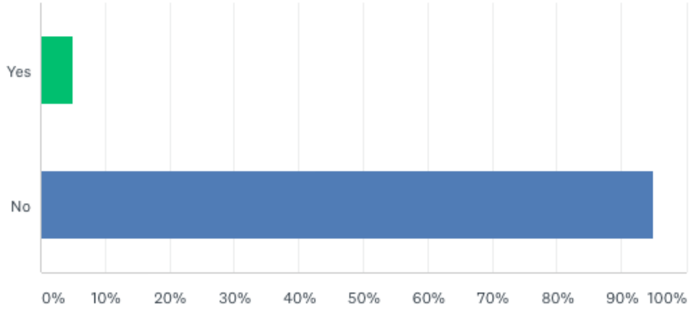
8/27/2017 8:50 AM View respondent's answers

Q5

Customize Export

Would you want money to change back to the imperial method?

Answered: 40 Skipped: 0



ANSWER CHOICES	RESPONSES	
Yes	5.00%	2
No	95.00%	38
TOTAL		40

Q6

Export

Why/Why not?

Answered: 39 Skipped: 1

RESPONSES (39) TEXT ANALYSIS MY CATEGORIES

Categorize as... Filter by Category Search responses

Showing **39** responses

I grew up with decimal - its what i know

9/10/2017 2:59 PM View respondent's answers

Because it's harder

8/28/2017 12:37 PM View respondent's answers

Decimal currency is easier to use.

8/28/2017 10:28 AM View respondent's answers

Too complicated to work out!

8/27/2017 4:20 PM View respondent's answers

Pointless nostalgia

8/27/2017 8:50 AM View respondent's answers

Appendix 4:

from: **Naomi Wray** <naomiwray314@gmail.com>
to: contact@dozenal.org
date: Mon, Dec 11, 2017 at 4:13 PM
subject: Research interest questions
mailed-by: gmail.com
: Important according to our magic sauce.

Naomi Wray <naomiwray314@gmail.com> 12/11/17
to contact

Dear Dozenalists,

I am a sixth form student in England studying and writing a dissertation for the Extended Project Qualification; my question is 'Is the decimal system appropriate for modern mathematics?'

Naturally, during my research, I came across the Duodecimal society of Great Britain and America. In doing so, I have read and used the works/articles of F. Emerson Andrews, Ralph Beard and Donald Goodman.

This has peaked my interest and I would much like to ask the society some questions to include as a convincing argument in my writing.

My main questions to begin with are:
What demanding needs of mathematics does the duodecimal system fulfill that the decimal system does not?
With all the advantages of base twelve, why is base six not the more likely choice?
Do any fundamentals of maths, ie theorems and axioms, change or is it simply the notation?

I would be very grateful for your response.

Thank you,
Naomi Wray
The Knights Templar School

from: **Donald P. Goodman III** <dgoodman@dozenal.org>
reply-to: Naomi Wray <naomiwray314@gmail.com>, contact@dozenal.org
to: Naomi Wray <naomiwray314@gmail.com>
cc: contact@dozenal.org
date: Mon, Dec 11, 2017 at 9:00 PM
subject: Re: Research interest questions
signed-by: dozenal.org
security: Standard encryption (TLS) Learn more
: Important according to our magic sauce.

Donald P. Goodman III <dgoodman@dozenal.org>
to me, contact

Ms. Wray:

I'd be delighted to do my best to answer your questions; I suspect some other members may attempt it, as well.

1.) What demanding needs of mathematics does the duodecimal system fulfill that the decimal system does not?

Well, it depends on what you mean by "demanding." Ultimately, mathematics is the same no matter what base is used; we could use base seven or nine and need to master the same concepts to do our work. Dozenal's real benefit is in the human interaction with mathematics; it makes it easier for us to judge scale, to quickly divide into fractions, to identify primes and perfect squares, and so forth.

> With all the advantages of base twelve, why is base six
> not the more likely choice?

A couple of reasons. First, base six is a bit small, and as a result relatively small numbers become lengthy and unwieldy. Second, and much more importantly, base six lacks 4 as a factor. While six is a *good* base, it's not the *best* because it doesn't include the quarter. A base intended primarily for human use should make halves, thirds, and quarters (halves of halves) easy to handle; as a bonus, it could include sixths (thirds of halves), eighths (halves of quarters), and ninths (thirds of thirds). Dozenal includes halves, thirds, quarters, and sixths trivially, as well as eighths and ninths with a little more difficulty. Six misses the quarters, most importantly.

> Do any fundamentals of maths, ie theorems and axioms,
> change or is it simply the notation?

It's solely the notation that changes; the beautiful thing about math is that *it's just math*, and works the same always and everywhere.

I hope this helps; and I hope you'll keep us updated on your project. I'm sure we'd love to get a look at the finished result.

...

--
Donald P. Goodman (#398)
President, Dozenal Society of America
http://www.dozenal.org

Appendix 5:

1	2	3	4	5	6	7	8	9	X	E	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do
11	12	13	14	15	16	17	18	19	1X	1E	20
do one	do two	do three	do four	do five	do six	do seven	do eight	do nine	do dek	do el	two do
10	20	30	40	50	60	70	80	90	X0	E0	100
do	two do	three do	four do	five do	six do	seven do	eight do	nine do	dek do	el do	gro
1	10	10^2	10^3	10^4	10^5	10^6	10^9				
one	do	gro	mo	do-mo	gro-mo	Bi-mo	Tri-mo				

Appendix 6:

Source:
http://www.dozenal.org/drupal/sites_bck/default/files/dsa_newscast_04_04.pdf