TGM

A Coherent Dozenal Metrology

Based on the System and Booklet
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Preface

TGM (Tim, Grafut, Maz) is a metric system specifically designed to take full advantage of the superior dozenal (base twelve) arithmetical system. As such, it may seem a bit esoteric upon first reading. However, experience has shown that the units presented by TGM, and the dozenal counting system which it was designed to exploit, are superior to any of the alternatives currently available for both daily and scientific use.

The "metric system" (SI is currently the most popular version thereof) has suffered from constant complaints regarding both its imposition on those countries which now employ it (this has always, historically, been by force of law, and never by choice), as well as for the inconvenient sizes of the units. Furthermore, its original myopically few set of prefixes (no higher than kilo, no lower than milli) having proven comically inadequate, it has been extended several times, and the extensions have resulted in a system which is esoteric in the extreme. Its units are so mismatched that standardized versions of the system must always make use of some power of the base units rather than the base units themselves; the original system was based on the *centi*meter, gram, and second (commonly referred to as "cgs"), while the current SI version is based on the meter, *kilo*gram, and second (commonly called "mks"). Finally, the system was based on measurements of what were regarded as clearly measured constants at the time (the volume of a given mass of water, the circumference of the earth, and so forth) which we now know were slightly but significantly mismeasured; as a result, units like the meter and the liter do not actually represent what they were designed to stand for. And, of course, its basis on inferior decimal arithmetic cannot be rectified without fundamentally and completely changing the entire system.

The imperial and customary systems, on the other hand, have a great many conveniently-sized units with a great many happy factors, including two with the most convenient of all numbers, twelve (the foot with its twelve inches and the troy pound with its twelve troy ounces). However, it suffers from the same problems as the metric system discussed above. It is chaotic in the extreme, with units of sometimes two, sometimes three, sometimes four, sometimes eight, sometimes twelve, and sometimes sixteen subparts; e.g., the volume system, with its tablespoons of three teaspoons, its cups of eight ounces, its pints of two cups, its quarts of two pints, and its gallons of four quarts, with the pints being sometimes sixteen ounces (the customary system, common in North America) and sometimes twenty ounces (the imperial system, still sometimes used in Great Britain). They do not suffer from the metric system's irrevocable basis in decimal; however, they are clearly not suitable for serious scientific work (though much great science has been done with them despite their unsuitability[1]), and they make even daily work much more complicated than it has to be

[1] The Apollo missions are probably the most spectacular example, though of course this could be multiplied

with their chaotic divisions and inconsistent relationships to one another.

TGM solves the problems of both systems. First, it eschews decimal division entirely, favoring the superior twelve for its divisibility and easier arithmetic. It bases itself on a number of fundamental physical realities, resulting in units conveniently sized for both daily and scientific work, and it maintains a strict 1 : 1 ratio between its basic quantities without sacrificing useful sizes in doing so. There is no need to further enunciate its many virtues here, as they are the subject of the bulk of this book.

TGM was invented by Mr. Tom Pendlebury of England, a member of the Dozenal Society of Great Britain. His system of an orderly set of prefixes denoting the powers of the dozen, and of representing this mathematically with either a superscript for positive powers or a subscript for negative ones, is brilliant and used throughout this work, though for the specific prefixes another system, called Systematic Dozenal Nomenclature, is used. All of the examples and exercises, as well as the answers thereto, are taken almost verbatim from Mr. Pendlebury's original work, except for those in Chapters 1 and 2. Furthermore, the basic structure of this book is also Mr. Pendlebury's doing; while his explanations have been extended and sometimes entirely replaced, the outline of this work explaining the TGM system is owed entirely to him.

Dozenalists everywhere owe Mr. Pendlebury a great debt of gratitude for providing them with a coherent, systematic, and scientific metric system for use with the dozenal base. His work in divising it was monumental, and his brilliance shines forth in every unit and every page of its exposition. May he rest in peace, and be ever remembered for this enormous and pivotal work.

nearly endlessly.

Part 1

The Dozenal System

Numbers are everywhere. They pervade all that we do, from our bank accounts to our medical appointments to the machines that we use every day. In the ever-increasingly large world of software, *everything* is numbers internally, even the things that don't look anything like numbers when the computer shows them to us. Numbers are nearly as pervasive and ubiquitous as words are, surrounding and informing nearly everything we do.

Yet we think very little about numbers, certainly much less than we think about words. Why is this? For numbers are nearly as important to us, and numbers have many interesting facets. For our purposes, the way that we write numbers is anything but written in stone; we have chosen one particular way of doing it, a way that most of us rarely if ever consider, but that doesn't mean that ours is the *only* way. There could very well be other and better ways of writing our numbers.

Just as we write words with letters, so we write numbers with digits. And just as we use letters to make words according to a certain *spelling*, we use digits to make numbers according to a certain *base*. We use the letters and spelling that we use for writing our words because they represent our sounds, however clumsily; why do we use the digits and base that we use for writing our numbers, and could there be a better or easier way that we haven't tried?

Chapter 1: Digits and Bases

Digits are to numbers as letters are to words. Some letters, of course, make up words all by themselves; think of "I" and "a." Most words, though, are made up of multiple letters, examples of which are numerous enough as not to require multiplication. Such a word includes "number"; such numbers include "144" or "2,368."

But what does the sequence of letters "number" mean? The reader, of course, understands English, so he knows in at least a general way what it means; but he knows this only in English. That same sequence of letters could mean absolutely anything, or indeed nothing, in another language without any contradiction. It only means number because we all *agree* that it means number, at least when we are all speaking English. There is nothing special about those letters combined in that way to give it that particular meaning; it just happens to be the way that all English-speakers have agreed to read them.

So also with digits. Some digits mean a number all by themselves; for example, "4" or "7." But most numbers have to be represented by certain digits arranged in certain ways. To represent the number "one hundred and forty-three," for example, we have to arrange the digits in this way:

$$143$$

We cannot change the order; we cannot introduce any additional digits (unless they are zero; and even then, if they are added to the right side, they must be separated from these digits by a decimal point). For all practical purposes, this is the only way to write the number "one hundred and forty-three" out of digits.

Translating Binary to Decimal								
Binary Digit	1	0	0	0	1	1	1	1
Decimal Trans.	128	0	0	0	8	4	2	1
								$\not d$143

Table 1: An example of a binary number and its decimal translation.

But *why* do those digits arranged in that particular way mean "one hundred and forty-three?" That is, why do they mean "one hundred, four tens, and three ones?" We've already seen the answer; they mean that only because we've all agreed that they mean that. We could just as easily have agreed that they mean "one sixty-four, four eights, and three ones," or "one two-hundred-fifty-six, four sixteens, and three ones," and they really would mean that, just by the fact of us all agreeing that they did. Numbers, like words, are *spelled* in a certain way. We have all agreed that we'll spell out numbers using these particular digits (that is, "0" through "9") and this particular *base* (that is, ten). That agreement is the only thing that compels us to spell our numbers the way we do.

The letters we use for spelling numbers are well known:

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

It is a closed and finite set; it can include no more digits and no less. The base we use for counting is also well known; we call it "ten" and we spell it, using our digits and base, "10." The digits are like our letters; the base is like our language. We speak *decimal* throughout most of the world, and spell it with these ten digits.

But what if we chose a different base? Computers, for example, work out things in base two; they speak *binary* rather than decimal. Because they only speak binary, not decimal, they only need two symbols for their language rather than our ten; those symbols are generally written $\{0, 1\}$, though computers themselves, of course, don't see them, knowing them merely as "off" and "on." We know how to say "one hundred, four tens, and three ones" in decimal; how do we say it in binary?

A demonstration of how to say it, and its translation into our current decimal language, is displayed in Table 1 on page 4. In brief, each digit in a number has a meaning beyond merely its own value. The first "1" in "10001111" doesn't mean "one"; rather, it means "one unit of one hundred and twenty-eight," because it's the eighth digit to the left of the number. (In decimal, a "1" in that position would mean "one unit of ten million," as in "10,000,000.")

The base determines what each number means when speaking a given language. For example, when speaking numbers in decimal, the second position means "tens," and the digit there isn't counting up ones, but rather how many tens to include in the final meaning. The third position means "hundreds," how many hundreds to include in the final number; in other words, how many "ten times ten" units to include. The fourth position means ten times the third position; the fifth means ten times the fourth; and so on. In binary, on the other hand, each position means *two* times the last position. So the one on the right in "10001111" means simply "one," but the one in the next position means one unit of two times one, or one unit of two; and the next one means one unit of two times that unit, or four; and the next means one unit of two times that, or eight. And so on, as long as is necessary.

The base does have an effect on what digits are needed, just as the language has an effect on what letters are used. The French don't use "w" (much), and binary doesn't use "3." A "number language," or base, only needs a number of digits equal to itself. So, for example, in binary (base two), we need only two digits, $\{0, 1\}$. In decimal, of course, we need ten digits: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. In hexadecimal (base sixteen), we need sixteen digits; typically, once we get past nine, we simply go into the letters of the alphabet, like so:

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

When we're speaking hexadecimal, therefore, "DEADBEEF" is a perfectly valid and acceptable number (translated into decimal, it means 3,735,928,559), and indeed this number is often used as a test of hexadecimal interpreters in computers.

Almost everyone on the planet (at least, everyone with wetware) speaks decimal; so shouldn't we just leave it at that? The answer to that is "no." While spoken languages are more or less interchangeable in terms of their utility for communication, at least considered in themselves, the same is not true for numbers. Numbers can be spoken in ways that are more or less simple for human beings to understand. Speaking in binary, for example, is difficult for people; relatively small numbers become extremely long, and it wastes resources insofar as human beings have the ability to easily handle numbers greater than two, unlike computers, which can recognize only "off" and "on." Speaking in sexagesimal (base sixty), on the other hand, would also be very difficult for people; while large numbers are relatively short, the number of digits necessary (sixty) is oppressive.

Furthermore, some bases can handle common fractions better than others can. The most common fractions are halves ($\frac{1}{2}$); thirds ($\frac{1}{3}$); and the halves of each ($\frac{1}{4}$; $\frac{1}{6}$; and the half of the quarter, $\frac{1}{8}$). Any base that can't handle these fractions quickly and easily should get a pretty hard look from someone trying to decide which number "language" is the easiest for people to speak.

Odd-numbered bases, as a first example, can't even conveniently represent halves (for example, in base seven, one half is $0.3333\ldots$), which is pretty damning given that it is such a common fraction. This provides grounds for eliminating all odd numbers as potential bases immediately. This narrows our field by half (no irony intended), but still leaves us with a lot of potential candidates for a number base.

Even bases, though, are also not all created equal. Those which are not divisible by three (such as eight, ten, and sixteen) make handling thirds needlessly difficult (in decimal, for example, one third is $0.3333\ldots$). This also makes handling a half of a third, a sixth, needlessly difficult (in decimal again, one sixth is $0.16666\ldots$). Those which are not divisible by four (such as ten and fourteen) make quarters difficult, though the fact that four is twice two mitigates this problem somewhat. For example, decimal keeps $\frac{1}{4}$ down to only two places, 0.25. However, half of a quarter, or an eighth, still winds up at three places, 0.125; a very common division thus winds up with three places, making things more difficult than they have to be.

The decimal base is clearly weak with regard to these most important fractions. While it does contain two as an even divisor, thus leading to a simple, single-place fraction for

Fractions in Various Bases					
	Eight	Ten	Twelve	Fourteen	Sixteen
2	0.4	0.5	0.6	0.7	0.8
3	0.2525...	0.3333...	0.4	0.4949...	0.5555...
4	0.2	0.25	0.3	0.37	0.4
5	0.14631463...	0.2	0.24972497...	0.2B2B...	0.3333...
6	0.12525...	0.16666...	0.2	0.24949...	0.2AAAA...
7	0.1111...	$0.\overline{142857}$	$0.\overline{186A35}$	0.2	0.249249...
8	0.1	0.125	0.16	0.1A7	0.2
9		0.1111...	0.14	$0.\overline{17AC63}$	0.1C71C7...
10		0.1	0.124972497...	0.15858...	0.1999...
11			0.1111...	$0.\overline{13B65}$	$0.\overline{1745D}$
12			0.1	0.124949...	0.15555...
13				0.1111...	0.13B13B...
14				0.1	0.1249249...
15					0.1111...
16					0.1

Table 2: A comparison of fractions for even bases between eight and sixteen.

the half, it skips three, four, six, and eight altogether. Every third number is a multiple of three; yet no power of ten, no matter how high one multiplies, is ever also a multiple of three. Similarly, every fourth number is a multiple of four; but one must get to 10^2 before the fourth becomes an even divisor. Every power of ten also misses six as an even factor, and one must wait until 10^3 (1000) before eight divides in evenly. All told, ten simply skips many of the most important fractions, and those it does catch it generally catches imperfectly. Ten is certainly a poor base from this perspective.

A full comparison of all whole-number, single-digit fractions in even bases from eight to sixteen is presented in Table 2 on page 6. This shows that all bases have irregular whole-number fractions; for example, the reciprocal of the base less one is always $0.\overline{1}$. However, it also shows that one base stands out in the regularity of the most common and important fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$): that is the base of twelve.

A full defense of the superiority of twelve as a number base (a number "language" referred to as dozenal, after the dozen) is beyond the scope of this work; it has been analyzed extensively in many sources, and those interested in further information on the topic should peruse some of the works cited in Appendix C on page 92. There, one can find many interesting and varied arguments for the superiority of the dozenal system over other candidates from the perspective of many different fields. For now, however, we will rest the argument as it stands, and move on to other considerations.

Once we've accepted the dozen as our base, we are faced with the problem of how we spell numbers and speak about them in our new language. That is the subject of our next section.

Exercises

1. Assuming that "A" is ten and "B" is eleven, translate the following numbers into decimal: (a) 2A, (b) 1B, (c) 16, (d) 3B, (e) AA, (f) BB.
2. Using "A" for ten and "B" for eleven, translate the following numbers into dozenal: (a) 143, (b) 130, (c) 47, (d) 18, (e) 23, (f) 34.

Chapter 2: Spelling in Dozens

Dozenal spelling and speaking require two considerations. First, we need to decide what new digits to use. As we discussed earlier, a number language needs a number of digits equal to its base; so, since we're talking here about the dozenal base, we need twelve symbols. However, in common use we only have ten; therefore, two more are required. Second, we need to decide how we will pronounce those new digits, and what words will be used for the powers of our base, for many of which there are currently no special words but which will be extremely important in our new number language.

There are many, many candidates for the two extra symbols; *The Duodecimal Bulletin* published an overview of all the varied proposals recently which shows the true scale of the issue.[2] A real analysis of the various proposals is beyond the scope of this little book; for our purposes, we will assume the symbology commonly employed by the Dozenal Society of Great Britain, which uses symbols devised by famous shorthander and dozenalist Sir Isaac Pitman, to be preferable. Fuller arguments on this topic can be found in works cited in Appendix C on page 92; a defense of the Pitman characters in particular has been mounted in my own *A Primer on Dozenalism*.[3] The symbols are simply ↊ for ten and ↋ for eleven; they blend well with our current digits, are easy to write, and are easy to integrate into existing fonts.

As for how these two new digits are pronounced, we will handle them one at a time. Currently we pronounce the number one greater than nine as "ten," and there seems no reason to change that. It's a single syllable, so there's no need to shorten it, and it's best to change as little as possible when changing our number system, so leaving ten as it is seems advisable. As for our second new digit, we currently pronounce the number one greater than ten as "eleven." This is acceptable when it's higher than our base, as we're accustomed to using multiple syllables for such numbers; however, now that eleven is lower than our base, it makes counting cumbersome to have a three-syllable word thrown in among many monosyllables and two disyllables ("zero" and "seven"). Tom Pendlebury recommended shortening this to "elv," which seems a perfectly good option, and at the very least as good as any other.

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[2] Michael deVlieger, *Symbology Overview* in 99 The Duodecimal Bulletin 13–14 (St. Louis, MO: 11↋6), available at http://www.dozenal.org/article/db4a211.pdf.

[3] Donald P. Goodman III, *A Primer on Dozenalism*, available at http://gorpub.freeshell.org/dozenal/blosxom.cgi/dozprim.html.

Finally, we have to pronounce our base itself. "Twelve" is fine, and sometimes should be retained; but since our system is called "dozenal" ("twelvinal" being too cumbersome), perhaps we should alter our word for twelve to match. Furthermore, using twelve in compounds often leads to tongue-twisters; pronouncing what in the dozenal system is written as "70" (seven twelves and zero ones) as "seventwelve" is clumsy and unappealing. Sir Isaac Pitman, an enthusiastic dozenalist, recommended using "zen" for this purpose, when twelve was seen by itself as well as in compounds, leading to "sevenzen," "fourzen," and the like. This is easily remembered, linked to the word "dozen," and has the advantage of historical lineage on its side. The Uncial system (explained shortly) uses "unqua" for twelve; abbreviating this to "qua," as in "sixqua" or "ninequa," is also an easy shorthand for such common numbers.

So the dozenal system of counting runs as follows (excluding zero):

1	2	3	4	5	6	7	8	9	↊	↋	10
one	two	three	four	five	six	seven	eight	nine	ten	elv	unqua

It's easy to go beyond this simple number line up to 10^2 (written as 100 in dozenal; translated into decimal, it means one hundred and forty-four). For seven dozen and six, we write "76" and say "sevenqua six"; for three dozen and ten, we write "3↊" and say "threeequa ten"; and so on. It's important to forget our habit of reading numbers in decimal when we do this; we're reading dozenal now. "30" does *not* mean "thirty"; it means "threeequa," three dozen, in decimal thirty-six. "10" does *not* mean "ten"; "↊" means "ten," while "10" means onequa, a dozen. It's also important not to translate them into decimal when reading them, either; "30" really does mean threeequa, not thirty-six written in a funny way. Getting used to *thinking* in dozenal is vital if we are to take real advantage of the system; and it takes only a little practice to do so.

But how do we go beyond ↋↋ (elvqua elv, decimal one hundred and forty-three)? We know that we would spell the next number in digits as "100," but how do we *say* it? Once again, Tom Pendlebury, in his original booklet on TGM, proposed a complete and admirably simple system for such pronunciation.[4] It is detailed in Table 3 on page 9.

The system is simple: there is a set of prefixes, one for each power of twelve. So "zena" is the prefix for 10^1; "duna" is the prefix for 10^2; and so on. These can even be combined; for example, "zenduna" is the prefix for 10^{12}, and "duntrina" is the prefix for 10^{23}. Each indicates in a simple manner how many zeroes are to be added to the number to make that number. Compare that to our present system, in which we have no such easy prefixes; rather, we have "thousand" for decimal 10^3, "million" for decimal 10^6, and no special words for anything in between. Furthermore, none of our current words give any indication of how many places the number referred to has; this often leads to confusion, particularly given the differences in the "short" system (used in America, in which a billion is a thousand times a million) and the "long" system (used in Commonwealth countries, in which a billion is a *million* times a million, and the American billion is called a "milliard").

This system works for negative powers of zen, also; that is, it works with *fractional parts*, what we commonly, and erroneously, refer to as "decimals." In decimal spelling, we

[4] Tom Pendlebury, TGM: A coherent dozenal metrology based on Time, Gravity and Mass (Dozenal Society of Great Britain), available at http://www.dozenalsociety.org.uk/pdfs/TGMbooklet.pdf.

Full Pendlebury System				
Prefix	Number	Zeroes	Exp.	Decimal
Zen	10	1	10^1	12
Duna	100	2	10^2	144
Trina	1,000	3	10^3	1,728
Quedra	10,000	4	10^4	20,736
Quen	100,000	5	10^5	248,832
Hes	1,000,000	6	10^6	2,985,984
Sev	10,000,000	7	10^7	25,831,808
Ak	100,000,000	8	10^8	429,981,696
Neen	1,000,000,000	9	10^9	5,159,780,352
Dex	10,000,000,000	ᘔ	$10^{ᘔ}$	61,917,364,224
Lef	100,000,000,000	Ɛ	$10^{Ɛ}$	743,008,370,688
Zennil	1,000,000,000,000	10	10^{10}	8,916,100,448,256
Zenzen	10,000,000,000,000	11	10^{11}	106,993,205,379,072
Zenduna	100,000,000,000,000	12	10^{12}	1,283,918,464,548,864
...				
Dunduna		22	10^{22}	1.144754599... x 10^{28}

Table 3: The full Pendlebury system of dozenal counting.

separate fractional parts from whole parts using a "decimal point," which is written like a period (at least in America; it is written in different ways, or even with an entirely different symbol, elsewhere). In dozenal spelling, we separate fractional parts from whole parts using a *dozenal point*, which we write ";".[5] While in decimal counting the decimal point is pronounced "point," in dozenal counting the dozenal point is pronounced "dit." This makes conversions much easier to speak about; rather than having to say "point six dozenal equals point five decimal," we can say simply, "dit six equals point five." Furthermore, the distinction makes it immediately clear which number language we happen to be speaking at the moment.

In the Pendlebury system, the same prefixes are used for negative powers of twelve as for positive ones, but they end in "i" rather than "a." So, for example, rather than have one zen*a* (10), we have one zen*i* (0;1). We can similarly talk of have ten dunis (0;0ᘔ) or fourzen-five trinis (0;45) or simply five trinis (0;005); or, if the occasion calls for it, we can simply say "zero dit zero zero five," as is common in our current parlance.

In addition to the Pendlebury system, other dozenalists have devised a system which accords with the standards of the International Union of Pure and Applied Chemistry (IUPAC). IUPAC has a system of numeric prefixes which are used to create names for elements based on their atomic number; these numeric prefixes are internationally acknowledged and well-known. Once expanded for use with dozenals, they can be used to build a system of prefixes which are unique as well as internationally recognized.

[5]This is often called a "Humphrey point" after its first proponent, H. K. Humphrey, one of the early members of the Dozenal Society of America.

Full Uncial System				
Prefix	Positive	Numeric	Negative	Numeric
Nil	Nilqua	10^0	Nilcia	10^{-0}
Un	Unqua	10^1	Uncia	10^{-1}
Bi	Biqua	10^2	Bicia	10^{-2}
Tri	Triqua	10^3	Tricia	10^{-3}
Quad	Quadqua	10^4	Quadcia	10^{-4}
Pent	Pentua	10^5	Pentia	10^{-5}
Hex	Hexua	10^6	Hexia	10^{-6}
Sept	Septua	10^7	Septia	10^{-7}
Oct	Octua	10^8	Octia	10^{-8}
Enn	Ennqua	10^9	Enncia	10^{-9}
Dec	Decqua	$10^{↊}$	Deccia	$10^{-↊}$
Lev	Levqua	$10^{↋}$	Levcia	$10^{-↋}$
Unnil	Unnilqua	10^{10}	Unnilcia	10^{-10}
Unun	Ununqua	10^{11}	Ununcia	10^{-10}
Unbi	Unbiqua	10^{12}	Unbicia	10^{-12}
. . .				
Bibi	Bibiqua	10^{22}	Bibicia	10^{-22}

Table 4: The Uncial system of dozenal counting.

This system is called the *Uncial* system, after the word *uncia*, which was used by the ancient Romans for their base-twelve fractional system. It is displayed in Table 4 on page ↊. We will use this system throughout this book.

The Uncial system has some advantages over Pendlebury's. Firstly, as mentioned before, it is international in its foundations, being derived from IUPAC with only a few small extensions to allow it to be used in dozenal. Furthermore, the Pendlebury system results in prefixes that differ only in their final vowel, which in many languages (including English) tends to get reduced to schwa. In other words, it is very difficult, in normal speech, to tell the difference between "trinaHour" and "triniHour." The Uncial system avoids this trouble by placing a *-qua* (pronounced "kwa") on the root for a positive prefix, but a *-cia* (pronounced "sya" or "sha") for a negative one. The positive and negative prefixes are thus easily distinguishable; there is no confusion between "unqua" and "uncia."

The Uncial system is also expandable in a more consistent way than the Pendlebury system is. By simply removing the suffix from the prefix (*-qua*, which mean "exponent"), we can combine these prefixes in a clearer and less ambiguous way. The Pendlebury system makes no distinction between *zen* as a prefix and *zen* as part of a larger prefix; so the same linguistic unit is used in both "zena" and "zenduna," though they are functioning in quite different ways. Removing the exponential suffix from the prefix in the Uncial system means that the number is acting on its own, not as a power of twelve; so the "biqua" meaning 10^2 and the two "bi" prefixes in "bibiqua" reflect their own different meanings.

Futhermore, the Uncial prefixes can be used in other contexts. With the Pendlebury prefixes, "trinal" as an adjective could mean "three" or "10^3"; in the Uncial system, "trinal" can only mean "three," for 10^3 can only be "triqual."

Regardless of the system of words that we use, all the normal rules of exponential arithmetic apply here. When multiplying, simply multiply the two numbers and add together any positive prefixes and subtract any negative. So two biqua (200) multiplied by three triqua (3000) is six pentqua (60 0000); six pentqua (60 0000) multiplied by three bicia (0;03) is onezen-six triqua (1 6000); more simply, biqua times triqua equals pentqua, while pentqua times bicia equals triqua. When dividing, do precisely the opposite; that is, subtract positive prefixes and add negative ones. For powers, multiply the prefix by the desired power, and for roots, divide it. So the square of triqua is hexqua, but the cube root of unbiqua is quadqua.

When writing these numbers, the full prefixes become rather cumbersome. Therefore, we can simply use a raised number for a positive prefix, or a lowered number for a negative one. So, for example, 5000 could also be written $^3 5$, and 0;04 could be written $_2 4$. This is a brilliant and compact notation, and eliminates the need for such long and cumbersome notation as 3.5×10^4. Even more often, however, these raised or lowered numbers will be attached to *units*; so, for example, one uncia of one foot (that is, one inch) could be written "1 $_1$ft," while a biqua feet (100 feet) could be written "1 ^{2}ft." This compact abbreviation is often very useful.

To keep the prefixes clearly separated from the units, unit names and abbreviations should begin with a capital letter, while the prefixes should be lowercase; so, for example, we may speak of hexquaFeet or triquaSeconds. (Whenever this is possible, that is; when working with the SI metric system, sometimes capital or lowercase abbreviations mean different units, so their native capitalization must be retained when the abbreviations are used.)

Any unit, from TGM or any other system, can take these prefixes and use this number notation; so one can easily speak of hexquaYears (the dozenal "million," written here ^{6}Yr) or biquaMeters (^{2}m).

Only one difficulty remains: how do we clearly identify a number as being written in dozenal or decimal? To start with, this is a dozenal book, so from here on a number should be presumed dozenal unless marked otherwise. From time to time a decimal number will be useful; these will be marked with the symbol "$\not d$" like so: $\not d$45. Of course, any number containing a decimal point (".") is clearly decimal, and need not be so marked; likewise, any number containing a uncial point (";") is clearly dozenal, and need not be so marked. When necessary, dozenal numbers will be marked with the symbol "$\not z$" like so: $\not z$45.

Now we've learned how numbers work; what base is the best; how to write numbers in our base; and how to talk about numbers in our base. That completed, we can move on to the really interesting part of this book, the metric system which it was written to describe: TGM.

Examples

In the decimal system, there are often several ways of reading a number; for example, "0.879" could be read as "zero point eight seven nine," as "eight point seven nine tenths," as "eight hundred and seventy-nine thousandths," and so on. The Uncial system is no different. Using the Uncial system, let's explore the possibilities by putting the following into words:

1. 67. We can do this in two ways:
   (a) *Six unqua seven*, or, abbreviated, *sixqua seven.*
   (b) Move the uncial point. *Six dit seven unqua*, written $^{1}6;7$.
2. 4↋79. We can do this in several ways:
   (a) *Four quadqua, elv biqua seven unqua nine.* The simplest and most straightforward, not unlike the decimal equivalent "eight thousand, five hundred and eighty-nine."
   (b) Move the uncial point. *Four dit elv seven nine quadqua.* This would be written $^{4}4;$↋79.
   (c) State the highest power and then recite the digits. *Four quadqua elv seven nine.*
3. ↊467 95↋2. Again, several possible ways:
   (a) *Ten septqua, four hexqua, six pentqua, seven quadqua, nine triqua, five biqua, elv unqua two.* Long-winded but complete and straightforward.
   (b) Group the digits. This could be done several ways according to the reader's preference; one example would be *ten four six seven quadqua nine five elv two.*
   (c) Move the uncial point. *Ten dit four six seven nine five elv two septqua*, written ^{8}↊;4679 5↋2.
   (d) State the major power, then the rest of the number. *Ten septqua four six seven nine five elv two.*
4. 0;67. Again, several possible ways:
   (a) Simply state the digits, as is common in the decimal system. *Zero dit six seven.*
   (b) Name the powers individually. *Six uncia, seven bicia.* Long-winded but legitimate.
   (c) Name the highest power, then list the remaining digits. *Six uncia seven.*
   (d) Move the uncial point. *Six dit seven uncia*, written $_{1}6;7$.
5. 0;0678. Again, several possible ways:
   (a) Simply state the digits. *Zero dit zero six seven eight.*
   (b) Name the powers individually. *Six bicia, seven tricia, eight quadcia.*
   (c) Group the digits; for example, group "678" and recite them as normal, then name the power. *Six biqua seven unqua eight bicia.* Confusingly put, but logical and legitimate.
   (d) Name the largest power, then recite the digits. *Six bicia seven eight.*
   (e) Move the uncial point. *Six dit seven eight bicia*, written $_{2}6;78$.

Naturally, some of these ways will be more popular than others, just as some ways of reading decimal numbers are common while others are rare. Ease of use and actual practice will determine which will become common.

Exercises

Read off the following numbers in at least two ways:

1. 4↋.
2. 592.
3. ↋↊↋↊.
4. 6978 4597;4598.
5. 0;0984 5876.

Part 2

The TGM System

TGM stands for Tim, Grafut, Maz, the system's primary units for time, length, and mass. These three are particularly representative of the system because these basic units are derived from real, physical realities that we commonly experience in the world around us.

The customary measures in Britain and her former colonies have been so hard to break out of precisely because their measurements are based on real, human-scaled, physical phenomenon. The foot is about the length of a human foot, the inch about that of the last length of a human thumb, the yard about that of a long pace, and so on. This is what is good about that system. However, the units don't relate to each other well; for the most part, they aren't arranged with the dozen as the radix. With occasional exceptions (like inches in a foot, or troy ounces in a troy pound), they use units like twos, threes, and sixteens; the British continue to use some units with even odder factors, like fourteen (there are fourteen pounds in a stone, the unit commonly used for weight when using customary units in England). So while the units themselves are conveniently and human scaled, they are not easily converted because they are not centered around the radix of the best number base; that is, the dozen.

The metric system, on the other hand, took a different approach. It regularized the conversions between the units to the radix of the number system (unfortunately sticking with the inferior radix, ten), but derived its basic units in such abstract and esoteric ways that it simply isn't human-scaled. The meter, for example, is far too long to be used as the common unit of length; the centimeter, too short; and the decimeter, for some reason or another, is rarely used. The meter was derived, theoretically, by taking one ten millionth of one quarter of the circumference of the earth from pole to pole; an interesting exercise, to be sure, but hardly the basis for the fundamental unit of an entire system of mensuration. Even if one likes the process, however, the fact remains that the metric system is based irrevocably upon an inferior radix, ten; we still need a measurement system based on the best number base, the dozen.

TGM attempts to walk the middle line between these problems. It tries to derive normal, human-scaled units, and it tries to derive them from universal physical realities that we all experience every day. As such, for each category of unit, such as "time" or "space," TGM will have a corresponding physical reality from which the units are derived. It further maintains a one-to-one (mathematicians will write this "1 : 1") correspondence in basic units of each type (except for the transition to electrical and light units, which due to scale must necessarily be based on a different ratio), and of course units are converted in factors of twelve. This allows TGM to be rooted in reality; produce reasonable, human-scaled units; and to conform to the best possible radix for a human counting system, the dozen.

That explained, let us proceed to the first category of units: time, upon which all others are based.

Chapter 3: Time

Fundamental Reality: The Mean Solar Day

Even schoolchildren can reliably recite the ancient mantra: every day, the sun rises in the east and sets in the west. We watch it happen every day; the sun stays up longer in the summer, down longer in the winter, but we always see it taking its appointed rounds, never varying too much, and ensuring that on average all our days are really equally long.

The *mean solar day*, the first fundamental physical reality relied upon by TGM, is basically just the average length of the day. In the units we're all familiar with (SI seconds), it comes to đ86,400.002 seconds long; in đ1820, or at least thereabouts, it was exactly đ86,400 seconds long. But it's important not to think of it in seconds; we're in TGM now, not SI or customary units. It's just the mean solar day; we should think of it as simply 1 mean solar day long.

It's easy to see that we already keep time partially in the dozenal system. We divide our day into two units of a dozen hours each; we divide each hour into a dozen units of five minutes each, each of which gets a number on our standard clock. This derives from our original division of the whole "day" into a twelve-hour day and a twelve-hour night, two separate and complete units; this is why we call midnight "midnight," even though by our current reckoning it's the beginning of another day. Even on digital clocks, the clock resets on "đ12," and goes back to zero when it should hit "đ24." The dozenal system offers great advantages even in this simple sphere. Only one number is needed around the whole circle; furthermore, to make a time in the afternoon rather than the morning, one simply adds 10 to it. So when we say 3;4, we know that we mean twenty minutes after three *in the morning*; if we meant in the evening, we'd add 10 to make it 13;4. In other words, just put a "1" in front to make it an afternoon time. In our decimal system, in "twenty-four hour time," 3;4 in the morning becomes đ15:20 in the afternoon. Despite it being three o'clock, there's no three in the number anywhere; in the dozenal system, however, three o'clock in the afternoon is still clearly three o'clock; it's just got a "1" in front of it.

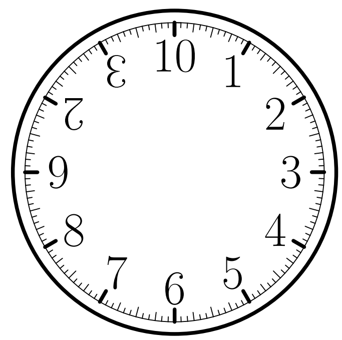

Figure 1: A dozenal clock in hours, unciaHours, and biciaHours

There's no reason to eliminate this customary dozenal division of the day, and TGM therefore retains it. The mean solar day is divided into two; each half is divided into twelve, which are called "hours." Each hour is in turn divided into twelve, making unciaHours; these are equal to five minutes. Each unciaHour is itself divided into twelve, making biciaHours; these are equal to a little less than half a minute, precisely đ25 seconds. These biciaHours are divided into triciaHours, and triciaHours into quadciaHours.

There are exactly 20 0000 quadciaHours in one mean solar day. This unit produces the most reasonably sized daily units when it is used as the base; therefore, it becomes the TGM fundamental unit of time, and has its own name: the Tim.

Tim (Tm) = 1 quadciaHour = $0.1736\bar{1}$s (đ25/144)

Divisions of the Hour			
1 unciaHour	$_1$Hr	5 minutes	
1 biciaHour	$_2$Hr	21 seconds	$\not{d}$25 s
1 triciaHour	$_3$Hr	2;1 seconds	$\not{d}2.08\overline{3}$ s
1 quadciaHour	$_4$Hr	0;21 second	$\not{d}0.1736\overline{1}$ s, or 25/144

Table 5: Divisions of the hour and the derivation of the Tim.

Tim Conversions			
Common	Tims	TGM	Seconds
1 Hour	1,0000 Tm	1 quadquaTim	3,600
1 Day	20,0000 Tm	2 pentquaTim	86,400
1 Week	120,0000 Tm	12 pentquaTim	604,800
$\not{d}$30 days	500,0000 Tm	5 hexquaTim	2,592,000
$\not{d}$365 days	50↊0,0000 Tm	5;0↊ septquaTim	31,536,000
$\not{d}$366 days	5100,0000 Tm	5;1 septquaTim	31,622,400

Table 6: Common conversions of time units to TGM Tims.

No more minutes, no more seconds, no more milliseconds or half-seconds. Now, we have Tims. There's certainly no problem with using our normal, customary units when they correspond to TGM ones; the hour, for example, is a natural division that TGM retains, and even employs when choosing its own basic unit. The unciaHour, a unit equal to five minutes, is also a useful period of time. But minutes and seconds don't conform to a dozenal division of units, and so must be scrapped.

That doesn't mean, though, that there aren't curious coincidences that will help us visualize the units. For example, $\not{d}$100 seconds is equal to 400 Tims, and of course 5 minutes is equal to 1000 Tims. The triciaTim is almost exactly one-tenth of a millisecond; to be exact,

$$1\ {}_3\text{Tm} = 0.1004694 \text{ milliseconds}$$

This means also, of course, that one biciaTim (which is one triciaTim multiplied by a dozen) is a bit over one millisecond and a fifth:

$$1\ {}_2\text{Tm} = 1.2056328 \text{ milliseconds}$$

These near correspondences will be very helpful to those who deal in such small periods of time.

Other common conversions can be stated, as are seen in Table 6 at 15.

It should be clear from this that much more even, easy to deal with numbers come from defining the time unit from the length of the solar day, rather than defining an arbitrary time unit and counting up from there.

Colloquial Time Expressions

TGM Unit	Value	Colloquial Name	Decimal Equiv.
quadquaTim	^{4}Tm	Hour	đ60 min
triquaTim	^{3}Tm	Block	5 min
biquaTim	^{2}Tm	Bictic	21 (đ25) s
unquaTim	^{1}Tm	Unctic	2;1 (2.08$\overline{3}$) s
Tim	Tm	Tick	0;21 (0.1736$\overline{1}$) s

Table 7: Some Colloquial Expressions for TGM Time Units with Decimal Equivalents

In the last few decades, science has made extreme refinements in the measurement of time. Previously, the most exact possible definition of time was the number of seconds in the tropical year of đ1900; now, the most precise measurements come from counting the number of vibrations given by certain radioactive elements, usually Cesium ↋1 (Cesium đ133). These refinements are equally a part of TGM:

Tropical Year	50↊5,9905;7291 Tm	đ31,556,925.9727 seconds
Cesium ↋1	3,8658,↊173 = 1 Tm	đ9,192,631,770 = 1 second

These refinements are extreme, of course, and are relevant only to scientists working in incredibly sensitive situations; even engineers in the vast majority of cases don't have to worry about differences this small. Fundamentally, the TGM unit of time, the Tim, is simply one quadciaHour, a fraction of the mean solar day.

Must we, however, speak of daily units in this way? When we want to wait for five minutes, must we say "I am waiting for one unciaHour" or "I am waiting for one triquaTim"? Naturally not; in these matters, as in all others, daily use will doubtlessly give rise to colloquialisms regarding these commonly used and extremely useful divisions of time. Far from discouraging these informal "non-coherent units" (for which see below), TGM encourages them; Tom Pendlebury even left spaces in his unit charts to allow TGM's users to fill in the gaps with units they have found to be useful. Table 7 on page 16 lists some such commonplace words for time that some TGM users have adopted. They are based partly on the clock, of course; the "tick," for the Tim, the smallest unit which the hand on a TGM clock "ticks" off.[6]

So our examples become, "I'll wait here for a block," or "I'll give you two blocks before I leave without you"; perhaps a race could be summed up as, "My horse won by an unctic"; programmers could assert that "this program runs almost a full tick faster." Surely these terms are just as easy to use as our current ones; yet they are more sensibly and regularly organized, and are in accordance with a good number base.

There are still other units of time with which we are familiar, and which are an unavoidable part of life on earth. Years, months, and so on continue to exist. But they are not part of TGM; they are *non-coherent* or *auxiliary units*, units which exist and will unquestionably

[6] A visual example of such a clock can be found at `http://gorpub.freeshell.org/dozenal/blosxom.cgi/clock.html`.

be used for the foreseeable future, but which do not fit into the coherent system which constitutes TGM. This is no matter; they can be used just as they are now. These time periods are not even numbers of units in either of the currently dominant systems, either.

Some such units, however, will need to be adapted for use with TGM. For example, we currently group years into groups of ten, called "decades." However, these decades are *not* all the same length. Some contain two, some three leap years, meaning that some are longer than others. Because leap years fall due every four years (more or less), and four is not a factor of ten, decades cannot regularly contain equal numbers of them.

But this is dozenal. Rather than the decade, we have the unquennium, a period of twelve years ("unqua,", twelve, plus "ennium," which we already know from our current word "millennium"). There are three units of four years in each unquennium, meaning that each unquennium contains the same number of leap years and consequently the same number of days (2653, to be precise, each four-year unit containing Ƨ19). But what of the leap day which is dropped every hundred years (except every fourth hundred years)? That dropped leap day actually does not come due every hundred years, but rather every Ƨ8 years; as such, dropping it every 100 (đ144) years, rather than every 84 (đ100) years, is actually closer to the truth (only 14 years off the actual date, rather than 24 years off). So every unquennium is the same length, containing the same number of leap days, except when it contains a year divisible by 100, which contains one less day. Then, rather than have the leap day after all every 294 (đ400) years, which we do now, we must instead omit another leap day every 800 years, perhaps the fourth year following the year divisible by 800; then we have an average year length over that period of 265;2ƧƧ6 days, which is so close to the correct figure of the tropical year as to make no real difference.[7] While the current decimal scheme is inaccurate by 0;00063 days per year, the above-outlined scheme is inaccurate by only 0;00003 days per year. Both of these errors are tiny; but the above scheme's error is tinier, adding up to only a day every 2400 years.

So, to sum up: a leap year every four years, except when the year is also divisible by 100, and except for the fourth year following a year divisible by 800 (in other words, we omit two leap days in a row). Simpler, yet equally or more accurate; this is the story of TGM and the dozenal system.

So to talk about these longer periods of time, we can discuss unquenniums rather than decades, biquenniums rather than centuries, triquenniums rather than millenniums; or, if we prefer, simply unquaYears, biquaYears, and triquaYears. (A un*cia*Year, of course, is roughly one month.) Indeed, any of the powers of unqua can have the *-ennium* suffix added to it to indicate that number of years (as we already do, irregularly, in decimal, in words like "mill*ennium*" and "bicent*ennial*").

In accordance with normal practice in other fields, a complete and exact record of a given time can be given by simply writing in the largest units followed by the smaller ones until we reach Tims. For example, this moment I'm writing this right now can be written:

11bƐ6y8m22d10;8hr

[7] This general scheme I found spelled out briefly at *Thoughts on the Leap Year* at http://www.dozenalsociety.org.uk/apps/leapdays.html. It is also explained in Robert Davies, *A Duodecimal Calendar*, 3Ɛ The Duodecimal Bulletin 8 (1184).

Which, in long form, is 11 biquenniums, Ɛ6 years, 8 months, 22 days, and 10;8 hours. (A lunch break.)

That is the TGM treatment of time; let's now try a few exercises to flex our TGM muscles before proceeding.

EXERCISES

1. Write the following times in dozenal numbers of hours:
   (a) A quarter to eight in the morning;
   (b) 08:50 hrs;
   (c) Five past two in the afternoon;
   (d) 22:40 hours;
   (e) 22 1/2 minutes past five in the morning.
2. Hong Kong time is onezen four hours (sixteen) ahead of California. Put the following California times into dozenal, and calculate the respective times in Hong Kong:
   (a) 2 a.m.;
   (b) 9:30 a.m.;
   (c) noon;
   (d) 5:45 p.m.;
   (e) 11:20 p.m.
3. A job took 3 days, 5 hours, and 20 minutes. How long is this in dozenal
   (a) In hours?
   (b) In Tims?

Chapter 4: Space

FUNDAMENTAL REALITY: MEAN GRAVITATIONAL ACCELERATION

From the Tim are derived all other units of TGM. The units involved in measuring space are no exception.

4.1 Length: The Grafut

Every schoolchild knows the story of Galileo at the Tower of Pisa. For centuries everyone had held to the common sense idea that heavier objects would fall faster than lighter ones; e.g., a bowling ball would fall faster than a feather. However, for whatever reason Galileo decided to climb to the top of a tower and actually try it, surprising many people with his result. Both a heavier ball and a lighter ball fall at the same speed; indeed, they even speed up as they fall at the same rate. This is due to what Newton would later call *gravity*.

Gravity exerts an acceleration on everything. All objects in the universe (that is, everything which contains matter, or has *mass*) are gravitationally attracted to one another. This acceleration varies, of course, depending on the size of the objects being attracted to one another and the distance between those two objects. So, while the pen on my desk is gravitationally attracted to my physical mass, that mass just isn't big enough to make the

Common Distances in Grafuts	
Mean distance from Earth to Moon	$3\ {}^{8}$Gf
Lightyear	$2\ {}^{13}$Gf
Radius of the electron	$1\ {}_{11}$Gf
Radius of geosynchronous orbit	$4\ {}^{7}$Gf
Ten times the polar diameter of Earth	$1\ {}^{8}$Gf

Table 8: Common distances as measured in Grafuts.

pen actually move. Of course, both my own mass and that of the pen are gravitationally attracted by the earth, which is such an enormous mass that it overrides all other gravitational forces on its surface.

The earth is so huge, in fact, that we have to get pretty far away from it to make any appreciable difference in its pull on us. Nevertheless, even on the surface of the earth differences in the pull of gravity are sometimes large enough to be measured. Gravity pulls harder when deep in a valley than when high on a mountain; it pulls lighter when on the equator than when at the poles. In terms of actual human experience of gravity, these differences are far too minute to have any effect on what we do or what we build. However, the fact remains that the pull of gravity on the surface of the earth is a *range*, albeit a very small one. So when we speak about the pull of gravity, we are generally referring to the *average* within that range, an average we call the *mean acceleration of gravity*.

In the common system, this mean acceleration caused by gravity's pull is measured as 32.1741 feet per second per second; in metric, it is measured at 9.80665 meters per second per second. This metric number is so close to ten, the base of the metric system, as to be intensely aggravating; in very loose work, sometimes it is even rounded to ten, but while it's close it's still far enough away that rounding to ten introduces serious error.

If we use Tims rather than seconds, we find that the mean acceleration of gravity is about Ɛ$\frac{5}{8}$ inches per Tim per Tim, or a little under 26 centimeters per Tim per Tim. In other words, the speed of a falling object increases a little less than a foot per Tim for every passing Tim. TGM takes this length and makes it the unit of length for the entire system. It is called the Gravity Foot, or more commonly the Grafut; its abbreviation is Gf.

We've already noted that gravity's acceleration varies very slightly over the surface of the earth, even though that variation is too minute to make much, if any, difference to us who live upon it. However, modern instruments are very accurate and can detect even these slight differences, and this is a good thing because it provides us with a *range* of figures for the accleration due to gravity, all of which are close enough to what we all experience every day for all our practical work. We can then select which of those figures is most convenient for the rest of our system, and which of those figures has been most accurately measured to ensure that we get the most precise value available, while still basing our unit of length on the *mean* acceleration of gravity; that is, on the figure which is so close to what we all know on a daily basis as not to make any difference.

As it happens, lots of natural phenomena come pretty close to exact figures when measured in Grafuts, as seen in Table 8 on page 19.

These are all approximations, of course. However, the last of them (ten times the polar diamter of Earth) is so close that, if we just say that 1 ^{8}Gf is equal to that distance, the length of the Grafut falls within the range of mean acceleration of gravity that we discussed earlier. This is very useful because the polar diameter of Earth has been measured *extremely* accurately; this accuracy will, in turn, rub off on our Grafut. So, if we consider ten times the polar diameter of Earth to be 1 ^{8}Gf, we can define the Grafut with great accuracy:

Grafut (Gf) = 0.295682912 m = 0.970088296 ft = 11.64105955 in

If all this sounds extraordinarily convoluted, consider how the meter was defined. Originally, it was intended to be one ten-millionth of one quarter of Earth's circumference. The Earth's circumference was calculated by measuring from Paris to Barcelona and then extrapolating from that. Naturally, this initial measurement was wrong, and consequently the benefits from the measurement being a fraction of Earth's circumference, if ever there were any, were gone. However, by then the meter was firmly established by force of law, and after having been forced by law to change to metric nobody was in the mood to change again. So practically, the meter was defined as the length of a piece of platinum bar in a vault somewhere in Paris, a totally arbitrary measurement that people went along with not because it made any sense, but because so many people were already doing it. In 1193, that platinum bar was measured with a laser; this resulted in the meter being defined in terms of the speed of light, which was declared to be $\not{d}$299,792,458 meters per second. This is quite precise, but hardly any simpler than the process we've followed here.

After a certain number of significant digits of accuracy, of course, further refinement is an exercise in gymnastics and nothing more; a neat display of our measuring acumen, but totally without effect in the practical world. However, since there's certainly no harm in such refinements, TGM can also define the Grafut in terms of the velocity of light:

Velocity of light = 4↊↋4 9923;08 Gf/Tm

Again, though, this is just a more refined description of what the Grafut already is, not a determination of it. It's nice to know the length of the Grafut with such precision, but in the end it's really just the acceleration due to gravity, a quantity that we all experience every day and every where, nothing more and nothing less.

So what does all this mean, practically, for the TGM system of measurement?

It means that measurements of length are done with the basic unit of the Grafut, which is slightly shorter than the standard English foot, about equal to 29.5 centimeters; this is very nearly exactly the length of standard, metric-sized A4 paper. The Grafut is divided, of course, into *unciaGrafuts*, each of which is a short inch; each unciaGrafut is divided into twelve biciaGrafuts, which are a bit more than 2 millimeters long. This can continue downward indefinitely, of course; the hexciaGrafut, for example, is just under a tenth of a micron. For another example, the pentciaGrafut ($_5$Gf) is just a touch bigger than one micrometer (a micron is 0;↊122 pentciaGrafuts), making it a convenient length for measuring cell size (for example, the average human cell is about 8;4↋9↋ $_5$Gf, or ↊ microns).

In the printing trade, too, TGM yields units that are almost identical to the traditional units involved. The printer's point, for example, is $\frac{1}{72.27}$ of an inch, and a pica is twelve points; an awkward number, to be sure, when related to the inch (which is why Postscript

Colloquial Distance and Length Expressions			
TGM Unit	Value	Colloquial Name	Decimal Equiv.
Gf	1 $_1$Gf	Gravinch (Unch)	0.9701 in; 2.4640 cm
Gf	3 $_1$Gf	Gravpalm	2.9103 in; 7.3921 cm
Gf	4 $_1$Gf	Gravhand	3.8804 in; 9.8561 cm
Gf	3 Gf	Gravyard (Trifut)	0.9701 yd; 0.8870 m
Gf	3 ^{3}Gf	Gravmile (triquaTrifut)	0.9524 mi; 1.5328 km
Gf	2 ^{3}Gf	Gravklick (Gravkay)	1.0218 km; 0.6349 mi

Table 9: Some Colloquial Expressions for TGM Length and Distance Units with Decimal Equivalents

defined the "big point" as simply d̸ $\frac{1}{72}$ of an inch), but useful for printers, who didn't deal with inches much, but simply with points and picas. This led to approximately six picas to the inch (exactly six to the inch, with Postscript's big points). In dozenal, $\frac{1}{72.27}$ equals 0;01Ɛ↊; when put into inches, 0;01Ɛ↊ inches comes out very closely to 0;002 Grafut (or, more simply, two triciaGrafuts; the exact number, worked out to four places, is 2;0793 triciaGrafuts). This yields some startlingly convenient measures:

$$\text{TGM point} = 2\ {}_3\text{Gf};\ \text{TGM pica} = 2\ {}_2\text{Gf};\ 6\text{ picas} = 1\ {}_1\text{Gf}$$

A single printer's point is almost exactly equal to 2 $_3$Gf; twelve of these, almost exactly equal to a printer's pica, is 2 $_2$Gf; this means that there are six picas in a TGM "inch" (really, an unciaGrafut, $_1$Gf). So developing new printers' units is a simple matter of rounding off 0;0020793 Grafut, or about 0.00036354 inches (0.00923395 millimeters), and making that the TGM point, equal to two triciaGrafuts. This rounding is tiny enough that we need not trouble ourselves with it; for most practical purposes, we can use TGM points and American printer's points interchangeably.

All in all, this is a remarkable correspondence, and means that printers and typesetters should have no problems adjusting to TGM.

For larger distance, the quadquaGrafut (^{4}Gf) works quite nicely. It is a little less than four miles (3;98 mi) and a little more than six kilometers (6;17 km). For those who desire more familiar lengths, there is the triquaGrafut (^{3}Gf), which is a little less than a third of a mile (0;398 mi), and a little more than half a kilometer (0;617 km). Three of these are a short mile (0;Ɛ5), dubbed a Gravmile; two of these are a long kilometer (1;03 km), dubbed a Gravklick or Gravkay. These simple relationships allow for easy conversions of travel distances and ease the transition to the new system.

4.2 Area and Volume: The Surf and the Volm

The Grafut, one of the three primary workhorses of the TGM system (we've already met the Tim; the third is the Maz, rounding out the three which give TGM its name), also yields units for area and volume. While the metric and traditional systems simply use the squares and cubes of their length systems (though the liter is theoretically a cubic decimeter, its use is now discouraged and it is not a standard unit of SI), TGM provides special units with definite values and independent names.

Area is measured by the square Grafut, the Surf (Sf). Obviously, a Surf is a little less than a square English foot (more precisely, one Surf equals 0;Ɛ362 ft^2). An unquaSurf (^{1}Sf) is quite close to a square yard, and a bit larger than a square meter (1;07 m^2).

$$\textbf{Surf (Sf)} = 0.9410713018 \text{ ft}^2 = 0.0874283848 \text{ m}^2$$

Volume is measured by the cubic Grafut, called the Volm (Vm). That makes it almost equal to a cubic foot (about Ɛ/10 ft^3), and a little more the twenty-five liters ($\not{d}$25 L). A Volm of water weighs about halfway between six imperial gallons and six customary gallons.

$$\textbf{Volm (Vm)} = 0.9129222555 \text{ ft}^3 = 25.8503556494 \text{ L}$$

Volume is a confused topic in both metric and customary measures. Metric attempts to define the liter as equivalent to a one cubic decimeter; however, SI discourages the use of the liter, and the fact that one cubic *deci*meter of water consists of one *kilo*gram of mass while being equal to a prefixless liter is beyond bizarre. Customary measurements are cobbled together as normal, equally chaotic as metric but at least honestly so. Ostensibly, a customary pint weighs one pound; as the old rhyme goes, "A pint's a pound the world around." Thanks to a British reform in Victorian times, however, a pint's *not* a pound the world around; indeed, to those following British Imperial measures, "A pintful of water's a pound and a quarter." Of course, neither of these have any relation to the cubic foot, which is normally used to measure volume; not to mention that volumes of wet and dry goods are measured using different units. All in all, the TGM system is a great simplification, not to mention rationalization, compared to both systems.

TGM's Volm ends up being a remarkably convenient unit, with many remarkably convenient subdivisions. One example is drink sizes and similar necessities. SI has proven totally inadequate in this field; people in metric countries routinely refer to the half-liter (a deprecated unit that SI rejects) as a "pint" for ordering drinks, for example, utilizing a rejected unit to approximate a truly convenient size. However, in TGM, 3 $_2$Vm equals just a bit less than the imperial pint, and a bit more than half a liter. (A liter, of course, is "a liter bit more than a quart.") Pendlebury suggests making this its own unit, and calling it the "Tumblol," and some have come to call it the "Pintvol." It can be combined in precisely the same way as our current pints are combined into quarts, which are combined into gallons, to approximate these sizes which have been found so convenient for practical work for so long. Two Tumblols (6 $_2$Vm) is one Quartol, which ends up being a little less than an imperial quart, a little more than a customary quart, and a little more than the old liter. Four Quartols (20 $_2$Vm, or 2 $_1$Vm) is the Galvol, which approximates the imperial and customary gallons closely enough to mimic their convenience, and which is equal to about four and a third liters. And, of course, the Oumzvol ("Oumz" pronounced to rhyme with "ounce") is 2 $_3$Vm, of which there are 16 (onequa six) in the Tumblol (or Pintvol).

It works for smaller divisions, as well. A teaspoon, which is often approximated in metric as 5 milliliters (a pretty close approximation, as the true value is 4;Ɛ190 ml), is almost exactly 4 $_4$Vm. This means that 10 $_4$Vm, or 1 $_3$Vm, is very close to one tablespoon (which is three times the teaspoon). We are here referring to customary, not imperial, teaspoons

Colloquial Volume Expressions			
TGM Unit	Value	Colloquial Name	Decimal Equiv.
Volm	4 $_{4}$Vm	Sipvol	1.0117 U.S. tsp; 0.0050 L
Volm	1 $_{3}$Vm	Supvol	1.0117 U.S. tbs; 0.0149 L
Volm	2 $_{3}$Vm	Oumzvol	1.0117 U.S. fl. oz.; 0.0299 L
Volm	16 $_{3}$Vm	Cupvol	1.1382 U.S. cups; 0.2692 L
Volm	3 $_{2}$Vm	Tumblol (Pintvol)	1.13818 U.S. pints; 0.538549 L
Volm	6 $_{2}$Vm	Quartol	1.1382 U.S. quarts; 1.077098 L
Volm	2 $_{1}$Vm	Galvol	1.1382 U.S. gallons; 4.308392 L

Table ↊: Some Colloquial Expressions for TGM Volume Units with Decimal Equivalents

and tablespoons. In metric countries, recipes are often scaled in "cups" or "eating-spoons" which do not have a standardized size; or they are measured in weights, which necessitates weighing them out on a scale rather than simply scooping them up with a normal spoon. TGM is an improvement on both systems.

All in all, the Volm is a very versatile unit which will serve us well. We will see how the Volm relates to mass and weight when we reach Chapter 5 on page 24.

4.3 Accleration and Velocity: The Gee and the Vlos

Naturally, this measurement of the Grafut also gives us units for accleration and for velocity. Accleration is measured in Gees, which are simply 1 Gf/Tm^2:

$$\textbf{Gee (G)} = 9.81005 \text{ m/s}^2 = 32.1852 \text{ ft/s}^2$$

Frequently, when doing calculations involving gravity we speak in "gees" even in the customary or metric systems; we simply convert "gees" into these awkward numbers in our formulas. In TGM, the Gee is the *unit* of accleration. That means it equals one; to multiply when it should be divided, or vice versa, or to forget to do either makes no difference in the final result. This prevents many accidental errors that frequently creep into calculations otherwise.

Simple motion can be measured either as a *vector* or as a *scalar*. When it's a *vector* (that is, when it includes directional information), it's called velocity; when it's a *scalar* (that is, when it doesn't), it's called simply speed. Either way, the unit, the Vlos, is the same; maintaing the normal 1 : 1 ratio, the Vlos is simply 1 Gf/Tm.

$$\textbf{Vlos (Vl)} = 1.7 \text{ m/s} = 5.6 \text{ ft/s} = 3.8 \text{ mph}$$

Tom Pendlebury, the inventor of TGM, calls this a "comfortable walking speed." Personally, I think this is optimistic on his part; it is, however, a very manageable brisk walking speed. Just as the quadquaGrafut and its fractions proved a useful distance for measuring walking, so also the Vlos is a useful unit for measuring walking speed. TGM is about a rational, dozenal system of human-scaled units, and the Vlos is another excellent example.

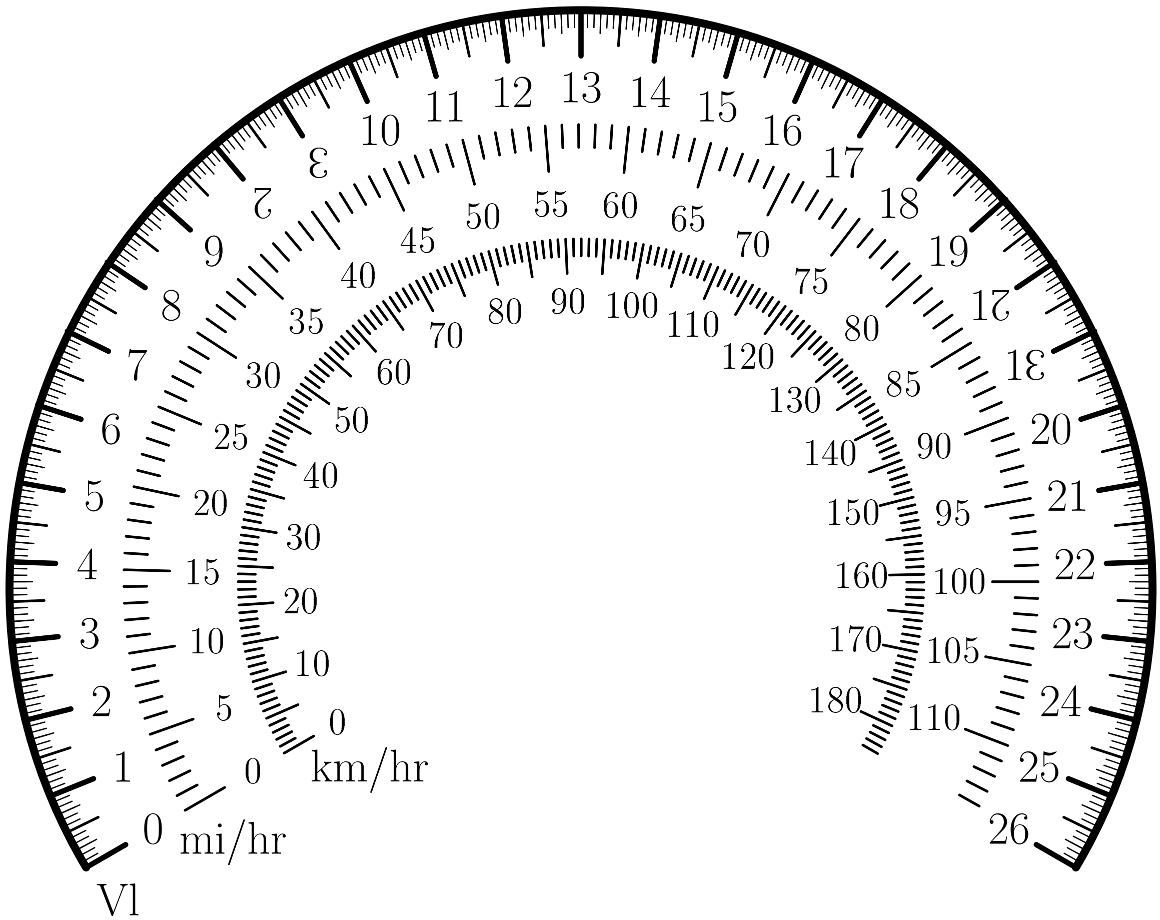

Figure 2: A speedometer calibrated in Vlos, mi/hr, and km/hr.

Its derivative units are interesting; 8 Vlos is only a little more than 30 mph, while 5 Vlos is only a little over 30 km/h. If anyone actually uses cassette tapes anymore, 4 biciaVlos is the speed that the tape has to move to produce comprehensible sound.

The Vlos yields even more interesting units regarding driving speeds. The standard highway speed limit in North America is typically d̸65 miles per hour ("mph," or, more correctly, "mi/hr"); this works out very closely to 15 Vlos exactly (more precisely, 15;089↊). 15 Vlos is also very close to the standard metric highway speed limit in North America where SI is preferred; in Canada, for example, it is typically cited as d̸100 kilometers per hour ("kph," or, again more correctly, "km/hr"). (It comes out to 104.2318 km/hr.) 15 Vlos, then, would provide a very convenient new speed limit for such cases without requiring people to change their habits much, and without confusing those who have not yet upgraded to new speedometers.

Even more interesting, d̸80 mi/hr comes out as almost exactly 19 Vlos (18;↋↋94 Vl), so those desiring a higher speed limit will have an equally convenient figure to argue for.

Further speed correspondeces are that d̸15 mi/hr is very close to 4 Vlos (3;↋2↋6 Vl), perhaps making a useful speed limit for parking lot travel lanes and narrow alleyways. d̸30 mi/hr is only slightly less than 8 Vlos (7;↊5↋0 Vl), a useful in-town speed limit. Another common in-town speed limit, đ25 mi/hr, comes reasonably close to 6;6 Vlos (6;68Ɛ2 Vl), providing another convenient estimation.

All in all, the Vlos is an extremely versatile unit, closely approximating common units in other systems with human-scaled dimensions of its own. Speed limits are a great example of this, as our current system of giving speed limits largely in increments of ten miles per hour could be easily replaced by giving them in whole Vlos, dividing them in half when necessary. Furthermore, these correspondences make the new units easy to work with, making the conversion smoother for those still using speedometers of the old system.

Examples

1. A car travels 4;8 ^{4}Gf (17.5 mi) in 0;7 Hr (35 min). What is its average speed in
   - (a) ^{4}Gf/Hr (mph), a. 4;8 ^{4}Gf/0;7 Hr = 8 ^{4}Gf/Hr 17.5 mi x 60 / 35 min = 30 mph
   - (b) Gf/Tm (ft/s) b. 4 8000 Gf/7000 Tm = 8 Gf/Tm (17.5 mi x 1760 x 3) / (35 x 60) = 44 ft/s
   - (c) Vlos? c. 8 Vlos.
2. A car runs over a cliff ↊8 Gf (145 ft) high.
   - (a) How fast is it falling after 10 Tm (2 s)?
     When it leaves the cliff its downward velocity is nil. G = 1 Vl/Tm (32.2ft/s^2), and acceleration is $\frac{\Delta v}{t}$. So after:

10 Tm	×	1 Vl	=	10 Vl
2 s	×	32.2 ft/s	=	64.4 ft/s

   - (b) How far has it dropped by then?

(Av. speed) 6 Vl (Gf/Tm)	×	10 Tm	=	60 Gf
(Av. speed) 32.2ft/s	×	2 s	=	64.4 ft/s

   - (c) How long before it drops in the sea?
     Distance equals average speed multiplied by time. In the usual algebraic calculations, therefore, $t = \sqrt{\frac{2d}{g}}$.
     $\sqrt{\frac{\text{↊8Gf}\times 2}{\text{1Gf/Tm}^2}} = 14\ \text{Tm} \qquad \sqrt{\frac{\text{145ft}\times 2}{\text{32.2ft/s}^2}} = 3\ \text{s}$
   - (d) What is its downward speed when it hits the water? $v = \frac{d}{t}$, so this is just a matter of plugging in the numbers, remembering that we've just calculated the time it takes to hit the water above.
     $\frac{\text{↊8Gf}}{\text{14Tm}} = 8\ \text{Vl} \qquad \frac{\text{145ft}}{\text{3s}} = 435\ \text{ft/s}$

Exercises

1. The base of a tank measures 3 Gf x 4 Gf (1 m x 1.25 m); it holds water to a depth of 1;6 Gf (0.5 m).
   - (a) What is the volume of water in cubic Gf (m^3) and also in Volms (liters).
   - (b) If a pipe empties the tank at 1 biciaFlo ($_2$Vm/Tm) (1 L/s), how long will it take the tank to empty?
2. A car increases speed from 6 ^{4}Gf/Hr to 14 ^{4}Gf/Hr (24 mph to 60 mph, or 40 to 100 km/h) in 20 Tm (4 s). What is the acceleration in:
   - (a) Vl/Tm (mph/s or km/h/s);
   - (b) Gf/Tm^2 (ft/s^2 or m/s^2);

(c) in terms of G?

Chapter 5: Matter and Force

Fundamental Realities: The Density of Water and Weight (Force to Mass Ratio)

Gravity is produced by matter, which is (more or less) anything that we can see and touch. It is *stuff*, physical stuff; all matter is attracted to all other matter through the force of gravity, and we can determine what physical things are matter and what physical things are not (such as energy) in part by whether they exert a gravitational pull on other matter. Matter occupies space, and so has *volume* (which we measure by the Volm[8]). Volume is simply how much space, in three dimensions, the matter occupies.

But volume only tells us how much *space* the matter occupies; it doesn't tell us how much matter there is. Matter can be packed more or less tightly within a given volume, meaning that there can be more or less matter within the same volume. Furthermore, gravity pulls more strongly on larger amounts of matter than on smaller amounts, so the more tightly the matter is packed, the more strongly gravity pulls on it (for the same volume). The pull of gravity on something is called its *weight*; so more tightly packed matter has a greater weight for the same volume than more loosely packed matter. For example, a Volm of lead will weigh more than a Volm of aluminum, which in turn weighs more than a Volm of water (which is why it sinks).

The amount of matter in an object is called its *mass*; how tightly packed that matter is is called its *density*; and (as already seen) how hard gravity pulls on an object is called its *weight*. All of these quantities are inextricably intertwined; as such, we must often define them in terms of each other. Nevertheless, we'll approach each in turn; simply remember the meaning of the terms, and there should be no trouble.

Force is the subject of Newton's famous second law, written mathematically as $F = ma$. In other words, force is the product of mass and acceleration. It is, obviously, dependent upon the unit of acceleration (the Gee, which we have already seen[9]), as well as upon the unit of mass (which we shall see shortly[↊]).

There is also the matter of *pressure*, which is the amount of force acting per unit area. This is likewise dependent upon mass and force, and will be addressed in turn.

5.1 Mass: The Maz

Water is by far the commonest liquid on the planet, and it's an easily manipulated fluid, easily contained and measured; therefore, it's convenient to use water as a base substance for determining various measurements, mass and density included. Mass, of course, depends upon density; however, density is defined partially in terms of mass. Consequently, it makes most sense to address mass first.

[8] *See supra*, Section 4.2, at 1↋.
[9] *See supra*, Section 4.3, at 21.
[↊] *See infra*, Section 5.1, at 24.

Mass is measured by the Maz, which is the mass of 1 Volm of pure, air-free water under a pressure of one standard atmosphere and at the temperature of maximal density (which is 3;↋915°C).

Maz (Mz) = 25.850355565 kg = 56.99028287 lb avoir

This is within a hair's breadth of 49 pounds, and only a little less than 22 kilograms. The unit is large in comparison with similar units in other systems, but it has at least one distinct advantage over them: it maintains the 1 : 1 ratio between basic units. In comparison, consider the units of SI. The basic unit of length is the meter, and consequently the basic unit of volume is the cubic meter; however, the original basic unit of mass was the gram, which was derived from the mass of one cubic *centi*meter of water. Now the basic unit of mass is the *kilo*gram, which has a prefix meaning "thousand." However, the kilogram's volume is about that of a cubic *deci*meter, which is only one thousandth of a cubic meter. Determining mass based on volume and density thus often introduces small errors which can end up being serious later on. We have no such difficulties in TGM.

In any case, smaller divisions of the Maz are often extremely conveniently sized. For example, massing people in unciaMaz might be convenient; the unciaMaz is about four pounds and one unqua ounces. The biciaMaz comes to $6\frac{1}{3}$ ounces, which is about 130 grams, a convenient size for massing smaller things; a can of beans, for example, might be two biciaMaz (2 ${}_{2}$Mz). The triciaMaz makes a good unit for spices and other ingredients generally added in small amounts; it totals just over half an ounce (specifically, 0;63↋↊ oz). For larger measures, the megaton is 1;0↋56 septquaMaz (^{7}Mz) (referring here to 1,000 times the metric ton).

For daily measures, however, such as massing persons, an auxiliary unit is probably appropriate: for example, the Poundz, or Kilg.

Poundz (Lbz) (Kilg (Klg)) = 3 ${}_{2}$Mz = 16;↋↋64 oz; 0;6567 kg

It's worth noting that this is almost exactly 17 ounces (đ19), or only three ounces more than a normal customary-imperial pound. It's also only a little bit more (just over three unqua grams) than the "metric pound" or a half kilogram, which is commonly used in metric countries for produce and similar items.

The Poundz is made up of 16 (onequa six) Oumz (pronounced to rhyme with "ounce"), another auxiliary unit that comes quite close to traditional values:

Oumz (Oum) = 2 ${}_{3}$Mz = 1;07↋8 oz; 25;↋048 g

It's also worth noting that 16 Oumz to the Poundz is much better than our current sixteen ounces to the pound; 16 is a dozen and a half, a very convenient number in reference to our base, something which sixteen simply doesn't share with ten.

The astute reader will have noticed a striking similarity between the Oumz and the Poundz and the Tumblol and Oumzvol found in Table ↊ on page 21; namely, that the larger units of 3 bicia are made up of 16 smaller units of 2 tricia. This is because of the 1 : 1 correspondence of TGM units; any substance of a density similar to water will have the same mass in Maz as its volume in Volm. So while "a pint's a pound the world around," the

TGM system works better, as there's no need to remember little rhymes about such things. For water or substances of similar density, the volume *is* the mass. This makes it quite easy to simply change "vol" to "mass" in the names of these auxiliary units and refer to amount of substance just as easily as to the space that it takes up.

So, for example, an Oumzvol of water masses an Oumzmass (which we better know as just the Oumz), and in turn weighs an Oumzweight (2 $_3$Mag, a unit which we'll meet in a moment); a Pintvol of water masses a Pintmass (a Poundz), and weighs a Pintweight (2 $_3$Mag); and so forth. These auxiliary units work just as well as the basic units do (whereby one Volm of water masses one Maz and weighs one Mag) due to TGM's insistence on a 1 : 1 relationship wherever possible.

Of course, our definition of the Maz includes a reference to density, for which we have not defined a unit. To density, then, we now proceed.

5.2 Density: The Denz

Density, again, is how tightly packed a mass is into a given volume. As such, it combines mass and volume into a single unit. Because TGM always maintains a 1 : 1 correspondence between basic units, density is no exception. The Denz is simply one Maz per Volm (1 Mz/Vm).

$$\textbf{Denz (Dz)} = 999.972 \text{ kg/m}^3 = 62.43 \text{ lbs/ft}^3$$

SI has no unit of density, using the composite unit of kilograms per cubic meter instead. Ostensibly, this was based on the density of water; however, as is easily seen from our definition of the Denz, it comes to an awkward number of kilograms per cubic meter, very close to an even thousand, yet still far enough away that rounding off can introduce significant errors.

This strange off-by-a-little error in SI also explains the deprecation of the old liter. The liter was, theoretically, a cubic decimeter, and was supposed to be the unit of volume in the metric system. (Why not the cubic meter, which would be more logical, and which was later adopted by SI?) However, at the time alterations in density based on temperature were not clearly measured, and consequently the volume of one kilogram of water at maximal density is not one cubic decimeter, but rather 1.000028 cubic decimeters. Since the liter has been deprecated, this is not much concern now; it does, however, explain certain differences in conversions between TGM units and metric units based on the kilogram as opposed to metric units based on the liter. Nevertheless, the entire issue is excluded from TGM itself; we need not bother with such business when we've adopted the new system.

5.3 Force: The Mag

Newton's famous three laws of motion includes the definition of force in his second law: force is the product of mass and acceleration, written mathematically as $F = ma$. A simple formula, it's immensely useful in all normal applications of physics and engineering. The original metric unit of force (in the "cgs" system) was the *dyne*, or one gram-centimeter per second squared ($\text{g} \cdot \text{cm/s}^2$). This has since been supplanted by the newton, which is one $\text{kg} \cdot \text{m/s}^2$. In the customary system, "poundals" are used, which equal pounds-feet per second squared ($\text{lb} \cdot \text{ft/s}^2$).

As these units make clear, force is what is necessary to accelerate a certain mass a certain amount. In TGM, as always, we maintain a 1 : 1 correspondence between these units, so the unit of force will be one Maz-Grafut per Tim squared ($\mathrm{Mz} \cdot \mathrm{Gf}/\mathrm{Tm}^2$), or one Maz-Gee ($\mathrm{Mz} \cdot \mathrm{G}$), which is the same thing. This gives us the unit of force, the Mag:

Mag (Mg) = 253.5932659 N = 1834.246667 pdl

It is the force required to accelerate one Maz by one Gee; that is, it is the force exerted by gravity upon an object of one Maz. Stated the other way, it is the force required to prevent one Maz from falling to the earth (strictly speaking, by holding something up against the force of gravity one is accelerating it equally to gravity, but in the opposite direction).

Because of the 1 : 1 correspondence in basic units in TGM, the *mass* of an object which is on the earth will always be equal to its *weight*; that is, gravity will pull on a mass measured in Maz with a force of equal quantity measured in Mags. So a small female of 2 Maz mass will also have a weight of 2 Mags (provided she is on the earth). As this explanation suggests, *weight* is the force by which an object is pulled by gravity; *mass*, on the other hand, is the amount of matter an object has. The two are very intimately related, but they are not the same thing.

For example, while mass and weight are equal (in TGM) while on Earth, they are *not* equal while on the moon. On the moon, gravity pulls only about a sixth as hard as it does on earth, because the moon has only about a sixth of the mass as earth. Therefore, our 2-Maz woman might weigh 2 Mags on earth, but she weighs only 0;4 Mags (one sixth of 2 Mags) on the moon. In either place, though, her mass is still 2 Maz. This is an important distinction to keep in mind when dealing with these units. The Maz is mass, the amount of matter in an object; the Mag is force, which includes weight, the force with which gravity pulls an object.

The old systems, however, do not clearly distinguish mass and weight. Both have separate units for the two; the metric system has the kilogram and the newton, while the customary-imperial system has the pound and the poundal. However, both the kilogram and the pound are often used to express *weight* as well as mass, which leads to confusion. When these two units are used in this way, an "f" is usually appended to their abbreviations; so we have kilograms-force (kgf) and pounds-force (lbf), rather than simply kilograms (kg) and pounds (lb). For these units, rather than $\mathrm{kg} \cdot \mathrm{m/s}^2$ and $\mathrm{lb} \cdot \mathrm{ft/s}^2$, we have $\mathrm{kg} \cdot 9.8\mathrm{m/s}^2$ and $\mathrm{lb} \cdot 32.2\mathrm{ft/s}^2$. In other words, just as $F = ma$, $W = mg$, giving us some very different units. This is why one kilogram of mass doesn't weigh one newton: because the units don't have TGM's 1 : 1 correspondence. Instead, one kilogram of mass weighs one kilogram-force, needlessly bifurcating the system.

These weight units correspond to the Mag as follows:

Mag (Mg) = 25.85931648 kgf = 57.01003812 lbf

TGM fortunately avoids this entire issue. Weight will always be equal to mass when they are measured in Mags and Maz, unless some additional force is being applied. When, for example, our 2-Maz woman is going up in an elevator, she weighs a bit more; that is, she really *is* heavier, even though her mass (the amount of matter she contains) is unchanged. When she's being launched in a spaceship, she is considerably heavier; that is, she weighs considerably more in Mags than she did before. But her mass remains the same. When she is falling, there is nothing opposing the acceleration of gravity, so she is *weightless*, just as she would be if in orbit[↋]; but her mass is still 2 Maz, just as it was before.

Weight is with us every day, and is more or less constant over the whole world. It therefore makes sense for weight and mass to be equivalent while on the earth, while still maintaining the real distinction between them. TGM does exactly that.

5.4 Pressure and Stress: The Prem

Related to force is *pressure* or *stress*, which is the amount of force applied per unit area. Most commonly we refer to *air pressure* or *water pressure*. As an example, atmospheric pressure is about 12;8 pounds-force per square inch (lbf/in^2, or psi), or 4↊779 newtons per square meter (N/m^2), which are typically called "pascals" (Pa). The pascal, obviously, is an extremely small unit, and commonly we speak of kilopascals (↊^{3}; abbreviated "kPa").

In TGM, maintaining as always the 1 : 1 correspondence, the unit of pressure is one Mag per Surf (1 Mg/Sf):

$$\textbf{Prem (Pm)} = 2900.582763 \text{ N/m}^2 \text{ (Pa)} = 0.42069339 \text{ lbf/in}^2$$

Remember, of course, that the Mag is simply Mz · G, or more verbosely Mz · Gf/Tm^2, which opens up many easy applications of the Prem. For example, because one Volm of water weighs exactly one Mag, one Grafut of water exerts a pressure of one Prem at its bottom. Hydraulic engineers would say that the pressure of water in Prem is equal to the head of water in Grafuts. The same works for any other liquid, once we've multiplied by that liquid's density in Denz. So a column of mercury 2;7 Gf high presses downward at $2;7 \times 11;7 = 2\text{↋};11$ Pm.

The astute observer will note that 2;7 Gf is approximately the same as 26 inches or 538 millimeters, which just happens to be very close to the current measurements for the pressure of the "standard atmosphere" (that is, normal air pressure on Earth's surface) in the current system. Air pressure at sea level varies according to many factors, especially humidity; however, it only varies within a specific range of possible values. As an example, originally the standard atmosphere was defined as the pressure of 26 inches of mercury; during metrification it was rounded off to the nearest centimeter, namely 64; and now it is quoted in millibars, which is one hundredth of one kilopascal (or one dekapascal), to give the value 705;3. All of these vary more or less; all of them are realistic as "standard atmospheres," because they fall well within the range of normal atmospheric pressure at sea level. If we convert that millibar figure to Prems, we wind up with 2↊;↋237 Prem; this is so close to 2↋ that rounding up to 2↋ only makes a difference of less than two millibars, putting it also well within the normal range of qualifying values for the "standard atmosphere." This gives us another non-coherent unit in TGM:

$$\textbf{Atmoz (Atz)} = 1015.203963 \text{ mb} \;/\; 761.465 \text{ mm} = 29.978 \text{ in}$$

That's millimeters and inches of mercury, of course. In terms of TGM, the Atmoz is simply 2↋ Prem.

[↋] That is, she is in *free fall*, which isn't quite the same thing but has the same effects as true weightlessness does. The distinction is not relevant here.

At	Pm	At	Pm	At	Pm	At	Pm	At	Pm	At	Pm
1/2	15;6	1/6	5;↊	1/↊	3;6	1/14	2;23	1/20	1;56	1/28	1;116
1/3	Ɛ;8	1/7	5	1/10	2;Ɛ	1/16	1;Ɛ4	1/23	1;368		
1/4	8;9	1/8	4;46	1/12	2;6	1/18	1;9	1/24	1;3		
1/5	7	1/9	3;↊8	1/13	2;4	1/19	1;8	1/26	1;2		

Table Ɛ: Some common fractions of the standard atmosphere in Prem.

But isn't 2Ɛ a rather clumsy number for such a commonly used figure as the standard atmosphere? This can be answered in two ways. The first is to reply with another question: clumsy compared to what? Compared to the thirty of 26 inches of mercury? Or clumsy compared to 64 centimeters of mercury? Or 705;3 millibars? 2Ɛ at worst is certainly no clumsier than these, so saying that it's clumsy doesn't given any value for comparison.

Furthermore, 2Ɛ is not only highly divisible, it is divisible by seven and five (since it is their product), the two numbers which the dozenal system doesn't normally handle well. A few examples of such common, easily handled divisions are listed in Table Ɛ on page 29.

The Prem is also the standard unit of *stress*, which is the average force per unit area (the pressure) on a deformable body on which internal forces are acting. It's especially important in engineering. As an example, a steel bar with one biciaSurf (1 ${}_2$Sf) cross-sectional area undergoing a force of one unquaMag (1 ^{1}Mg) endures a stress of one triquaPrem (1 ^{3}Pm).

Examples

1. A bar of iron measured 2 x 3 x 40 ${}_1$Gf (đ50mm x đ75mm x 1m). What is its mass? The density of iron is 7;Ɛ Dz (đ7900 kg/m^3).
   $$200\ {}_3\text{Vm} \times 7;\text{Ɛ Dz} = 13↊0\ {}_3\text{Mz} = 1;3↊\ \text{Mz}$$
   $$0.00375\ \text{m}^3 \times 7900\ \text{kg/m}^3 = 29.625\ \text{kg}$$
   Note here that uncia × uncia × uncia = tricia.
2. What is the pressure of water on the floor of the tank in Exercise 1, Chap. 4 (found on page 23)?

Vol. of water		=	16 Vm
Weight	16 Vm × 1 Dz × 1 G	=	16 Mg
Base area		=	10 Sf
Pressure	16 Mg / 10 Sf	=	1;6 Pm
Vol. of water		=	0.625 m^3
Weight	0.625 m^3 × 1000 kg/m^3 × 9.80665 m/s^2	=	6129.156 N
Base area		=	1.25 m^2
Pressure	6129.156 N / 1.25 m^2	=	4903.325 N/m (pascals)

   Note the shorter method of solving this question in the TGM system; because of the 1 : 1 correspondence, 1;6 Gf of depth means that the pressure at the base will be 1;6 Pm.
3. A man "weighs" 3 Mz (đ75 kg) and is sitting in a car which decelerates from 14 Vl to 8 Vl (đ100 to đ50 km/hr) in 16 Tm (3 seconds). By what force does he feel himself thrust

forward? (In English we "weigh" usually to measure mass, not weight. Remember the distinction explained in Section 5.3 on page 26. Mass is the amount of matter in an object; weight is a measure of force.)

$\frac{14-8\text{Vl}}{16\text{Tm}}$	=	4 / 9	=	0;54 G
Force	=	3 Mz × 0;54 G	=	1;4 Mg
$\frac{100-50\text{km/hr}}{3\text{s}\times 3600\text{s/hr}}$	=	50 / 10,800	=	4.63 m/s^2
Force	=	75 kg × 4.63 m/s^2	=	347 N

EXERCISES

1. A bar of aluminum "weighs" 0;4 Mz (2 kg). The density of aluminum is 2;8 Dz (đ2700 kg/m^3).
   (a) What is the volume of the bar?
   (b) If it is 2;3 Gf long (đ750 mm), what is its cross-sectional area?
2. A "weight" of 3 Maz (đ75 kg) is on one end of a piece of rope, which passes over a large pulley. A 5 Mz (đ125 kg) "weight" is on the other end, and at first held from descending, then let go. What is the acceleration of the system? Formula: $a = F/m$. Hint: Mass to be moved is the sum of the masses (ignore the rope and pulley), but driving force is their difference multiplied by G.
3. A man "weights" 3;26 Mz (đ83 kg) and each of his feet covers an area of 0;36 Sf (0.0255 m^2). What is the pressure in Prems (N/m^2) on his feet when standing evenly on both?
4. A hotwater tap in a kitchen is 14 Gf (4.7 m) lower than the surface of the water in the filler tank in the house loft. What is the pressure in Prems (N/m^2) at the tap?
5. A metal bar of cross-section 7 ${}_3$Sf (0.6 in^2) was loaded till it broke. This took 3 ^{2}Mg (10.5 tons). What is the tensile strength of the metal in Prems (lb/in^2)?
6. What is the approximate equivalent in avoird. (metric) of
   (a) the unciaMaz,
   (b) the biciaMaz,
   (c) the triciaMaz, and
   (d) the quadciaMaz?

Chapter 6: Work, Energy, Heat, and Power

FUNDAMENTAL REALITIES: ABSOLUTE ZERO AND THE SPECIFIC HEAT OF WATER

WORK, ENERGY, HEAT, AND POWER are fundamentally interrelated concepts; consequently, in TGM their units are likewise interrelated. Like all basic units in TGM, they maintain a 1 : 1 correspondence to one another, facilitating conversion and other routine chores. First we will address work and energy; then we will move on to heat; and finally we will discuss power.

6.1 Work and Energy

Work is not only something that we have to go to every day to make our money; it is also a scientific term with a very specific physical meaning. Work is force over a distance. Take, for example, an automobile. Automobiles are very large, and it therefore takes a large amount of force to move them. When your automobile breaks down, you might have to push it. You can push until the veins pop out of your neck, but if you don't actually move it, you've done no work. When you *have* moved it, though, you've done a lot of work, even if you've only moved it a tiny distance. It is, in other words, simply force exercised over a distance; the same unit is used for measuring the amount of energy required to do a given amount of work.

Because work is force over distance, TGM's unit of work is Mags over Grafuts:

$$\textbf{Werg (Wg)} = 74.983195487 \text{ N}\cdot\text{m} = 55.3 \text{ lbf}\cdot\text{ft}$$

The Werg is used to measure work and energy of whatever kind.

The 1 : 1 correspondence which is TGM's strength serves it in good stead here, as well. For example, raising six Maz to a height of eight Grafut requires fourqua Wergs of work. This is also called *kinetic energy*; that is, it's the energy required to propel the item upward, as well as the amount of work that was actually done. The object has *potential energy* while it is being held up; namely, the potential energy which would become work when it falls. The amount of this potential energy is equal to the work done to hold it up, fourqua Wergs. When that same object is dropped, fourqua Wergs of work is done on it by the earth to bring it to the ground.

This equivalency between energy and work is not unique to TGM. Work is really just a specific type of transfer of energy; it is unsurprising, then, that levels and changes of energy can be measured by means of the same unit that measures work. Other types of energy transfer cannot be measured in this way; but when energy is transferred resulting in a change of actual position, work has been done, and Wergs are what is used to measure it.

It's all very simple; and that is all there is to work and energy in TGM.

6.2 Heat

We all know what heat is; it's what comes from the sun on a warm summer day, and what comes out of the barbecue grill on the Fourth of July. Physically, however, heat is a bit more complex, but still easy enough to understand: it's any transfer of energy to a body by anything other than work actively performed on it.

Heat—energy transferred from one body to another without the performance of work—should not be confused with *temperature*, which is a way of measuring how much heat is present. Strictly speaking, *temperature* is defined as the concentration of kinetic energy relative to heat capacity, but we needn't worry ourselves too much with that right now. For now, it is enough to know that heat is *energy*, while temperature is something else. Energy is measured, as we've seen, in Wergs or joules (newton-meters); temperature is measured in other units. Traditionally, temperature has been measured in *degrees*, either Fahrenheit or Celsius. Scientifically, it is measured in *kelvin*, Celsius-sized degrees with a zero point at absolute zero, or *rankine*, Fahrenheit-sized degrees with the same zero point.

Traditionally, heat has been measured in quantities necessary to raise a certain quantity of water by a certain unit. The most common units are displayed in Table 10 on page 30. This was designed to make the *specific heat* of water equal to one, but it yielded problems of its own.

Traditional Units of Heat Measurement		
Unit	**Quan. of Water**	**Temp. Unit**
British Thermal Unit (BTU)	1 lb	1°F
Centigrade Heat Unit (CHU)	1 lb	1°C
Old Calorie (cal)	1 g	1°C
Cal or kcal.	1 kg	1°C

Table 10: Traditional units for heat measurement based on raising a quantity of water through a unit of temperature.

The specific heat of a substance is the amount of heat required to raise that substance through a given unit of temperature. So by definition, measuring heat in units formed as above gives the specific heat of water as one. However, it gives no idea as to how much energy is required to accomplish this raising of temperature, which is usually exactly what we need to know. Furthermore, it ignores one of the vagaries of specific heat: namely, that the energy required rises the closer one gets to the freezing and boiling points, meaning that even the specific heat of water is not equal to one at certain temperatures.

SI solves this problem by simply ditching the idea of the specific heat of water being one and measuring all energy, including heat, in joules. So the specific heat of water (assuming a kilogram of water) is 4185.5 J; that is, 4185.5 joules raises one kilogram of water through one degree Celsius. Kilograms, degrees, joules; the relationship between these units is opaque to the casual user. On the other hand, the old system of using calories to make the specific heat of water equal to one isn't transparently related to the system of energy. Thus, the worst of both worlds is encompassed without ever tripping over the best.

We can do better than this, combining the notion of the specific heat of water being equal to one while still using the same units for heat as we do for energy. We can do this by determining what the specific heat of water is and defining our temperature unit based on that, leaving our energy system as it is.

Raising 1 Maz of water from freezing to boiling requires 6Ɛ7;7 biquaWerg at a pressure of 2Ɛ Prem (one TGM standard atmosphere, the Atmoz). Converted into decimal, this means 1003.6 biquaWergs to raise 1 Maz of water through 100.054 degrees (kelvin). This means that 1 biquaWerg raises 1 Maz of water by almost 0.1 kelvin on average, thus avoiding the irregularity of the present systems. Since the specific heat of a substance varies with its actual temperature, we have a small range of possible values to use as our base point; if we set 1 biquaWerg as raising 1 Maz by 0.1 K, we can then calculate that 1 Werg raises 1 Maz of water through 0;001249 K, and make that our unit of temperature. This is a basic consequence of TGM's insistence on maintaining a 1 : 1 correspondence of basic units to one another.

Calg (Cg) = 0;0012497249 K = 0;0021Ɛ05914°F

("Calg" is from "*cal*orific *g*rade.") More simply, we can equate the units in biquaCalgs and decikelvins; biquaCalgs, or even triquaCalgs, are probably the units that would be most frequently actually used:

biquaCalg (^{2}Cg) = 0.1 K = 0.18°F

Note that this is *not* basing the TGM unit of temperature on one-tenth (1/↊) of a kelvin. The TGM unit of temperature, the Calg, is equal to the increase in temperature of one Maz of water when one Werg of energy is applied to it, and has nothing to do with kelvins. The 0.1 K equivalency is to make it easy to convert from kelvins to Calgs and vice-versa; the equivalency is well within the range of the actual specific heat of water between freezing and boiling, so no liberties are taken with the data.

This equivalency *does*, in fact, make conversion from Celsius and kelvin scales very easy: simply multiply the Celsius or kelvin temperature by ten and dozenize. Remember, of course, that the zero point will vary; a Celsius temperature will zero at the freezing point of water, while a kelvin temperature will zero at absolute zero, that temperature so low that even molecular motion stops. Normally Calgs should be likewise counted from absolute zero (which is 16Ɛ7;6 biquaCalgs below the freezing point of water), but there is nothing to prevent the Calg from being counted from some other zero point, so long as that zero point is recalled.

We are accustomed to neat ranges between water freezing and boiling in our current Celsius and Fahrenheit systems; however, these ranges are not particularly useful for any kind of precision work. For example, in the Fahrenheit system water freezes at 28° and boils at 158°, a range which is actually more useful than is generally supposed. The difference is 130, or đ180, which is a highly divisible number, and not coincidentally also the number of degrees in a half-circle. It gives even divisors much more frequently than the poorly chosen 84 (đ100) degrees of the Celsius scale. In that scale, of course, water freezes at 0° and boils at 84°. However useful these scales may be in the kitchen, though, they aren't particularly helpful anywhere else. The freezing and boiling points of water vary greatly with different conditions, particularly pressure; the range is great enough that the generalization allowed isn't always useful.

Still, for popular use, there is no harm in ensuring that the numbers we use to record day-to-day outdoor temperatures and things of that nature are reasonably sized. Because we've selected the size of the Calg rationally, though, as equal to the increase in temperature of one Maz of water when one Werg of energy is applied to it, we cannot arbitrarily say that water freezes at 0 and boils at 100, as the inventor of the current Celsius system did. However, it is still convenient to count the freezing point of water as zero; because we equated 0.1°K with one biquaCalg, this means we can simply take the temperature in the Celsius scale, multiply it by ↊, and dozenize it to get the temperature in biquaCalgs, with the freezing point of water as the zero point. For convenience's sake, we can call these decigrees.

Another idea, which yields some more conveniently-sized temperatures for day-to-day uses, is the tregree, which is one dozen times the decigree; that is, the tregree is a triquaCalg zeroed at the freezing point of water, while the decigree is a biquaCalg zeroed at that same point. Convert centigrade to tregrees by multiplying by $0.8\overline{3}$ instead of by ↊.

Some frequently used, relatively standard temperatures are listed in Table 11 on page 33. Calling the first of these systems "decigrees" helps us remember that, though these are indeed biquaCalgs, their zero point is the freezing point of water, not absolute zero. Calling the second "tregrees" reminds us that we are dealing with triquaCalgs, not biquaCalgs; and also that the zero point is the freezing point of water, not absolute zero.

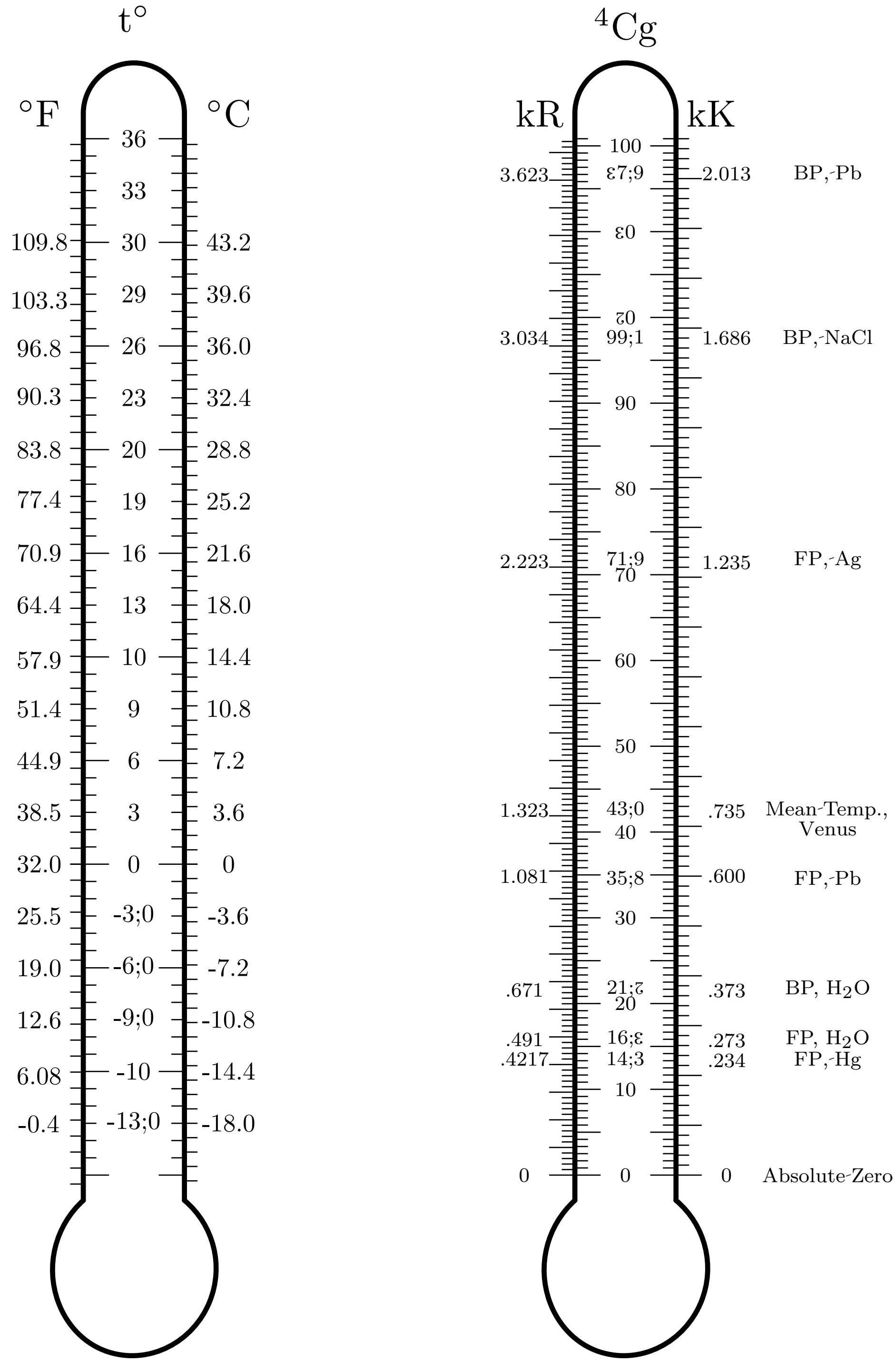

Figure 3: Thermometry: left, comparisons between Celsius, Fahrenheit, and Tregrees; right, comparisons between quadquaCalgs, kilorankines, and kilokelvins.

	Decimal		**Dozenal**	
Standard	**d̸°F**	**d̸°C**	**d°**	**t°**
Freezing Point	32	0	0	0
Room temp.	68.72	20.4	150	15;0
Blood temp.	98.42	36.9	269	26;9
Boiling Point	212	100	6Ɛ4	6Ɛ;4
B.P. at 2Ɛ Pm	212.09	100.05	6Ɛ4;6	6Ɛ;46
°C and °F equipoint	-40	-40	-294	-29;4
Absolute Zero	-459.67	-273.15	-16Ɛ7;6	-16Ɛ;76

Table 11: Some frequently used temperatures in degrees, decigrees, and tregrees.

Converting decigrees to biquaCalgs is as easy as converting Celsius to kelvin: simply add the number of heat units between absolute zero and the freezing point of water. In Celsius-kelvin, this number is 273.15; in decigrees-biquaCalgs, it is 16Ɛ7;6. To convert tregrees, simply moved the uncial point one to the left; that is, add 16Ɛ;76 instead.

Decigrees are abbreviated d°; tregrees are abbreviated t°.

6.3 Latent Heat

Latent heat is the extra heat which is added to a substance which does not actually raise the temperature of that substance, but instead melts or vaporizes it. For example, when boiling a pan of water, we can only raise that water's temperature to the boiling point; beyond that, we can continue adding heat all we want, but the water will not get warmer. Instead, it will start to *vaporize*; namely, turn to steam (which *can* get hotter, but that's a different question). If we turn the heat down, we can keep the water at a very high temperature without it vaporizing; we call this "simmering." Or we can turn the heat up and make it boil more, which will turn it into steam more quickly. The extra heat that we have to add to water which is already at its hottest temperature (the boiling point) in order to make it vaporize (turn to steam) is called its *latent heat*.

Every substance has a different latent heat of vaporization, but that of the most common substance, water, is commonly used as a benchmark. In TGM, the latent heat of water is measured in Wergs per Maz. Boiling point, of course, varies with pressure, so we typically measure latent heat at the standard atmospheric pressure; in TGM, one Atmoz, or 2Ɛ Prem.

$$\text{Latent heat of vaporization of water at 2Ɛ Pm} = 3;162314\ {}^{5}\text{Wg/Mz}$$

Notably, this is roughly five times the amount of heat required to bring the water from freezing to boiling; in other words, this is not a negligible quantity.

Similarly, at the lower end of the heat spectrum there is a corresponding *latent heat of fusion*; that is, the amount of heat to change a frozen substance into a liquid. Every sustance has this; but once again, that of water is usually taken as a convenient starting point.

$$\text{Latent heat of fusion of water at 2Ɛ Pm} = 5;6690\ {}^{4}\text{Wg/Mz}$$

One should note that this number will be substantially the same regardless of the atmospheric pressure; or, in other words, it takes a great change of pressure to make a significant change in the latent heat of fusion.

6.4 Power

Power is the rate at which work is performed; in TGM terms, it is the number of Wergs done every Tim. In the metric system, this unit is called the watt (W); in traditional systems, the unit is called ergs per second, horsepower, or foot-pounds per minute, as well as a number of other, more creative units (including Btu/hr and similar constructions). In TGM, there is the Pov:

$$\textbf{Pov (Pv)} = 2↋↋;↊208 \text{ J/s (W)} = 0;6↋4↊ \text{ hp}$$

The SI conversion factor to watts will be observed to be extremely close to 300; as such, this allows for some very close correspondences to watts in the TGM system. For example, one ${}_2$Pv (one biciaPov) is less than one thousandth below precisely three watts, so saying 1 ${}_2$Pv = 3 W is accurate enough for almost all practical purposes.

One will also note that the Pov is only a little above a half a horsepower, allowing some easy conversions between TGM and the traditional systems, as well.

Power also has to be applied over a given area; this measure is called *power density*, or more precisely *surface power density* or *specific power*. This is measured in SI in watts per square meter; it is measured in TGM in Penz, or Povs per Surf.

$$\textbf{Penz (Pz)} = \text{Pv/Sf} = 2↊38;0↋60 \text{ W/m}^2$$

Note that this is just under 5 kW/m^2.

Examples

1. A mass of 8 Maz (4 cwt, 148 kg) is raised by rope and pulley to a height of 40 Gf (40 ft, 13 m) above the ground. If the other end of the rope is attached to some load,
   (a) How much work can be done by its descent to ground level?

TGM

Force = mass × acceleration (gravity)
Earth's gravity = 1 G; mass = 8 Mz
Force = 8 Mg
Potential energy = 8 Mg × 40 Gf = 280 Wg

SI (all numbers in decimal)

Force = mass × acceleration (gravity)
Force = 200 kg × 9.8 m/s^2
Force = 1960 N Potential Energy = 1960 N × 15 m = 29400 J

Traditional (all numbers in decimal)

Force = mass × acceleration
Force = 4 cwt × 112 lbs
Force = 448 lbf
Potential Energy = 448 × 48 ft = 21504 ft-lbs

(b) How much potential energy would the same mass have at the same height above the surface of the moon? (Luna's gravitation: 0;2 G.)
0;2 G is $\frac{1}{6}$, so simply take the final answers in the last question and divide by six:

TGM	SI	Traditional
280 / 6	29400 / 6	21504 / 6
54 Wg	4900 J	3584 ft-lb

2. A car "weighs" 30 Maz (16 cwt, 6Ɛ4 kg) and is travelling at 8 Vlos (26 mi/hr, 40 km/hr). What is its kinetic energy? (E = mv²)

TGM

$$30\text{Mz} \times (8\text{Vl})^2 = 30 \times 54 = 1400\text{Wg}$$

SI

$$6\text{Ɛ}4\text{kg} \times 40\text{km/hr} = 6\text{Ɛ}4 \times 11;37\text{m/s}^2 = 6\text{Ɛ}4 \times 128;\text{Ɛ}8 = 86448;\text{Ɛ}2\text{J}$$

Traditional

$$\text{Mass} = \text{weight/gravity}$$

$$16\text{cwt} \times 94\text{lb}/28;24\text{ft/s}^2 = 52;72\text{lbf}$$

$$52;72\text{lbf} \times 26\text{mi/hr}^2 = 52;72 \times 38\text{ft/s}^2 = 52;72 \times 1154 = 2\text{Ɛ}099 ft - lb$$

This example shows the immense simplication of setting G = 1 and 1 hr = 10000 Tm.

3. A drum of oil weighing Ɛ Maz (5 cwt; 18Ɛ kg) is lifted 6 Gf (6 ft, 2 m) in 16 Tm (3 sec). What was the power required to lift it?

	TGM	
Force	Ɛ Mz × 1 G	Ɛ Mg
Energy	Ɛ Mg × 6 Gf	50 Wg
Power	50 Wg / 16 Tm	3;4 Pov
	SI	
Force	18Ɛ kg × 9;97 m/s²	1502 N
Energy	1502 N × 2 m	2Ɛ04 J
Power	2Ɛ04 J / 3 s	Ɛ41;4 watts
	Traditional	
Force	5 cwt × 94 lbs × 1 g	3Ɛ8 lbf
Energy	3Ɛ8 lbf × 6 ft	1Ɛ40 lb-ft
Power	1Ɛ40 lb-ft / 3 s	794 lb-ft/s

4. An electric motor has a power rating of 7 Pov (3 kW). How much work can it do in
   (a) 1 unciaHour (Ɛ min.);

TGM		SI	
7 Pv × 1 ³Tm	7 ³Wg	3 kW × 420 s	1060 kJ

(b) ↊ hours?

TGM		SI	
7 Pv × ↊ ^{4}Tm	5↊ ^{4}Wg	3 kW × ↊ hr	26 kWh
		26 kWh × 2100 s/h	90 MJ

Finding the answer in the traditional system is left as an exercise for the reader.

5. A 1 unquaPov (5 kW) immersion heater is in a water cistern 1;6 Gf × 3 Gf (0;49 m × 1 m). The thermostat is set to cut out at 360 decigrees (42°C), and the temperature of the water is before the heater is switched on is 130 d° (16°C). How long before the termostat cuts out? (π is 3;18.)

TGM		
Vol.	3 × 3;18 × 0;9^{2}	5;37 Vm
So mass of water:		5;37 Mz
Temp. Rise	360 - 130	230 ^{2}Cg
Heat Req.	5;37 × 230	ƐƐ0;9 ^{2}Wg
Time	ƐƐ909 Wg / 10 Pv	ƐƐ09 Tm

SI		
Vol.	1 × 3;18 × 0;24^{2}	↊5;72 L
So mass of water:		↊5;72 kg
Temp. Rise	42° - 16°	28°C
Heat Req.	↊5;72 × 28°C × 2512 (spec. heat)	14;↊0 MJ
Time	14;↊0 MJ / 5 kW	1Ɛ48 s

Exercises

1. A fork-lift truck lifts a box of goods weighing 16 Mag from a height of ↊ Gf up to 16 Gf.
   (a) How much energy has it spent on doing this?
   (b) What potential energy (from ground level) has the box of goods at that height?
   (c) If it then falls, at which velocity does it hit the ground? (Tip: acceleration is at 1 G from zero to final, so the average velocity is equal to half the final velocity.)
   (d) What is its kinetic energy on hitting the ground? ($E = mv^2/2$.)
   (e) Assuming it is unbroken (!) and it takes a force of 8 Mag to push it aside, how much energy is spent in moving it 20 Gf?
   (f) Where has this energy gone?
   (g) Do the whole problem again in metric using: đ450 kg, 2.5 to 3.25 meters, g = 9.8 m/s^2, side push 2 kN × 8 m.
2. A container was closed up at normal atmospheric pressure when the temperature was 130 decigrees (đ18°C). The building in which it was stored caught fire during which its temperature ran to 1300 d° (đ216°C). Assuming it remained intact and sealed, what was the internal pressure
   (a) in atmospheres, and
   (b) in Prem (N/m^2),

at that temperature? (Formula: $P_2 = P_1 \times T_2/T_1$. Absolute temperatures must be used.) Work to three significant figures.

Chapter 7: Angles, Rotation, Radiation, and Perspective

Fundamental Realities: Pi and the Radian

Although many of our common units contain an element of the dozenal system in spite of centuries of decimal and metric interference, some have little dozenal element, and some lack even a decimal one. The most commonly encountered of these are minutes and seconds of time, with their units of sixty; similar, however, are minutes and seconds of angle. Sixty is a multiple of both ten and the dozen; therefore, minutes and seconds cause an equal amount of trouble in both dozenal and decimal.

This system must, therefore, be corrected. First we will address TGM's contribution to angular measurement, then proceed to rotation and radiation, and finally to perspective.

7.1 Angles

First things first: it is expedient to eliminate the messy system based on the $\not{d}$360° in a circle. While this figure does provide many factors, and in fact combines the best features of both ten and twelve (since it is a multiple of both), it neither fits into a coherent dozenal system of units nor do its advantages override this problem. There are too many degrees, which are too small; we are frequently forced to deal with over one hundred degrees at a time, even when utilizing negative numbers to avoid numbers greater than one hundred and eighty. Furthermore, there is a better system already commonly used: that of the radian.

Most people know about the number pi, written π in mathematical notation. In mathematical terms, π is the ratio between the diameter of a circle and its circumference; that is, it's the result of dividing the circumference of a circle by its diameter. It is an *irrational* number, which means that it repeats to infinity; it is furthermore a *non-repeating* irrational number, because it has no discernible pattern. In decimal, of course, π is well-known as 3.141592, and so on as long as one cares to go. In dozenal, π is equal to 3;184809493Ɛ, and again so on as long as one cares to go.

π is, obviously, an inconvenient number, and no one particularly likes dealing with it. However, it is a natural number and is unavoidable when dealing with natural systems; furthermore, it is a cornerstone of geometry and engineering, without which we could do very little mathematically.

π also gives us the *radian*, which is a useful measure for angles handier than the cumbersome degree. Since π is the ratio of the diameter of a circle to its circumference, we know that a circle of a given radius (which is equal to half its diameter) rolls through a certain constant distance every time it travels its radius; that is to say, a circle one Grafut in radius rolls through a certain portion of its circumference for every Grafut it travels. That portion of its circumference we call the *radian*, and the angle beginning at the center of the circle and extending outward to cover that portion of its circumference is an angle of one radian. This is a natural $1 : 1$ ratio, and exists whether the radius is measured in Grafuts, feet, meters, or anything else.

Degree to unciaPi Correspondences											
$0^{\text{v}}1$	$0^{\text{v}}2$	$0^{\text{v}}3$	$0^{\text{v}}4$	$0^{\text{v}}5$	$0^{\text{v}}6$	$0^{\text{v}}7$	$0^{\text{v}}8$	$0^{\text{v}}9$	0^{v}↊	0^{v}↋	$1^{\text{v}}0$
15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°

Table 12: Degree to unciaPi correspondences.

When the circle has traveled over the whole of its circumference, it has traveled 2π Grafuts, and therefore turned through 2π radians; when it has turned through half of its circumference, it has traveled π Grafuts and therefore turned through π radians. This means that an angle representing a whole circle, conventionally represented by ∡360°, can instead be represented as 2π radians; a half-circle, normally represented by ∡180°, can instead by represented as π radians; and so on. Typically these angles are easily recognized as radians by the presence of π as a term, and consequently the word "radians" is often omitted when speaking about them. It is an admirable system well-based in the nature of the circle.

But TGM has benefits to offer here, as well. While in decimal angles like one-third of a circle (∡120°) require strange locutions such as ${}^{2\pi}/_{3}$, TGM is able to leverage the power of the dozenal system to make these portions of the circle easier to deal with. Refer back for a moment to the chapter on time[10]; what is the clock but a dozenal division of the circle? This provides a very easy and powerful way to deal with radians that the decimal system simply cannot match.

The clock, of course, divides only half of the day into twelves; so let us imagine the whole day charted on a single large circle, with two dozenal divisions (that is, 20 [twenty-four in decimal parlance] parts). The top half of the circle is 0;0–1;0, the bottom half 1;0–2;0. Remember that half of a circle in radians is π, and a full circle is 2π. Let us, then, say that the 1;0 on the circle is in fact $1;0\pi$, and the 2;0 is $2;0\pi$. A quarter of a circle (∡90°) is then simply $0;6\pi$ rather than ${}^{\pi}/_{2}$; a third of a circle is $0;8\pi$ rather than ${}^{2\pi}/_{3}$. The angle, in fact, is simply equal to its dozenal division of the semicircle multiplied by π, making angular calculations much simpler.

Indeed, this is so simple and powerful that TGM applies a new name to such angles; namely, *unciaPis*. This is simply a new unit, the Pi, which is equal to π radians, divided by twelve and labeled with *uncia* in the normal, regular way. The TGM protractor, then, is a marvel of regularity and ease, particularly when compared to the old one. It is pictured in Figure 4 on page 39.

Looking at the protractor, remember that each of the numbers on the outer rim is in fact simply the number of unciaPis. Using the simple "$^{\text{v}}$" symbol, nothing more than a superscripted lowercase "v," to replace the normal uncial point, we have a convenient notation for a powerful concept. So, for example, ∡120° is equal to $0^{\text{v}}8$; the "Pis" is understood because of the angle symbol replacing the uncial point. "$0^{\text{v}}8$" can also be read "8 unciaPis," or simply "0;8 Pis." The superscripted "v" indicates an angle measured in Pis.

This system has been applied to the most common angles for practical work in Table 12 on page 38; the superior simplicity of the system stands out immediately.

[10] *See supra*, Chapter 3, at 14.

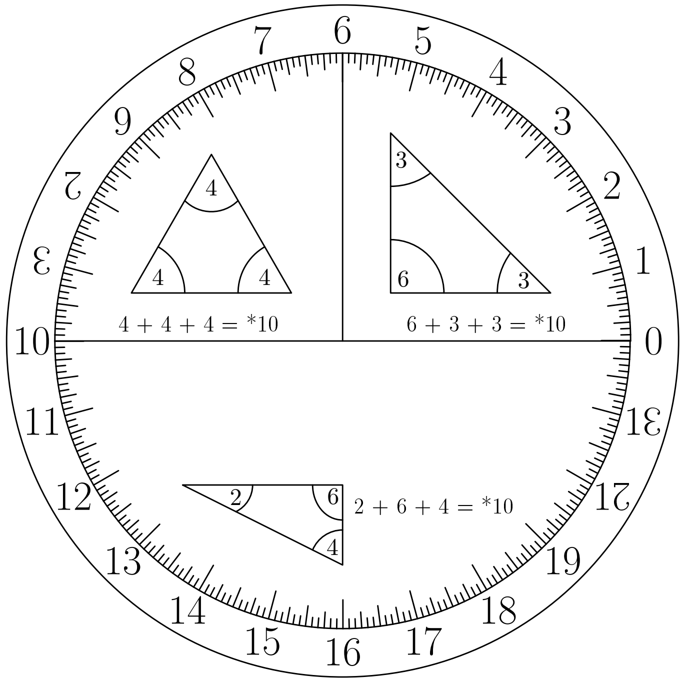

Figure 4: The TGM Protractor, labelled in unciaPis.

UnciaPis can, of course, be further divided like any other TGM unit. $0^{\text{v}}04$ (four biciaPis, or 0;4 unciaPis, or 0;04 Pis) is equal to 5°, for example, and $0^{\text{v}}08$ is therefore equal to $\not\partial$10°. The TGM protractor, therefore, provides for all the major divisions of a traditional protractor precisely and accurately, in addition to providing a more convenient and powerful notation.

A few examples demonstrating the power of this notation should not be unwelcome.

1. Opposite angles measured in unciaPis differ simply by 1;0. So, for example, the angle opposite $0^{\text{v}}3$ is $1^{\text{v}}3$; the angle opposite $1^{\text{v}}8$ is $0^{\text{v}}8$. There is no need for cumbersome calculations of adding or subtracting $\not\partial$180°.
2. The sum of any triangle's angles will no longer be the strange $\not\partial$180°, but instead a simple $1^{\text{v}}0$; that is, a single Pi.
3. The *supplementary angle* (that is, the angle which, added to the angle in question, will equal a full semicircle, or $\not\partial$180°) is simply the angle in question subtracted from $1^{\text{v}}0$. So, for example, the supplementary angle of $0^{\text{v}}4$ is equal to $1^{\text{v}}0 - 0^{\text{v}}4$, or $0^{\text{v}}8$.
   As such, dozenal complements are simply fractions which, when summed, equal $1^{\text{v}}0$. Trigonometry thereby becomes much simpler. Trigonometric functions of supplementary angles are equal; so $0^{\text{v}}1$ and 0^{v}↋ (and, for that matter, $1^{\text{v}}1$ and 1^{v}↋) will have the same numerical value for their trigonometric functions (such as sine and cosine).
4. The system combines the best features of degrees (no need to be tossing around factors of π all the time) and radians (no need to deal with the cumbersome degree measurements like 90° and 180°). So while the mathematician is still dealing with normal numbers, such as $0^{\text{v}}4$, he is also dealing with 4 unciaradians. Put another way, the number of Pis multiplied by π is equal to the number of radians.

The simplest example of this is the angle of the semicircle; that is, đ180°. This is 1 Pi, or 10 unciaPis. As such, we are dealing with π radians. The full circle, of course, is 2 Pis, or 20 unciaPis, and is equal to 2π radians. The quarter circle is 0;6 Pis, or 6 unciaPis, and is equal to $0;6\pi$ radians, or 6 unciaRadians. It's that simple; the easy-to-use degree measurements are equal to the radian measurements without the factor of π.

5. The unciaPis of longitude match up with the hours of the solar day, and so with the basic Standard Time zones around the world. In astronomy, the unciaPis match up with sidereal hours of right ascension (something astronomers will understand and find extremely useful). In traditional units, by contrast, one hour, one minute, and one second of right ascension equals đ15 degrees, đ15 minutes, and đ15 seconds of angle.

TGM totally eliminates the necessity of adding, subtracting, multiplying, and dividing by cumbersome numbers like đ180 and đ360, and even avoids the necessity of using π all the time, while still maintaining easy compatibility with the radian system. It is the best of both worlds.

Exercises

1. Write the following angles in Pi notation; e.g., $15° = {}^{\mathrm{v}}1$, $5° = {}^{\mathrm{v}}04$, $240° = 1^{\mathrm{v}}4$:
   45°, 15°, 10°, 5°, 20°, 25°, 65°, 75°, 80°, 22.5°, 2.50°, 7°30', 1°15', 120°, 190°, 270°, 300°, 325°
2. Write down the complements, supplements, opposites, and negatives of:
   (a) ${}^{\mathrm{v}}3$
   (b) ${}^{\mathrm{v}}5$
   (c) ${}^{\mathrm{v}}4$
   (d) ${}^{\mathrm{v}}16$
   (e) ${}^{\mathrm{v}}04$
   (f) ${}^{\mathrm{v}}18$
   (g) ${}^{\mathrm{v}}6$
   (h) ${}^{\mathrm{v}}8$
   (i) $1^{\mathrm{v}}4$
   Note that two angles are complementary if their sum is a right angle (that is, a multiple of 6; e.g., ${}^{\mathrm{v}}2 + {}^{\mathrm{v}}4 = {}^{\mathrm{v}}6$. Two angles are supplementary if their sum is equal to a semicircle; e.g., ${}^{\mathrm{v}}2 + {}^{\mathrm{v}}↊ = 1^{\mathrm{v}}0$. Two angles are opposite if their difference is equal to a semicircle; e.g., $1^{\mathrm{v}}2 - {}^{\mathrm{v}}↊ = 1^{\mathrm{v}}0$. Two angles are negatives if their sum is equal to a full circle; e.g., $1^{\mathrm{v}}↊ = -{}^{\mathrm{v}}2$.
3. Find the third angle in the following triangles:
   (a) ${}^{\mathrm{v}}3$, ${}^{\mathrm{v}}3$
   (b) ${}^{\mathrm{v}}5$, ${}^{\mathrm{v}}4$
   (c) ${}^{\mathrm{v}}4$, ${}^{\mathrm{v}}4$
   (d) ${}^{\mathrm{v}}1$, ${}^{\mathrm{v}}7$
   (e) ${}^{\mathrm{v}}16$, ${}^{\mathrm{v}}24$
   (f) ${}^{\mathrm{v}}8$, ${}^{\mathrm{v}}24$
4. The sum of the angles of any polygon is always $(n-2)\pi$, $1^{\mathrm{v}}0$ for triangles, $2^{\mathrm{v}}0$ for quadrangles, $3^{\mathrm{v}}0$ for pentagons, $4^{\mathrm{v}}0$ for hexagons, and so on. What is the sum for:

Rotational and Radiational Unit Prefixes			
	Radius	Radius^2	Radius^3
Multiplied by	Rada	Quara	Cubra
Divided by	Radi	Quari	Cubri

Table 13: Rotational and Radiational Unit Prefixes

(a) An octagon;
(b) A heptagon;
(c) A rectangle;
(d) A parallelogram;
(e) A gable end wall?

What is the angle of a regular:
(a) Octagon;
(b) Hexagon;
(c) Heptagon;
(d) Nonagon?
(e) Do these again in decimal, giving answers in degrees, minutes, and seconds.

7.2 Rotation and Radiation

Units concerned with rotation and radiation often involves multiplying or dividing by the radius, and sometimes also by the square or cube of the radius. TGM therefore offers some easy ways to referring to such multiples or quotients in the form of prefixes, which can be seen displayed in Table 13 on page 3Ɛ.

In abbreviations simply use the initial letters; these prefixes can be further abbreviated in the same way as the numerical prefixes, by superscripting the multiples and subscripting the quotients, or alternately by capitalizing multiples and lowercasing quotients. Pendlebury frequently called this putting them "upstairs/downstairs," a particularly apt comparison. For example, to refer to a quaraMaz, write either $^{\mathrm{Q}}$Mz or QMz; for a quariSurf, write $_{\mathrm{Q}}$Sf or qSf.

These prefixes can also be added, subtracted, or cancelled out in the normal way, as well. For example, any one divided by itself cancels out to simply 1; so $^{\mathrm{Q}}$Mz divided by $^{\mathrm{Q}}$Mz is simply 1 Maz. R×R and R/r = Q, while r×r or r/R = q, and q/R = c, and so on.

This also gives some very convenient abbreviations for different concepts. For example, moment of intertia, which in SI measurements requires the cumbersome kg·m^2, is simply QMz, and angular acceleration, in SI rad/s^2, is simply rG. To find torque, the product of moment of inertia and angular acceleration, simply find the product of Q×r (R) and the product of Maz×Gee (Mag), making RMg. The *radaMag*, therefore, is the TGM unit of *torque*, or angular force. In the traditional systems this is measured in "pound-feet" or "newton-meters" to distinguish it from energy, measured in "foot-pounds" or "meter-newtons" (the latter of which, of course, is simply joules). The advantage of the radaMag over this cumbersome system should be evident.

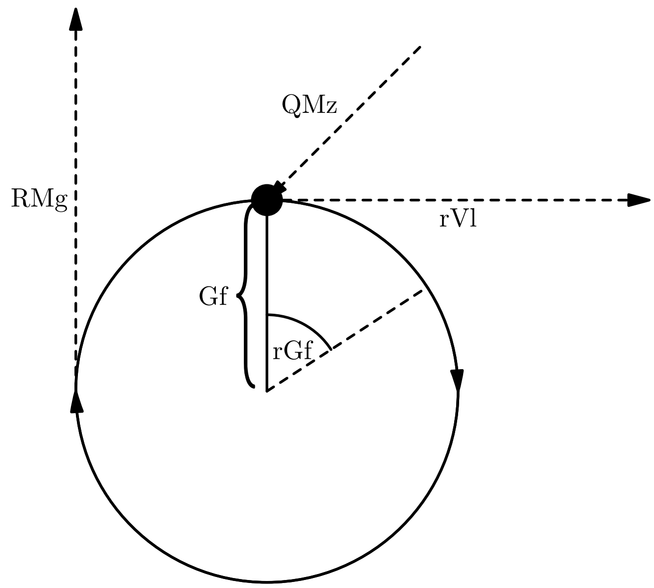

Figure 5: A graphical demonstration of some radial and rotational units.

Figure 5 on page 40 demonstrates a few of these concepts. Depicted is the standard situation of a ball tied to a rope being swung around a central point. The rope is the solid line connecting the rim of the circular path to the center and the ball is the black disc. The rope is, as depicted, one Grafut long; this is not necessary, but it makes the demonstration simpler. The *mass* of the ball is one Maz (a *large*, or at least massive, ball); this mass gives it a moment of inertia of one Maz multiplied by the square of the radius, or one quaraMaz (QMz). Its *displacement*, or change of position, is along the rim of the circle; its displacement is therefore one Grafut along the edge of a circle with a radius of one Grafut; that's a Grafut divided by the radius, or radiGrafut (rGf). Its *velocity* is constantly changing direction, of course; right now it is at the top of the circle, so its velocity is directly to the right. Since its velocity is *angular*, rather than a straight line, it is equal to one Grafut *along the circumference* per Tim, or one Grafut per Tim divided by the radius, giving us radiVlos (rVl). The *force* which gives it this velocity is angular force, or *torque*; this is the force multiplied by the radius, or the radaMag (RMg).

Of course, there are many other possible units given rise to by these prefixes; but this should suffice for graphical demonstration.

Using these prefixes, we find some interesting correspondences. The *radiGrafut*, or radifut (rGf), turns out to be another name for the radian, since it means literally one Grafut along the circumference of a circle divided by the one Grafut of the radius. It's easier to use the symbol "rGf" than the traditional "Rn" or "rad" for radians because the prefix for "radi" makes for easy multiplying and dividing.

Solid angle, essentially a three-dimensional angle such as a pyramid or cone shape, or an angle extending from the center of a sphere, is traditionally measured in *steradians* (Sr). In TGM, this is simply the *quariSurf* (qSf).

Angular velocity is measured in *radiVlos*; because the radiGrafut is a radian, the radiVlos comes to one radian per Tim.

We have already seen the units for *moment of inertia*, the inertia which tends to preserve rotational energy, which is proportional to the square of the distance from the center of the rotation. This is the *quaraMaz* (QMz), the mass of the object multiplied by the square of the radius (that is, the square of the distance to the center of rotation).

The *quaraPov* (QPv) is the unit of *radiant power*; that is, power eminating uniformly outward in all directions. Light from a burning bulb is the most common example, but it is also relevant to stars and other radiant energy sources. This type of energy decreases in proportion to the square of the distance from its source; that is, to the square of the radius of the sphere. The square of the radius is the *quara*.

The *quaraPenz* (QPz) is the unit of *radiant power density* or *radiant intensity*. It relates to power density the same way that the quaraPov relates to the Pov.

Exercises

5. The radius of a lorry's roadwheels is 2 Gf (2 feet).
   (a) Through what angle (in radians) do the wheels turn for every 10 Gf (twelve feet) that the lorry travels?
   (b) If it is going at 7;Ɛ Vlos (d̸30 mi/hr), what is the angular velocity of the wheels in radiVlos (radians per second)?
   (c) Since 2π equals 6;35 (6.28), what is it in revolutions per Tim (per second)?
   (d) How many revolutions per biciaHour (minute)?
6. A torque of 7 radaMag (d̸400 lb-ft) is applied to a flywheel having a moment of inertia of 6 quaraMaz (d̸340/32.2lb-ft^2).
   (a) What will be the angular acceleration in radiGee (radians per second per second)?
   (b) How much work will have been done by the end of the second revolution? (Angular acceleration is equal to torque divided by moment of inertia; work is equal to torque times radians.)
7. The earth is 8;2 decquaGrafuts ($1.5\times10^1$1 meters) from the Sun.
   (a) What is the square of this distance? (Square the number, double the prefix.)
   (b) The intensity of the Sun's radiation: 16;ƐƐ ^{18}QPz (unoctquaquaraPenz) (3.13×10^{25} W/Sr). What is the power density in Penz (watts/m^2) at the Earth's orbit?

7.3 Reciprocal Units

Frequencies are usually expressed against time, as so many per Tim (or per second, minute, hour, and so on). Sometimes, however, when speaking about light and other electromagnetic radiation, it is measured in inverse wavelengths, as so many per meter. This is the measurement used for lenses. The convergence, or strength, of a lens is measured in *dioptres*; two dioptres means a focal length of half a meter, while three means of a third of a meter, and so on.

Other measurements are also expressed inversely in this way. So, for example, the fineness of a grating depends on how many perforations are present per unit of area; in TGM terms, how many per Surf. The compactness of a solid is measured in how many molecules are present per Volm. In TGM, the prefix "Per" expresses this inverse relationship when attached to a unit. The TGM equivalent to dioptres, for example, is the PerGrafut or Perfut, which is equivalent to 1/Gf.

Perfut (PGf) = 1/Gf = 3.382 dioptres, or per meter

Another example is Rydberg's constant (mathematically depicted as R_∞), a number commonly employed in spectroscopy, equal to 1;1058↋4↊↋ hexquaPerfut (^{6}PGf).

This prefix should not be abused; it is not used for simply replacing the division marker ("/"). Rather, its use should be limited to a plain integer numerator, most typically simply 1, to preserve its meaning as marking an inverse unit.

7.4 Perspective and Angular Size

Perspective should be familiar to any artist, but science demands that it not only be familiar, but also measured with the same precision as any other physical phenomenon. Perspective measures the differences in appearances which occur based on changes in position. So, for example, when we see an object retreating from us or coming towards us, the object *appears* to shrink or to grow, although in reality it remains the same size. When it is twice as far away as originally, it looks half as big; when it is half as far away, it looks twice as big. The area, however, appears only a quarter as large when the object is twice as far away, because both the height and width of the object has reduced by half.

Simply put, lengths vary in proportion with distance; area varies in proportion with the square of the distance. TGM handles this elegantly, as well.

One Grafut viewed at a distance of one biquaGrafut spans an angle of one biciaRadian; at one triquaGrafut it spans an angle of one triciaRadian; at one quadquaGrafut, one quadciaRadian; and so on. This has immediate and easy practical applications. For example, Earth's moon, Luna, is four hexquaGrafut (4 ^{6}Gf) wide and three octquaGrafut (3 ^{8}Gf) distant, so it spans one and a third biciaradiGrafut (1;4 $_2$rGf) (that is, it has an angular diameter of that number) when looking at it from the surface of the earth. This is easily calculated by dividing the numbers ($4/3 = 1;4$) and then subtracting the prefixes ($6 - 8 = -2$).

In comparison, the same calculation in one of our old systems of measure, SI metric, involves d̸3500 kilometers wide, 380,000 kilometers distant, and 31′40″ angular diameter.

As another example, the sun *appears* to have the same diameter as Luna when viewed from Earth; that is, its angular diameter is 1;4 biciaRadian. However, this is because it is much more distant and much bigger. Its actual diameter is about ↊;↋ ^{8}Gf; we can calculate its distance from Earth by utilizing this simple TGM correspondence:

$$^{8}\text{↊};\text{↋}/_{2}1;4 = 8;2\ ^{\text{↊}}\text{Gf (decquaGrafut)}$$[11]

We're using simplified figures here—we cannot expect nature to provide all its measurements in integer units—but this does yield the approximate distance of Earth to the Sun, and it's been found convenient to use this as a unit for interplanetary distances. Astronomers call it an "astronomical unit," abbreviated "au," and it's a long, unique number in all its precise glory. In TGM it is called the *Astru*:

Astru (Au) = ≈ 8;2 $^{↊}$Gf = 820 7↊42 1224;2↋47 Gf

The International Astronomical Union defines the astronomical unit in terms of meters, of course; specifically, that it is equal to exactly d̸149,597,870,700 meters. This figure and the

[11] Read: "Octua ten dit elv divided by bicia one dit four equals eight dit two decquaGrafut."

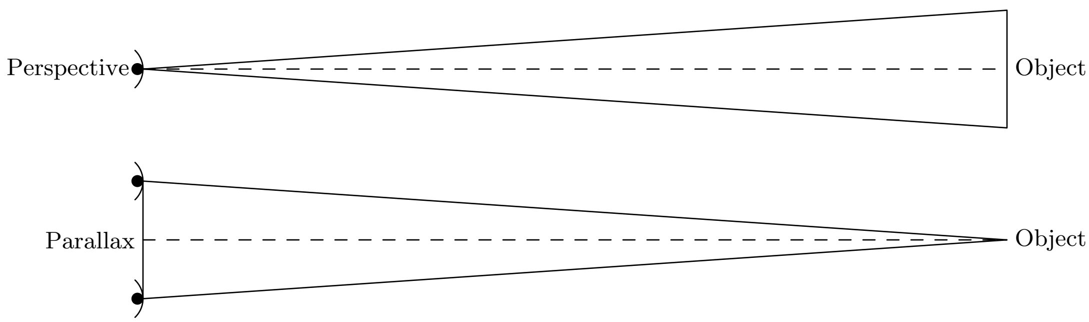

Figure 6: Parallax and perspective compared and contrasted

Grafut figure given above for the Astru are exact, or *very* nearly so; but at these ranges, any inaccuracy is so small that it's equally safe to consider it as equal that figure. With such huge numbers the difference is hardly worth considering.

7.5 Parallax

Parallax is the same as perspective, but considered from the opposite direction. That is, while with perspective we're dealing with the different apparent sizes of distant objects, with parallax we're dealing with the same object looked at from disparate positions close by. It's chiefly useful for determining extremely long distances, distances that makes the Astru look like a trip next door; that is, interstellar and intergalactic distances. Indeed, it was parallax which first demonstrated how far away the stars really are.

The bottom line with parallax is that we look at a distant object and note its position; we then look at it from another position relatively close by; we then note the apparent difference in position from these two different vantage points and use that to calculate the distance. Because parallax is measured along what we call the "celestial sphere"—that is, along the giant imaginary circle that surrounds our planet—we measure parallax in angles; so in TGM, we measure it in Pi and the radian.

It sounds complicated, but we do it literally every waking moment without even thinking about it. Our eyes are in two different places, and we only gain depth perception by mentally gauging the distance of objects based on their different apparent position in each eye. It speaks well for the power of our brains that we do this and put together a single picture from two (one from each eye) without ever even realizing it.

Interstellar distances are so vast that they require a pretty significant distance between vantage points to measure any parallax at all. So we acquire the greatest difference we can: the different positions of the earth on either side of the sun, six months apart. These positions are, of course, two Astru apart, as well; this means that the difference of angular position divided by two is the parallax relative to one Astru.

This principle gives us some useful information; e.g., a parallax of one hexciaRadian means a distance of one hexquaAstru; and a parallax of one septciaRadian means a parallax of one septquaAstru. In other words, there are three simple steps to determining distance in Astru from parallax:

1. Take the parallax in radians and divide by two; for example, 14 $_6$Radian divided by two equals 8 $_6$Radian.

2. Take the reciprocal of the resulting number; that is, divide it from one. For example, $1/8 = 0;16$.
3. Make the exponential prefix positive rather than negative. So, after step 2, we had ${}_{6}0;16$. Make it positive: ${}^{6}0;16$. Moving our uncial point, we have 1;6 ^{5}Au.

Naturally, the smaller the parallax, the greater the distance to the object.

Currently, we measure parallax in *seconds*; that is, in $\not{d}\frac{1}{120}$ of a degree. An object which shows one second of parallax is said to be one *parsec* (*par*allax *sec*ond) distant. A parsec is about 6;9344 ^{13}Gf (untriquaGrafut), or 3.0856×10^{16} meters; that's equal to about 9Ɛ449 Astru. It is, naturally, an enormous distance.

But in TGM there is no need for seconds or parsecs; we simply divide by two; take the reciprocal; and raise the prefix of the parallax from negative to positive, and we have the distance in Astru.

Exercises

1. A car is 6 Gf wide and 5 Gf high (6 ft, 5 ft). When it is at a distance of 2 ^{2}Gf (93 yards), what is:
   (a) its angular width and height in Pi (radians); and
   (b) its angular area in qSf (steradians)?
2. The nearest star, Proxima Centauri, has a parallax of Ɛ;00 ${}_{6}$Pi. What is its distance in Astru?

Chapter 8: Electromagnetism

Fundamental Reality: Permeability of Free Space

Electricity and magnetism are often and easily misunderstood, so a little explanation is probably in order before forging ahead. The word "electromagnetism" is often used to describe these two phenomenon together, for while no one ever got burned by magnetism, nor will magnetism all by itself run the computer this book is being prepared on, the two are really just different aspects of the same thing.

8.1 A Brief Explanation of Electromagnetism

All matter, as we know, is made up of tiny particles called *atoms*. These atoms sometimes join together to form *molecules*, but sometimes they remain by themselves and form what we call *elements*, such as hydrogen, iron, and uranium. These atoms are themselves made up of even tinier particles, which come in a variety of different sizes and types. The quantum physicists have a huge variety of great arguments about these types, but at least three of them are agreed upon by everybody, quantum physicist or not: protons, neutrons, and electrons.

Atoms themselves are like tiny solar systems (many physicists would disagree, but it's still helpful to think of them that way), with a large mass in the center and lots of little things spinning around it. The mass in the center is made up of *protons*, particles which have a positive electrical charge, and (usually) *neutrons*, particles which have no electrical charge. This mass we call the *nucleus*. The "planets" which circle around the nucleus are called *electrons*; they are considerably smaller than either protons or neutrons and they carry a negative electrical charge. And this is where the wonder of electromagnetism is made.

The term "electrical charge" tends to conjure up images of static electricity bolts and television hums, and these are, of course, examples of electromagnetism. However, all electrical charge is, at its root, an attraction or repulsion between two types of thing. Positive and negative charges attract one another; negative and negative, or positive and positive, charges repel one another. Magnetism, similarly, is just an attraction and repulsion; north and south poles attract, while north and north or south and south repel.

The source of electrical attraction are these protons and electrons. An atom has a certain number of protons, different depending on its type, and this gives it a certain positive electrical charge. It wants to have an equal number of electrons circling its nucleus in order to have a neutral charge. This is desirable because it has just enough positive charge to attract a number of electrons equal to its number of protons, but not enough to attract more and too much to attract less.

However, all electrons are not created equal; that is, not all orbits around a nucleus are created equal. Electrons orbit the nucleus in little shells, and each shell can only hold a certain number of electrons. So when an atom has the correct number of electrons—that is, that number is the same as its number of protons—it still isn't necessarily stable. It's only really happy when it not only has the right *number* of electrons, but when those electrons also completely fill its outer shell.

When an atom is not in this situation—which is most of the time—its outer-shell electrons tend to go wandering to other nearby atoms, where they might fill up the outer shell. An atom with more electrons than protons, due to this tendency, is called a *negative ion*; an atom whose electrons have gone walkabout in this way is called a *positive ion*. They are called this, naturally, because they have negative or positive charges.

Moreover, these moving electrons are called *electricity*; and that is what electricity is: simply the movement of these tiny electrons from one place to another. Specifically, electrons (and thus electricity) flow from areas of greater negative charge to areas of greater positive charge. This sounds like a law of nature, but it's really just another way of saying what we discussed earlier: negative and positive attract one another. Negatively charged electrons are attracted by positively-charged areas; they travel from places where there are more to places where there are less electrons.

Certain atoms are more given to this sort of thing than others. Those who have just an electron or two extra on their outer shell, and thus have a great tendency to lose those electrons to other atoms, are called *conductors*. When they lose these extra electrons, they become positive ions, and thus are great attractors for moving electrons. The best conductors are the *metals*, things like iron, gold, and silver. The shininess that characterizes metals in their pure forms is, in fact, a swarm of electrons; we call this *metallic luster*.

Elements which tend to attract the wandering electrons let loose from positive ions are called *insulators*. These elements are typically only an electron or two short of having a full outer shell, and thus easily grab an electron or two and become negative ions. Since electrons, being negatively charged, are repelled by other negative charges, these insulators resist the flow of electricity.

Elements with a half-full shell are called *semi-conductors*, and they tend to let their

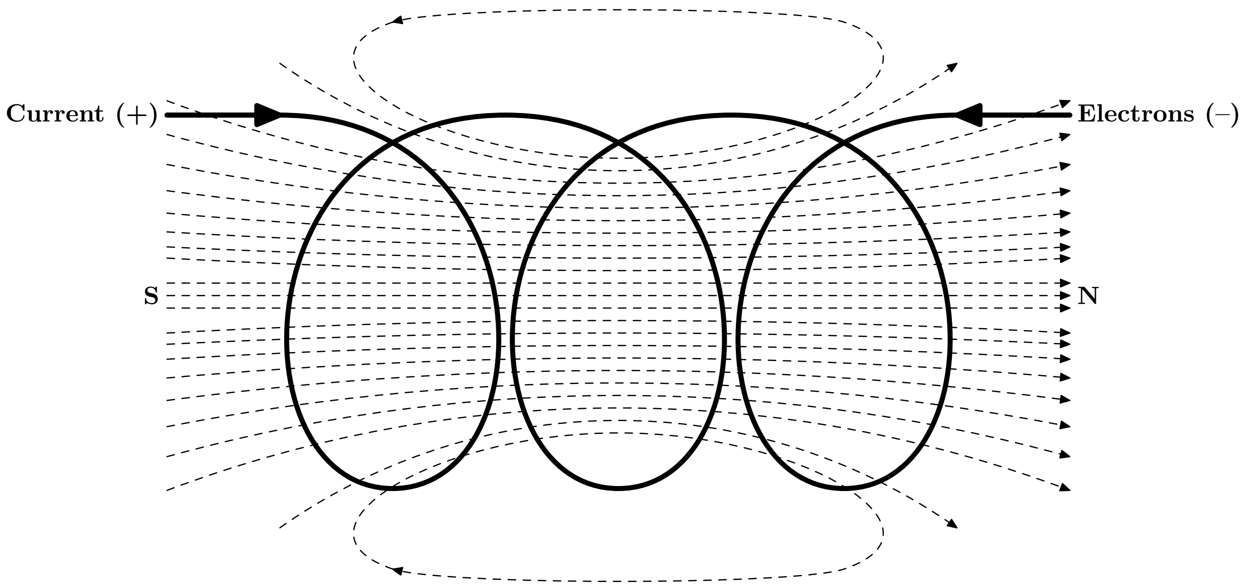

Figure 7: A simplified diagram of electromagnetism in a coiled wire; solid line is wire, dashed the field lines of the magnetic field.

electrons go or grab onto new ones equally well. Elements like carbon, silicon, and germanium are included in this group, and they are very important for applications like transistors.

Areas with a net positive charge are said to have *positive potential*; areas with a net negative charge have a *negative potential*.

When electricity flows, it doesn't just move electrons; it effects the space all around the flow. In the same way that Luna can feel the gravity of Earth over such a vast distance of empty space, the attractive or repulsive force of electricity is felt over empty space (though not nearly so much of it). The force of electricity is carried along invisible lines, which we call *lines of electric flux*. These lines also cause a twisting of space, giving rise to another force which we call *magnetism*; and this force is called *magnetic flux*.

Magnetic flux often cancels itself out as it interferes with itself, and it is also often extremely weak, such that it wouldn't normally be noticed. However, if the electrical current is sent in a spiral motion (say, by wrapping the wires that it's flowing through into a coil), the magnetic flux reinforces itself and creates an *electromagnet*. The magnetic forces in such a case will typically be very evident, and the flux will organize itself into *magnetic poles*, one north and one south. The more electricity and the more turns, the greater the flux. If you send the electricity through the wire in the opposite direction, the magnetic poles will reverse.

Just as some elements conduct electricity, some conduct magnetism, though fewer and less dramatically. Strictly speaking, since electricity is flowing in every atom (meaning that every atom has moving electrons), every atom is a tiny electromagnet. But normally the current is spinning in different ways in different atoms, and is facing in different directions, so this force is cancelled out. However, in these magnetic conductors, the atoms form themselves such that they reinforce one another's magnetic flux, and thus become large magnets. (In most substances the matter will disorganize itself again after a time, losing this net magnetism.) For this reason, putting a magnetic conductor, such as iron, inside an electromagnet greatly increases its magnetic flux; the magnetic conductance of the iron reinforces the electromagnetism.

Just as electricity gives rise to magnetism, so magnetism gives rise to electricity; that is, just as moving electrons cause magnetism, magnetism will cause electrons to move. When magnetic flux (say, from magnetized iron) affects an electrical conductor (like copper), an electrical flow is caused in the conductor. This is called *induction*.

Electromagnetism is like light; that is, it travels at the speed of light, and it travels great distances. When it does this, it is called *electromagnetic radiation*. Such radiation gives rise to "on the air" radio and television broadcasts, as well as radiant heat and many other effects.

There is one more type of electron movement; this last is not really a type of electricity. Sometimes electrons are not moving from one atom to another; sometimes they break completely free of atoms and go flying off on their own. This is typically the result of radioactive decay, and is called *beta radiation*. Such electrons are still negatively charged, and they are still effected by flux lines; CRTs make their pictures using such free electrons.

8.2 Electromagnetic Measurement

Having described the basic theory of electromagnetism, we can now move on to what parts of it we measure and how we measure them.

To visualize what these units and measurements describe, it can be helpful to picture a wire as a water pipe, and the electricity as the flow of water within it. A certain *amount* of water flows through a pipe at a certain *pressure*; similarly, a certain *current* of electricity flows through a wire at a certain *potential*. Because electricity is not water, we measure potential as a *potential difference*; that is, the difference in electrical potential between two different points.

The unit for current in the SI metric system is the ampere; however, it bears little relationship to other SI metric units. For example, the force between two parallel conductors (e.g., wires) placed one meter apart and carrying a current of one ampere is 2×10^{-7} newtons per meter; needless to say, this is not a particularly helpful unit.

In TGM, the force for two wires carrying one ampere of current placed one Grafut apart is 2×10^{-7} newtons per Grafut of length. That force is approximately 4;0↊ ennciaMag ($_9$Mg). To make that force equal to exactly one ennciaMag, the current must be 0.49572 amperes (or "amps"), and this provides a convenient figure for our unit of current.

$$\textbf{Kur (Kr)} = 0.495722069 \text{ amp} = \text{TGM Unit of Current}$$

This means that six hexciaKur ($_6$Kr) is almost exactly equal to one microamp (0.9960979).

As we will recall from above,[12] the TGM unit of power is the Pov, while the SI unit is the watt. These units not only measure work over time, but also electrical work over time; so dividing the unit of power over the current gives us our unit of electrical potential. In SI metric, this is the volt; in TGM, it is the Pel (from "potential electric," and from the Latin *pellere*, to drive):

[12] *See supra*, Section 6.4, at 34.

Pel (Pv/Kr) (Pl) = 871.260799 V = Unit of Electric Potential

This measurement means that one triciaPel ($_3$Pl) is 0.5042 V, a little more than half a volt. A twelve-volt car battery is only a little less than two biciaPel.

Just as clogs sometimes come into a water pipe, impeding the flow of water, so also resistance in a circuit can impede the flow of electrons. This is called, unsurprisingly, *resistance*, and it is measured in the amount of potential per unit of current. In SI metric, this is volts per ampere, and it is called the "ohm" (symbol Ω); in TGM, it is Pel per Kur, and it is called the Og.

Og (Pl/Kr) = 1757.559033 ohms (Ω) = Unit of Resistance

At first glance the Og seems enormous, and in comparison with the ohm it is; however, it is a more practically-sized unit all the same. Units like kilohms and megaohms are common in practice, because the ohm is really too small.

As water flows over time, it accumulates at the end; so also does electrical charge. With water flow, we multiply the volume of water by the time it has been flowing; with electricity, we multiply the rate of flow (the current, or Kur) by the time it has been flowing (in Tims). So in SI metric, this is amperes times seconds, the coulomb (symbol "C"); in TGM, it is Kur times Tim, the Quel (symbol "Ql").

Quel (Ql) = 0.0860628591 C = Unit of Electrical Quantity

It's worth noting that an unquaQuel is 1.0327543098 coulombs, very close to one. This is a convenient equivalency for conversion purposes.

A Quel has a charge equal to about 2;↊↊45 unquadqua (10^{14}) electrons; stated conversely, an electron has a charge equal to about 4;1691 unpentciaQuel ($_{15}$Ql). In particle physics and some other disciplines, the charge of an electron is treated as a unit; this is typically called the electron-volt (symbol "eV"), and is equal to 1.602176×10^{-19} joules.[13] In TGM, we have the electron-Pel, equal to 4;1691 unpentciaWergs ($_{15}$Wg).

The electron-volt and the electron-Pel are exactly equal in terms of their actual charge; they are merely expressed in different units. The conversion factors between them are the same as those between volts and Pels.

The "official" SI derived unit of electrical quantity may be the coulomb, but other units have also been found to be useful. Battery capacity, for example, is typically measured in "amp-hours," or ampere-hours, rather than the coulomb's ampere-seconds. By the usual, if inconvenient, expedient of our current time-measuring system of multiplying by six tens, we easily find that one amp-hour is equal to d̸3600 amp-seconds, or coulombs. It's easy enough to approximate amp-hours to TGM units; since one ampere is about equal to two Kur, one amp-hour is equal to about two KurHours, which (because an hour is a quadquaTim) can also be expressed as two quadquaQuel (2 ^{4}Ql).

Capacitance is another aspect of electricity which we have not yet discussed. When one takes two conductors, usually metallic plates, and places in between them an insulator, one

[13] It is really a unit of energy, and consequently is cited in joules (in SI metric) and Wergs (in TGM); however, since it is linked to a single volt and a single Pel, converting it is a simple matter of multiplying by one, and makes no difference in the figure.

has a *capacitor*. One then attaches a current to the capacitor; the insulator refuses to permit electrons to flow from one plate to the other, which results in a build-up of electrons on one plate and a dearth of them on the other. The amount of electrical quantity necessary to charge the capacitor to the potential of the current source is its *capacity*; the phenomenon is called *capacitance*.

In SI metric, capacitance is measured in farads (symbol "F"), or coulombs per volt (C/V). In TGM, it is measured in Quel per Pel, which is given its own unit name, the Kap:

$$\textbf{Kap (Kp)} = 98.77967559\ \mu\text{F} = \text{Unit of Capacitance}$$

The Kap is most easily attached to the microfarad because the farad is an enormous unit. In practice, we typically deal with microfarads or even picofarads; the Kap is a more reasonably sized unit.

Of course, like most physical processes, charging a capacitor does not happen entirely at the same rate. Rather, it starts off quite quickly, then slows as one approaches the limits of the capacitor. Also, no matter how good a conductor a substance is, it will have *some* resistance, and this will also effect the rate of charge. Thus, we speak of the *time factor* of the capacitor circuit; this is the theoretical time to charge the capacitor if charging continued at its initial rate. It is determined by multiplying the resistance by the capacitance. In SI metric, multiplying ohms by farads will give seconds; in TGM:

$$\text{Og} \cdot \text{Kap} = \text{Tim}$$

In reality, of course, this isn't the time to full charge, but the time to 77;03% (perbiqua) of the full charge. That number is simply 0;7703 multiplied by one biqua to yield the perbiqua; it is produced by the equation:

$$1 - e^{-1}$$

The "*e*" is the Euler number, the base of the natural logarithm, and it crops up in situations as diverse as capacitance and population growth. It's a repeating fraction that can't be exactly expressed in any number base; in dozenal, it is 2;8752 3606, and so on; in decimal, 2.718 281 83, and so on.

Examples

1. A room is lit by four 18 biciaPov (đ60 watt) lamps, and heated by a 6;8 Pov (3 kilowatt) heater. The mains supply is 340 triciaPel (đ240 volts). What is the current when:
   (a) the four lights only are on;
   (b) the heater only; and
   (c) all on?
   What is the resistence of:
   (d) one lamp;
   (e) the heater?

a.	4 × 0;18 / 0;340 Pv/Pl = 2 Kur	4 × đ60 / đ240 W/V = 1 amp
b.	6;8 / 0;340 Pv/Pl = 20 Kur	đ3000 / đ240 W/V = 12.5 amp
c.	2 + 20 = 22 Kur	1 + 12.5 = 13.5 amp
d.	0;340 / 0;6 Pl/Kr = 0;68 Og	đ240 / 0.25 V/A = đ960 ohm
e.	0;340 / 20 Pl/Kr = 0;018 Og	đ240 / 12.5 V/A = 19.2 ohm

2. A car battery is 2 biciaPel (đ12 volt) and has a capacity of 64 KurHour (đ38 amp-hours). The dipped headlights are 0;106 Pv (37.5 W) each. Two sidelights, two tail-lights, and two number-plate lights are 0;02 Pv (6 W) each. The car is left with the dipped-headlight switch on. How long before the battery becomes flat?
   Total Povage: $2 \times 0;106 + 6 \times 0;02 = 0;31$ Pv.
   Total Wattage: $2 \times 37.5 + 6 \times 6 =$ đ111 W.
   Current: 31 / 2 $_2$Pv/$_2$Pl = 16;6 Kr. đ111 / đ12 W/V = 9.25 amp.
   Time: 64 / 16;6 KrHr/Kr = 4;14 Hour. đ38 / 9.25 A-hr/A = 4.108 hours.
3. A 4 unciaKap capacitor (8 microfarads) is connected in series with a 60 Og resistor (0.5 megaohms) across a 200 triciaPel (đ200 volt) DC supply. Calculate:
   (a) the time constant;
   (b) the initial charging current;
   (c) the time taken for the potential across the capacitor to grow to 180 $_3$Pl (160 V); and
   (d) the potential across the capacitor, and the current, at 20 Tim (4 seconds) after connection to the supply.

Solution

a.	Time Constant T = RC	
	$60 \times 0;4$ OgKp = 20 Tm	$0.5\times10^6 \times 8\times10^{-6}$ ohm-farad = 4 s
b.	Initial current I = V/R	
	0;2/60 Pl/Og = 0;004 Kr	đ200/(0.5×10^6) V/ohm = đ400 μA
c.	$v = V(1 - e^{-t/T})$	
	180 $_3$Pl = 200 $_3$Pl(1 - $e^{-t/20}$)	đ160 V = đ200 V(1 - $e^{-t/4}$)
	$e^{-t/20}$ = (200 - 180) / 200 = 0;2	$e^{-t/4}$ = (đ200 - đ160)/đ200 = 0.2
	ln 0;2 = -1;96 = -t/20	ln 0.2 = -1.61 = -t/4
	t = 37 Tm	t = 6.44 s
d.	$e^{-t/T} = e^{-20/20}$ = 1/e = 0;45	$e^{-4/4}$ = 1/e = 0.368
	v = 0;2 × 0;77 = 132 $_3$Pl	v = đ200 × 0.632 = 126.4 V
	$i = Ie^{-t/T}$ = 0;004 × 0;45 = 158 $_5$Kr	i = đ400 × 0.368 = đ147 μA

8.3 Magnetism

Doubtlessly every child has played with magnets; and we've already seen that magnetism is a result of the organization of electrical charge. To be more specific, magnetic flux is current at right angles. If current is straight, the magnetic flux rotates around it; if the current is rotating, the magnetic flux will be straight. A long coil of rotated electrical current is called a *solenoid*.

In other words, the force that magnets produce (*magneto-motive force*) is determined by the product of the number of turns of a coil and the amount of current flowing through those turns. In TGM, this unit is called the Kurn:

Kurn (Kn) = 1 Kr × 1 turn = 0.496 ampere-turns (At)

In other words, a Kurn is about one half the value of an ampere-turn (the SI metric unit, meaning one ampere of current multiplied by the number of turns of the coil).

The strength of a magnetic field is the magneto-motive force per unit of distance; in other words, Kurn per Grafut.

Magra (Mgr) = 1 Kn/Gf = 1.677 At/m

This property is typically represented by the symbol "H." The unit of distance (Grafut for TGM, meters for SI) refers to length along the flux.

Further properties of magnetism are the magnetic flux and the magnetic flux density. These are measured in Flum and Flenz, respectively. In SI metric, magnetic flux is measured in webers, while flux density is measured in webers per square meter. The symbol for flux density is "B."

Flum (Fm) = Magnetic Flux = 151.26 Wb

Flenz (Fz) = 1 Fm/Sf = 1730.1 Wb/m^2

These units mean that one triciaFlenz (1 $_3$Fz) is equal to 1.001 Wb/m^2, or very close to one, providing an easy means of mentally converting between the two systems.

The density of the magnetic flux (the Flenz) for a given field strength (the Magra) depends upon the *permeability* of the substance. Permeability, represented by the symbol "μ," is equal to the flux density divided by the magnetic field strength.

Meab (Mb) = 1 Fz/Mgr = 1032 Wb/At·m

When we defined the Kur, we noted that it gave a force of only 1 ennciaMag (1 $_9$Mg) between two conductors one Grafut apart and each carrying 1 Kur. This means that the flux density of that current is 1 ennciaFlenz (1 $_9$Fz).

The circle of flux from one of these currents passes through the other; each circle of flux has a circumference of 2π Gf, giving H = $1/2\pi$ Kn/Gf. Applying that equation, we find that the *permeability of free space* is equal to 1 $_9$Fz divided by $1/2\pi$, which equals 2π $_9$Mb.

Permeability varies for different substances, sometimes greatly.

Examples

4. An electromagnet made of mild steel has a mean effective length of 2 Gf (đ60 cm). The poles confront each other across a gap which is 2 biciaGf (4 mm) wide. Through the gap is a conductor carrying a 4 Kur (2 amp) current and engaging with flux for a length of 1;6 unciaGf (3.7 cm). The magnetising coil has 1000 turns (about đ1700).

What is the magnetising current required to produce a force of 9;5 quadciaMag (0.12 newtons) on the conductor?

Force on the conductor is equal to the product of the flux density (B), the length of the flux engagement (L), and the current in the conductor (I). In other words, the equation is F = BLI. We already know the force we want (9;5 $_4$Mg), the length of the engagement (1;6 $_1$Gf), and the current involved (4 Kr). So we rearrange the equation and arrive at B = F/LI.

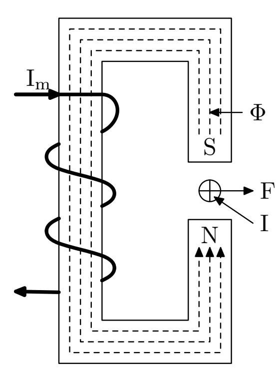

Figure 8: A diagram of an electromagnet for Example 4.

B = ${}_4$9;5 / (0;16 × 4) = 1;6↊ ${}_3$Flenz | B = 0.12N / (0.037 m × 2 A) = 1.62 Wb/m^2

But the permeability of air is 2π ${}_9$Meab ($4\pi\times10^{-7}$ Wb/At·m). So at this point, it's easy to find the magnetizing gradiant for the air gap:

${}_3$1;6↊ / ${}_9$6;35 = 3 ^{5}Magra | 1.62 / ($4\pi\times10^{-7}$) = 1,289,000 At/m

And from there, we can easily find the magneto-motive force for the gap:

^{5}3 × ${}_2$2 = 6000 Kurn | 1,289,000 × 0.004 m = 5157 At

And thence we use the magnetizing gradient for the core of the electromagnet. This is something that varies from substance to substance, and as mentioned above is often found in the form of a chart. For mild steel, the material in question, the value for 1;6↊ ${}_3$Fz is more or less ↋00 Kn/Gf (about đ2600 At/m; for simplicity of calculations, though, we'll use đ3000). Taking these values, we can now derive the magneto-motive force for the core:

↋00 × 2 Gf = 1↊00 Kn | đ3000 × 0.6 m = đ1800 At

And the total magneto-motive force:

6000 + 1↊00 = 7↊00 Kn | đ5157 + đ1800 = đ6957 At

And from there, finally, we calculate the necessary magnetizing current:

7↊00 Kn / 1000 T = 7;↊ Kr | đ6957 At/m / đ1700 T = 4.1 A

The *relative permeability* (μ_r) of a material is simply its *absolute permeability* (μ) divided by the permeability of free space (μ_o). Both are measured in Meabs in TGM; as such, dividing the two removes the unit, and the relative permeability is a plain number, really a simple ratio.

8.4 Inductance

We have already met the unit of electric potential, the Pel, and its SI metric equivalent, the volt. There is another property of electricity, however, which is very similar but not identical, *electromotive force*. This is simply whatever makes electrons and ions flow the way they do. It is also measured in Pel or volts, and it is often produced by inductance.

Inductance is the generation of an electrical current by a magnetic field. Just as flowing electrons generate magnetic fields, magnetic fields tend to get electrons flowing. When we produce an electric current by moving a conductor along a magnetic field, we are said to be *inducing* a current. In more complex arrangements, this is essentially how a generator produces electricity.

The relevant formula is E = BLv; that is, electromotive force equals the product of the flux density, the length of the motion, and velocity of the conductor. When the flux density is 1 Flenz, the length of the conductor actually within the field is 1 Grafut, and the conductor is being moved at a velocity of 1 Vlos, an electromotive force of 1 Pel is induced so long as the movement lasts.

But the Flenz is 1 Flum/Surf, and the Vlos is 1 Grafut per Tim. If we reduce out these units, we find that in the above case the conductor cuts across 1 Flum per Tim. This is called *rate of change of flux*, and the formula is $E = \Delta\Phi/\Delta t$ (change in flux divided by the change in time). A change of 1 Flum, then, generates 1 Pel.

It needn't be the conductor that's moving, however; if one moves the field of flux, the conductor can remain stationary and a current will still be generated. The two must merely be in motion relative to one another; it makes no difference which one is doing the moving relative to anything else.

TGM provides a unit for inductance, of course: the Gen (Gn). In SI metric, inductance is measured in henrys (H), or Volt-seconds per ampere (V·s/A).

Gen (Gn) = Inductance = 305.131777 henry

Examples

5. An iron core has 200 (300) turns wound on it. A change of current from 4 to 5;6 Kur (2 to 2.8 amps) increases the flux from 4 to 4;5 hexciaFlum (d̸200 to d̸220 μWb). What is the inductance?
   To solve in TGM, let change occur during 1 Tim. Then ΔΦ / Δt is 5 septciaFlum per Tim (4;5 - 4 = 0;5 $_6$Fm; this is the same as 5 $_7$Fm). So this change of flux induces 5 $_7$Pl in each turn of the wire. Since there are 200 turns, the total electromotive force ("emf") induced by this change in flux is 200 × 5 $_7$Pl, or ↊00 $_7$Pl, more conveniently expressed as ↊ $_5$Pl. But we've already seen that inductance is the electromotive force divided by the change in current per unit time, which is 1;6 Kr/Tm (5;6 - 4 = 1;6). So ↊ $_5$Pl divided by 1;5 Kr/Tm yields an inductance of 6;8 $_5$Gn.
   To solve in SI metric, let change occur during 1 second. Then ΔΦ is d̸20 μWb/s, including d̸20 μV in each turn of the wire. Total electromotive force is therefore d̸20 × d̸300 = d̸6000 μV. Using the formula from the TGM solution, we divided 6 mV (the equivalent of d̸6000 μV) by the change in current per second (2.8 - 2 = 0.8) and get 7.5 mH.

8.5 Alternating Current

Inductance is a fascinating phenomenon that is behind most of our electrical generation; however, it's important to note that current cannot just keep rising forever. That is, it could, but only if it had a change of flux to move across in the same direction forever. Remember that changing the direction of an electrical current changes the poles of the magnetic field it generates; so also changing the poles of the magnet will change the direction of the current induced; and finally, so also will changing the direction of motion change the direction of the current induced. Because the direction of the current switches back and forth, current behaving in this way is called *alternating current*.

Furthermore, most of our electricity is generated by a turning wheel; this produces a *sinusoidal* pattern of magnitude for the current. In other words, the amount of current which is induced rises and falls along a sine curve (hence the name "sinusoidal"), like a spot on a steadily turning wheel (which in fact it is). In technical terms, an alternating current is always proportional to the sine of the angle turned to at a given moment.

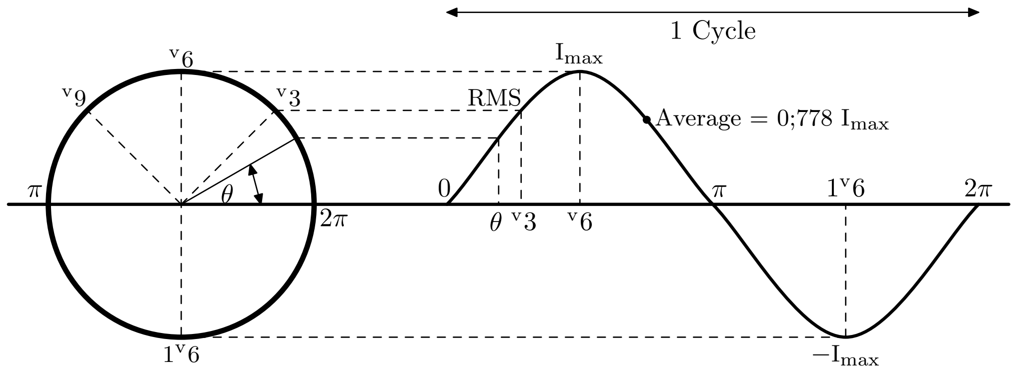

Figure 9: A simplified diagram of alternating current with several significant points identified

Since the current reverses direction every half-cycle (that is, every half-turn of the wheel), the average current for a full cycle is zero. However, the first half comes out at $2/\pi$ multiplied by the maximum current, or 0;778 (0.637) of the maximum current level of the cycle. For the second half of the cycle, the answer is the same, but negative.

Alternating current is different from direct current because we are not dealing with a flow of electrons from one point to another. Rather, we're dealing with a vibration pulsing like a wave down a wire, with the electrons at each point in the wire vibrating more or less depending on the amplitude of the wave. This is not as easy to understand as normal electrons-flowing-like-water direct current, but no matter; simply knowing that it's different is sufficient for non-experts.

In practice, though, a current is measured not so much by how much the electrons are flowing or vibrating, but by how much work it can do. The work doable by a current is proportional to the current's square, and squares of a negative number are positive, which means that our negative maximum current during the second half of the alternating current's cycle doesn't hurt us any. Working through the algebra, this means that the square root of the average of the squares of all the values of the current over time is used as the measurement for the current. This is confusing, of course, for those of us who are not electricians, but it gives us an easy number to work with. The resulting figure is called the *root mean square* ("RMS").

For a sine wave like alternating current, the RMS is equal to half the square root of two, multiplied by the maximum current of the cycle, which is also the sine of $0^{v}3$ and $0^{v}9$; that is, 0;859↊ (0.7071).

The *form factor* of the current is the RMS over the average current; mathematically:

$$\frac{\pi\sqrt{2}}{4} \approx \frac{↊}{9}$$

$$1;13↋3 \approx 1;1400$$

The *peak factor* is the peak current divided by the RMS; mathematically:

$$\frac{2}{\sqrt{2}} = \sqrt{2} = 1;4↋79(1.4142)$$

Examples

6. A transformer has 2600 turns (d̸4600) in its primary winding, and 80 (d̸100) in its secondary winding. An alternating current at 9 cycles per Tim (d̸50 cycles per second) and having a peak value of 90 Kur (d̸50 A) is sent through the primary, giving a maximal flux of 1;6 quadciaFlum (0.0108 Wb).
   (a) What is the average rate of change of flux?
   The change is from 1;6 to -1;6 ${}_4$Fm in half a cycle (0.0108 to -0.0108 Wb).

$$\frac{3\,{}_4\text{Fm}}{8\,{}_2\text{Tm}} = 4;6\,{}_3\text{Fm/Tm}$$

$$\frac{0.0216\,\text{Wb}}{0.01\,\text{s}} = 2.16\,\text{Wb/s}$$

   (b) What is the average electromotive force induced in the secondary winding?
   Each turn receives an electromotive force, so the total is $n\Delta\Phi/\Delta$t.
   80 × 4;6 ${}_3$Fm/Tm = 300 ${}_3$Pl
   d̸100 × 2.16 Wb/s = 216 V
   (c) What is its RMS value? (Form factor = add one nineth)
   300 + 40 = 340 ${}_3$Pl
   d̸216 + d̸24 = d̸240 V
   (d) What is the self-inductance of the primary winding?

$$\text{L} = \frac{-\text{E}}{\Delta\text{I}/\Delta t} = \frac{-n\Delta\Phi/\Delta t}{\Delta\text{I}/\Delta t} = -n\Delta\Phi/\Delta\text{I}$$

   L = -2600 × 1;6 ${}_4$Fm / 90 Kr = 5 ${}_3$Gn
   L = d̸-4600 × 0.0108 Wb / d̸50 A = 0.994 H
   (e) What is the mutual inductance of the secondary in respect to the primary?
   M = -80 × 1;5 ${}_4$Fm / 90 Kr = 1;4 ${}_4$Gn
   M = d̸-100 × 0.0108 Wb / d̸50 A = 0.0216 H

8.6 Electric Force

An alternating current sent into a capacitor looks for all the world like it's passing right through the insulator and out the other side. This isn't what's really happening, however. The electrons are not passing through; as we've seen, in an alternating current electrons aren't really flowing at all, just vibrating with a greater or lesser frequency according to the strength of a wave passing through them. What's really happening is that the current is passing into one plate, then into the other, reversing direction according to its cycle.

However, *electric force* does pass through; that is, the attraction or repulsion brought on by the current. A negative charge on one plate will repel electrons out of the opposing plate, which gives it a positive charge, and the same happens in the opposite direction, giving the other plate a negative charge.

Just as materials have a permeability for magnetic force, so they have a similar property for electric force. This property is called *permittivity*. In SI metric, this is measured in coulombs per square meter over volts per meter; in TGM, the unit is the Mit.

$$\textbf{Mit (Mt)} = \frac{\mathrm{Ql/Sf}}{\mathrm{Pl/Gf}} = 334.073\ \frac{\mathrm{C/m^2}}{\mathrm{V/m}}$$

The Mit itself is a composite unit, the ratio between the unit of electric flux density and the unit of electric field strength. The former, *electric flux density*, is measured in Quenz:

$$\textbf{Quenz (Qz)} = \mathrm{Ql/Sf} = 0.984381\ \mathrm{C/m^2}$$

Remarkably, this relationship is nearly unity; that is, the Quenz is nearly equal to a single coulomb per square meter. The latter, *electric field strength*, is measured in Elgra:

$$\textbf{Elgra (Egr)} = \mathrm{Pl/Gf} = 2946.6\ \mathrm{V/m}$$

This strange-looking unit name is derived from the term "*el*ectric *gra*dient."

8.7 Absolute and Relative Permittivity

Absolute permittivity is the permittivity as described above; its symbol is ϵ and the equation for finding it is $\epsilon = \mathrm{D/E}$, or the electric displacement field divided by the electric field. Free space has an absolute permittivity (for which see Table 19 on page 95), the symbol for which is ϵ_o.

Relative permittivity is the ratio between the absolute permittivity of the material divided by ϵ_o, and it is symbolized by ϵ_r.

$$\epsilon_o = \text{↋;49061497↊ octciaMit}$$

Or, of course, just under one septciaMit. In SI metric, the figure is $8.854187817\times10^{-12}$ C/V·m. It's a fact of nature that the product of the permeability of free space and the permittivity of free space will equal the reciprocal of the square of the speed of light; or, mathematically:

$$\mu_o\epsilon_o = \frac{1}{\mathrm{c}^2}$$

Since we've already seen that μ_o is equal to 2π enncia,[14] and that the speed of light (as seen in Table 19 on 95) is 4↊↋4 9923;07↋↋ Vlos, it's easy to see how we were able to arrive at this figure for ϵ_o.

EXAMPLES

7. The effective area of a capacitor is 3;6 Surf (0.3 m^2), and the dielectric is mica 1;9 triciaGrafut thick (0.3 mm) with a relative permittivity of 6. What is the capacity?

$$\mathrm{C} = \frac{\epsilon_o\epsilon_4\mathrm{area}}{\mathrm{thickness}} = \frac{1_7\mathrm{Mt}\times6\times3;6\mathrm{Sf}}{1;9_3\mathrm{Gf}} \qquad \mathrm{C} = \frac{8.9\times10^{12}\mathrm{C/V\cdot m}\times6\times0.3\mathrm{m}^2}{0.0003\mathrm{m}}$$

$$\mathrm{C} = {}_4 10\ (1\ \mathrm{triciaKap}) \qquad \mathrm{C} = 5.34\times10^{-8} = 0.05\ \mu\mathrm{F}$$

EXERCISES

[14] *See supra*, Section 8.3, at page 4↊.

1. The element in an electric iron has a resistance of 68 $_3$Og (đ80 ohms) and is connected to a 340 $_3$Pl (đ240 V) supply.
   (a) What is the current? (Kur = Pl/Og)
   (b) What power is consumed? (Pv = KurPel)
   (c) How much heat is produced in 1 $_1$Hr (5 min)? (Wg = PvTm)
   (d) How much electrical energy is consumed in one hour?
2. A capacitor has a working area of 0;24 Sf (0.019 m^2). Its dielectric has a relative permittivity of 6, and is 7 $_4$Gf thick (0.1 mm).
   (a) What is its capacity? (C = $\epsilon_o \epsilon_r$area/thickness). Use ϵ_o = 1 $_7$Mt (8.9×10^{-12} SI units.)
   (b) If connected in series with a resistor of 6 Og (đ10 kilohms) across a supply of 16 $_3$Pl (9 V), what is the time constant? (T = RC)
   (c) What is the initial charging current? (I_{max} = E / R)
   (d) What is the current at the instant when the ellapsed equals the time constant? (i = $I_{max}(1 - e^{-t/T})$; i and t are instant current and time, e^{-1} = 0;45 (0.368))
3. A 2 biciaPel (đ12 volt) car battery has a capacity of 64 KurHour (đ38 amp-hour). The car, with battery fully charged, is put away in the garage but with the interior roof light left on. The resistance of the roof lamp is 20 $_3$Og (đ24 ohms). How long before the battery is flat? (Kur = Pel/Og)
4. A wrought iron core has a mean cross-sectional area of $_3$Sf (0.0004 m^2) and an effective length of 9 $_1$Gf (đ22 cm). The air gap between its poles is 1 $_2$Gf (2 mm). If the supply current is to be 0;5 Kr (0.25 A), how many turns must be wound on it to give a flux of 12 $_6$Fm (0.0007 Wb)?
   Method: First find flux density B, same for both core and air gap. Divide this by μ_o and multiply by length of air gap to find Kurns required for air gap. Assume that the magnetic field strength is 23 ^{2}Kn/Gf (đ6500 At/m). (Remember that this number varies by material and flux density.) Multiplied by length of core gives Kurns for core. Divide total Kurns by current, and you're there.
5. A transformer has 1400 (đ1600) turns in the primary winding and 100 (đ100) in the secondary.
   (a) What is the turns ratio? (N_2/N_1)
   (b) If the primary is supplied with an alternating current at 340 $_3$Pl (đ240 V) RMS, what will be the RMS emf in the secondary? (Multiply by turns ratio.)
   (c) What will be the peak values of emfs in primary and secondary? (Peak factor = $\sqrt{2}$ = 1;50 (1.41)).
   (d) If a current of 4 Kur (2 A) is drawn from the secondary, what current will the primary draw? (Multiply by turns ratio.)

Chapter 9: Counting Particles

Fundamental Reality: Mass of an Atom of Carbon-10

VOLUME AND MASS are necessary and useful measures of things; however, for certain items an absolute number of particles is much more useful. Specifically, when counting very small things—such as atoms and molecules—mass and volume are so small that using an absolute number is typically easier.

Determining what number to use, however, can be a bit tricky. Take one example of a very small object, sodium chloride, typically known as common table salt. Sodium is a metal which explodes on contact with water; chlorine is a deadly poisonous gas. Each molecule of table salt contains one atom of sodium and one of chlorine. The *mass* of the molecule, however, is not contributed by each atom equally. By mass, it has 13 parts sodium to 23;6 parts chlorine, because the chlorine atom is considerably more massive than the sodium. Consider also dihydrogen monoxide, chemically written H_2O. This contains two hydrogen atoms for each oxygen atom, with the mass of two hydrogen atoms for every mass of an oxygen atom, in ratio.

This observation led to the use of *gram-atoms*, the number of atoms which makes up the same number of grams as its atomic weight, or *gram-molecules*, the number of molecules that equals the same number of grams as the molecule's weight. As it turns out, all gram-atoms and gram-molecules equal the same absolute number of particles. In SI metric, that number is called the *mole*, formally defined as the number of elementary particles as there are atoms in twelve grams of carbon-10 (₫12).

(Incidentally, this is another example of the illogic of the SI metric system. The number ten is enshrined as central, yet twelve grams are used as the basis for the mole; and the kilogram is the basic unit of mass, yet the mole is based on the gram. What sense is there in this system?)

How many particles is this? This is "Avogadro's number," so named after Amodeo Avogadro, who first proposed a relationship between number of particles and volume, though he did not find the value of the number named for him. Decimally, it is 6.02204×10^{22}, obviously an enormous number; its symbol is N_o, N_A, or L.

In TGM, we must be more rational than this. The TGM unit of mass is the Maz, not the gram (or kilogram), a unit about 23 times the size of the kilogram. The TGM mole is called the Molz (symbol Mlz), and is that amount of substance which contains as many elementary particles as there are atoms in one unqua Maz of carbon-10; this equals about 12Ɛ62;432 moles.

One quadciaMolz ($1\ _4$Mlz) is only a little more than a mole (1;2Ɛ62 mol).

The TGM "Avogadro's Number" is similarly much larger, equalling 1;439Ɛ4 bibiqua (^{22}1;439Ɛ4). In TGM, this is referred to as the Em (abbreviated M):

$$\textbf{Em (M)} = {}^{22}1;439Ɛ4$$

By taking the value of one Maz and dividing it by this figure (in other words, by taking its reciprocal), we can get the TGM version of the *unified atomic mass unit*, abbreviated "u" in SI metric; in TGM, it is called the emiMaz (mMz):

$$\frac{1\text{Mz}}{^{22}1;439Ɛ4\,\text{particles}} = 8;9Ɛ82\ _{23}\text{Mz}$$

The emiMaz and the atomic mass unit are identical, and the "u" symbol does not conflict with any TGM unit, so there's no reason why "u" couldn't be used in TGM, as well. However, using "mMz" makes it clearer which system is being employed, reducing the chances of errors, and it can cancel out to "Mz" when necessary.

Examples

1. A Molz of sodium carbonate means 1 Em of Na_2CO_3 molecules. This consists of 2M atoms of sodium, 1M atoms of carbon, and 3M atoms of oxygen.
   To find the mass, we use mMz to cancel out the M by m:

   $$2\text{M} \times 1\mathcal{E}\text{mMz} + 1\text{M} \times 10\text{mMz} + 3\text{M} \times 14\text{mMz} = 3\text{Ƨ}\text{Mz} + 10\text{Mz} + 40\text{Mz} = 8\text{Ƨ}\text{Mz}$$

   (Of course, there is no need to write such things out in full every time.)
   The sodium atom has Ɛ electrons, carbon has 6, and oxygen has 8. So the Molz of sodium carbonate has 2M × Ɛ + 6M + 3M × 8 = 44M electrons. For each electron there is a proton in a nucleus, so there are 44M protons.

9.1 Particles and Gases

Of course, numbers of particles and their interactions become especially important in gas calculations. These leads to several auxiliary TGM units.

Gases differ from solids, and even from liquids (which are also fluids), in that the atoms of gases have no necessary connection to one another, but are free to float around, and they do. The higher the temperature, the more active the atoms of the gas, which causes an increase in volume if the gas is uncontained, or of pressure on the inside of the container if it is. These considerations lead to the universal gas formula:

$$\text{RT} = \text{pV}$$

T (temperature) must be in absolute units, of course; namely, Calg or kelvin. Degrees Celsius, decigrees, or tregrees simply won't do here, as there is nothing special in nature about the freezing point of water, on which these scales are based. R is the constant ratio between T and pV (the product of pressure and volume).

R is called the *gas constant*, and in TGM units it equals 1;ƐƧ781Ƨ2 PmVm/Cg (8.3144621 J/mol·K). Clearly, this is extremely close to a simple 2; indeed, simply using 2 is less than 1% (per gross) deviation from the true value.

Also available as an auxiliary unit is the *standard gas volume*, in TGM called the Avolz. Scientifically, this is defined as the volume of one Molz of a gas at the freezing point of water and standard atmospheric pressure.

Avolz (Avz) = 10Ɛ41;Ƨ Volm = 578.2844 m^3

Like the gas constant, this comes very close to being equal to a nice, round number; in this case, 11 ^{3}Vm. Indeed, like the gas constant, this is less than 1% away from the true figure. So for practical purposes, 11 ^{3}Vm is a perfectly reasonable approximation of the standard gas volume.

In TGM, a quick and very accurate way of estimating RT from degrees Celsius is the following:

1. Subtract two degrees from the Celsius temperature.
2. Convert the new temperature figure to biquaCalgs.
3. Multiply that temperature by two.

In this way TGM makes even complex calculations of this sort considerably easier.

Examples

2. Find RT for đ20°C.
   đ18°C = đ180 d° = 130 d°. Add 1700 = 1830 ^{2}Cg. × 2 Wg/CgM = 3460 ^{2}Wg/M.
   By the exact method the figure is 3463.
   At ice point, 1 Molz of gas occupies 1 Avolz at 1 Atmoz (2Ɛ Pm). At 0;6 Atz it occupies 2 Avz, at 3 Atz, 0;4 Avz, and so on.
3. 2 Mlz of gas occupy 2 Avz. The pressure is 2Ɛ Pm. Using the formula RT = pV, and assuming R = 2 PmVm/CgMlz and Avz = 1;1 quadquaVolm, calculate the temperature in Calgs.
   2Mlz × RT = 2Ɛ Pm × 2;2 ^{4}Vm. So

$$T = \frac{2\mathcal{E} \times 2;2\text{Pm} \cdot^4 \text{Vm}}{2\text{Mlz}(2\text{PmVm/CgMlz})} = \frac{2\mathcal{E} \times 1;1}{2} = \frac{31;\mathcal{E}^4}{2}\text{Cg} = 16\mathcal{E}6^2\text{Cg}$$

   Compare this with the exact ice point, 16Ɛ7;6 ^{2}Cg. Try it in metric: 2 moles occupy 4.483×10^{-2} m^3. The pressure is 101.325 N/m^2. R = 8.3143 J/K·mol.

9.2 Molvity and Molmity

In chemistry, when working in the SI metric system, one routinely refers to solutions in terms of a number, either whole or fractional, followed by an "M," e.g., "1M" or "0.5M". This "M" indicates that the preceding number (the *coefficient*) is the number of moles dissolved into one cubic decimeter (1 dm^3) of solution. This is yet another example of metric silliness. The basic mass unit of SI is the kilogram, but its basic unit of amount, the mole, is based on the gram, and in actual practice when dealing with moles we use a nomenclature based on the cubic decimeter, when the basic unit of volume is the cubic meter.

In any event, *molarity* describes the number of moles of substance per cubic decimeter, while *molality* describes the number of moles per kilogram.

In TGM, of course, the system is much more sensible. Molarity is called *molvity*, and equals one Molz per Volm, while molality is called *molmity*, and equals one Molz per Maz.

Molv (Mlv) = Mlz/Vm = đ1000 mol/L = 999.972 mol/dm^3

Molm (Mlm) = Mlz/Mz = đ1000 mol/kg

This does away with all the trouble about 1M and 0;4M solutions (though remember that a 0;4M solution would be quite difficult to write precisely in the decimal SI metric system, since it is a third). Solutions are identified simply by their molvity. E.g., a 1M solution is a 2 quadcia solution:

2 quadciaMolv $({}_4\text{Mlv})$ = 0.09644 mol/dm^3

9.3 Electrolysis

Many atoms and molecules become *ions* when in solution. We discussed ions in Section 8.1 on page 45; in short, what this means is that atoms and molecules in solution tend to let their electrons go roaming. They borrow or lend out their electrons from or to nearby atoms and molecules. To accentuate this, it is possible to run an electric current through such a solution in order to draw those particles *out* of the solution; because opposites attract, all negative ions will be attracted to the *anode* while positive ions will be attracted to the *cathode*.[15] This can result in many interesting effects; gold-plating, for example, is often done by electrolysis, as gold in solution will deposit on the cathode in the solution. This is called *electroplating*. Electrolysis can also have some more mundane effects; for example, it can separate oxygen and hydrogen from water.

This process, too, has been reduced to some very precise mathematics susceptible to precise measurement. Electrolysis will draw one atom out of the solution for every electron which passes through the circuit (any chosen point on the circuit will do fine for measurement) divided by the electrical valency of the atom. The *valency* of an atom is how many electrons it has available to form chemical bonds; for most elements (not all) this essentially means the number of electrons in its outer shell. Carbon, then, has a valency of four; hydrogen has a valency of two; oxygen has a valency of two. The formula can be written thus:

$$n_a = \frac{n_e}{v}$$

More precisely, let I equal current, t equal time, and e equal the charge of an electron; the number of electrons necessary to pull an atom from solution is then:

$$n_e = \frac{It}{e}$$

We can also calculate the mass released from the solution with these same variables, if we let a equal the relative atomic mass of the atom:

$$m = \left(\frac{It}{e}\right)\left(\frac{a}{v}\right)$$

The answer here will be in emiMaz, or atomic mass units (eMz, or u). To get the answer in Maz, simply divide this by Em; to get it in grams, divided it by N_o.

Examples

4. A current of Ɛ Kr (5.5 A) flowed through a solution of copper sulfate for 1 hour. How much copper was deposited:
   (a) number of atoms;
   $\frac{It}{e}$: Ɛ Kr × ^{4}1 Tm / $_{15}$4;16 Ql = ^{19}2;8
   5.5 A × 3600 s / 1.6×10^{-19} = 1.24× 10^{23}
   So number of copper atoms = $\frac{n_e}{v}$:
   ^{19}2;8 / 2 = ^{19}**1;4**
   1.24×10^{23} / 2 = **0.62×10^{23}**

[15] The cathode is always the electrode out of which current flows; it is *not* always negative in polarity. But these complications don't matter much for our discussion here.

(b) in Maz;
Multiply by the atomic mass of copper:
$^{1\text{Ɛ}}53;67$ / 9 = $^{1\text{Ɛ}}7;09$ mMz or m_u / $^{22}1;44$ (Em) = **5;23 $_4$Mz**

(c) in grams?
$63.55 \times 0.62{\times}10^{23} = 3.94{\times}10^24$ m_u / $6.02{\times}10^{23}$ (N_o) = **6.54 g**

The atoms mass of copper is 53;67 (63.55), and its valency is two. e = 4;16 $_{15}$Ql = $1.6{\times}10^{-19}$ C.

9.4 Acidity

Electrolysis can also happen in reverse. That is, instead of the electrical current causing a chemical reaction, the chemical reaction can cause an electrical current. We call devices which utilize this effect *batteries*.

A special kind of battery is used to measure the *acidity* of a substance. Acidity is current measured on a sliding scale between 0 and 14, with 7 being a perfectly neutral substance (like water). The two electrodes, one of glass and one of calomel, are placed into the solution to be tested. A voltimeter (Pelimeter?) then measures the resulting potential difference (Pels or volts) between the two electrodes. We currently call the resulting measurement the substance's *pH*.

pH is technically defined in the following way:

$$-\log_{\text{Ɛ}} a_{\mathrm{H}+}$$

where $a_{\mathrm{H}+}$ is the activity of hydrogen ions in the solution (it is this activity which gives rise to acidity), while its converse, alkalinity, can be produced by a number of molecules, especially hydroxide (HO). This definition is extraordinarily strange, giving rise to positive mantissa with negative characteristics. For example, a pH of $2{\times}10^7$ is 6.699, not 7.301 as one would expect.

Still, the system is so thoroughly ingrained in chemistry that there is little point in changing it. However, it should be noted that the mole being based on the gram introduces still another complexity. The liter is the volume of one kilogram of water (almost equal to one cubic decimeter), which in combination with a gram-based mole means that pH readings are hiding a factor of Ɛ^3. Curing this factor requires adding 3 to the pH when finally read, and this is the method TGM takes.

The TGM unit of acidity is, then, $-\log_{\text{Ɛ}} a_{\mathrm{H}+} + 3$, and this is called the *decHyon*. It must be remembered, though, that $a_{\mathrm{H}+}$ is the activity of hydrogen ions in a one Molv solution, not in a 1M SI metric solution.

For those who *are* interested in dozenalizing this system, the *dozHyon* (zH) can be used. This is equal to

$$-\log_{10} a_{\mathrm{H}+}$$

Multiplying by the dozenal log of Ɛ will convert decHyons to dozHyons. So zH equals dH×0;Ɛ153, which equals pH + 3 × 0.9266. If you don't understand this, that's not a problem; but TGM must include logarithmic scales for those who need them. It may seem obtuse, but it's not more so than necessary; logarithmic scales really are complex.

Some common pH numbers in decHyons are water, a pH of 7 and a dH of Ɛ; vinegar, a pH of 4 and a dH of 7; and phenol, a pH of 9.886 and a dH of 10;Ɛ77.

EXAMPLES

5. pH of a solution is 2.15 (dH 5.15 = dH 5;1↊). Find its arithmetic value.

$$↊^{-5;1↊} = {}_{5}1;918$$

That is, 1;918 $_5$Mlv.

EXERCISES

1. What is the mass of one molecule of ethanol, C_2H_6O?
   (a) In unified atomic mass units?
   (b) In emiMaz?
   (c) In Maz?
   (d) In kilograms?
   Atomic weights: H = 1; C = 10; O = 14. Use: 8;↊ $_{23}$Mz and 1.7×10^{-27} kg for a.m.u.
2. What is the mass of
   (a) 2 Molz of ethanol in Maz,
   (b) 2 moles in kg?
3. How many electrons in
   (a) 1 Molz of ethanol in terms of M,
   (b) in full,
   (c) 1 mole?
   H has 1 electron, C 6, and O 8.
4. A quantity of gas occupied 8;8 triquaVolm (1.5×10^{-2} m^3). If the temperature is zero decigrees (0°C) and the pressure is one and a half atmospheres, how many Molz (moles) does the quantity represent? (Metric $V_o = 2.24\times10^{-2}$ m^3).
5. What is the R_ZT in Exercise 4? b) Divide your answer by 2↋ Pm, then by 1;6. What do you notice about these two results?
6. What is the decimal RT in exercise 4? (R = 8.3 J·K^{-1}·mol^{-1}).
7. 4↊ $_4$Mz (đ58 g) of sodium chloride, NaCl, is dissolved in water to make 3 $_1$Vm (5 dm^3) of solution. What is the Molvity (molarity)? (Atomic weights: Na 1↋ (đ23), Cl 2↋ (đ35)).
8. A metal plate of total area 0;35 Sf (đ250 cm^2) is chromium-plated by a current of one unquaKur (6 A) for 1 hour.
   (a) What is the mass deposited (in Maz and grams)?
   (b) What thickness is the deposit?
   (Chromium: atomic weight 44 (đ52), valency 6, density 7;2 Denz (7.2 g/cm^3)). Mass deposited:

$$\frac{It(\frac{a}{v})}{N_o e}$$

   Note: N_oe is the metric unit, the *faraday* (F) = 9.6485×10^4 C. The TGM counterpart is, of course, the *Emelectron* (Me) = 5;7497 ^{9}Ql. (a/v)/Me or (a/v)/F is the *electro-chemical equivalent*. For chromium: 44/(6×^{9}5;7497) = $_9$1;6624 Mz/Ql; in decimal, 52/(6×96485) = 0.0898 mg/C.

9. The hydrogen ion concentration of a solution is 2 hexciaMolv (6.7×10^{-4} mol/dm^3). What is the acidity in:
   (a) dozHyon (zH) (Dozenal common log. with "-" removed),
   (b) decHyon (dh) (zH divided by zlg ↊ (0;↋153)),
   (c) pH (Decimalize dH and subtract 3)?

Chapter ↊: Reckoning by Ratios

The most common ratio we deal with is the *half*; that is, we're constantly dealing with halving things and doubling things. Yet we have more ways of referring to such operations than most people realize. Mostly this is due to long tradition in given fields; but it will still do well for us to examine a few of them, just to see what we mean.

In traditional paper sizes, we start out with a *folio*; halving this gives a *quarto*, and halving a quarto gives an *octavo*. (And what's more, none of these words actually give a certain indication as to size; they merely indicate the number of times the basic, flat sheet has been halved!) Metric paper sizes are hardly better; we begin with an A1 sheet, and halving it makes an A2, and halving it again makes an A3, and so forth. In acoustics, every gain of three decibels indicates a doubling of loudness; that is, three decibels is twice as loud as the quietest sound a human being can hear, while six decibels is twice that, and nine decibels is twice that, so that nine decibels is eight times louder than zero decibels. In photography, there is a similar progression, with film of sensitivity 15 DIN being twice as sensitive as film of sensitivity 12 DIN. These are logarithmic scales, and the "plus 3" pattern derives from the fact that $\log_{↊} 2 = 0.30103$; simply 3 makes a reasonably close estimation.

Music is perhaps the most confusing: we speak constantly of octaves, which comes from the word for eight, yet each octave consists of unqua semitones, and each octave represents a doubling of frequency of the resulting sound. That is, the frequency of the C above Middle C is twice the frequency of Middle C itself, even though it's octave away. The C above that, two octaves from Middle C, is *four* times the frequency. And so on. TGM offers us a better way.

↊.1 Doubles

TGM does away with all this conflicting nomenclature and gets back down to the basics. When we double something, TGM says that we are *doubling* it. This obvious improvement it calls a *Double*. To double twice, we refer to 2 Doubles; another doubling makes 3 Doubles.

No more increasing three decibels; when the loudness doubles, the new reading is a Double of the old one. No more octaves; doubling the frequency is simple a Double of the old one.

Doubling can also be negative. For example, if we go *down* an octave, we've lost a Double. Subtracting a Double is, of course, simply halving, while adding a Double is doubling. These can be combined arithmetically; for example, if we increase a signal by 5 Doubles and then decrease it by 3 Doubles, we have doubled it five times, then halved it three times, making a total of $5 - 3 = 2$ Doubles, or four times the original.

Doubles can also be multiplied and divided, and fractions can be taken from them. Because this scale is logarithmic, not arithmetic, this involves exponents. So a *half-Double* is an increase by $\sqrt{2}$; a *quarter-Double* is an increase by $\sqrt[4]{2}$, a *third-Double* by $\sqrt[3]{2}$, and so on. And at this point, the mathematically astute reader will have noticed that we're simple talking about logarithms to base two. And once we do this, we find that the logic of doubling and halving makes a lot of sense, and gives rise to patterns that are not typically visible in other bases. These base-two logarithms make them visible, and so we'll give them some more extensive treatment now.

Ɛ.2 Dublogs: Logarithms to Base Two

These logarithms in base two, expressed in dozenal notation, are called *dublogs*, and they work in the following way. For the mathematically disinclined, logarithms are really nothing more than another way of expressing exponents, like the familiar 2^2 or 3^3 (two squared or three cubed). The relationship looks like this:

$$(b^x = y) = (\log_b y = x)$$

These two equations are equivalent. The leftmost equation is voiced as, "b to the power of x equals y." The right equation is voiced as "the logarithm of y to base b equals x." They are simply two different ways of expressing the same thing, useful in their own ways and their own fields.

In school, we learn mostly about "base-ten logs"; that is, logarithms to base 10. On decimal calculators, pressing "log" without specifying a base means a base-ten log. There are also *natural* logs, abbreviated ln, which are logs to the base of e, a constant we've seen earlier but which is not important here. On a dozenal calculator, the "log" without a specified base is a log to base unqua (d̸12). But just as a number can be raised by any number as an exponent, so we can have a log to any number as a base.[16]

So when we speak about logarithms to base 2, we're speaking about equations which look like this:

$$(2^x = y) = (\log_2 y = x)$$

As an example, let's do something simple: what is the logarithm of 2 to base 2? When we ask this, we're submitting *2* (the logarithm of 2) as the value of y in the above equations; so we're saying that $\log_2 2$ equals something. We're also saying that $2^x = 2$. To solve for x, one can use a table of base-two logarithms; one could work it out by hand; or one could use a calculator. For the purposes of this example, I will calculate the answer using `dozdc`, a free-software RPN calculator[17], which uses "dlg" as its abbreviaton for "dublog"[18]:

```
2 dlg =
```

[16] Well, the base can be any positive number not 1, and you can't take the log of a negative number, but that's good enough for now.

[17] Available at `http://dozenal.sourceforge.net`.

[18] Of course, simply entering `2 2 logb =` would have done the same thing, as `logb` can take the logarithm in any base.

The answer which `dozdc` gives me is "1," which is, of course, the correct answer; we can check it by plugging the answer into x on either side of the equation:

$$\left(2^1 = 2\right) = (\log_2 2 = 1)$$

That's really all there is to logarithms; they sound like very complicated and involved mathematics, but in reality they are just another way of writing exponents.

So dublogs are a system of logarithms to base 2; one will see many of them in the units appendix tables, as well.[19] They can be written in two ways. One way is the *straight dublog*. Straight dublogs are simply the answer as it emerges from the calculator or the dublog tables, without further manipulation. The vast majority of the time these are sufficient, as well as *always* when the answer will be manipulated by computer or calculator.

For those situations when the fractional part must be positive, straight dublogs can be written in a special notation. (Again, computers and calculators do not understand this; it's solely for human benefit.) Take, for example, -5;2058. Subtract the fractional part from one (1 - 0;2058 = 0;9Ɛ64), then change the whole number (-5) such that adding this fractional part to it will yield the answer. In this case, -6. The negative sign is then put above the whole number: $\overline{6}$;9Ɛ64, which means -6 + 0;9Ɛ64, which equals -5;2058.

In additional to straight dublogs, there are *mixed dublogs*. Once again, these are solely for human benefit and will not be understood by computers and calculators. They were designed to make working with dublog tables easier; given the advent of easy computers and calculators to handle the matter, they are now little used, but are still included in the units table and must therefore be explained.

Mixed dublogs are constructed by counting the powers of unqua rather than taking the actual dublog. As an example, the dublog of 4 is 2. So to write the dublog of 400, one can take the dublog of 4 (which is 2) and then note how many powers of unqua the actual number is off, which in this case is 2. One would then write the mixed dublog so: 2,2;0000, which means that the dublog of the number is 2;0000, but that the number is in fact two powers of unqua higher than that. Similar, the mixed dublog of 0;04 is $\overline{2}$,2;0000.

Mixed dublogs are converted to the actual dublog values by use of another number called an *adjuster*. There is no need to further explain the concept in this work; those interested can consult Mr. Pendlebury's original text.[1↊] For now, one can get the true dublog by determining the source of the part *after* the comma by reversing the dublog (if $\log_2 x = 2$, then $2^2 = x$, so clearly $x = 4$), raise it to the power of unqua specified *before* the comma (here, by 10^2, so $4 \cdot 10^2 = 400$), and then take the dublog of that by use of a computer or calculator ($\log_2 400 = 9; 2057$).

This gives us a brief background in logarithms and specifically in logarithms to base 2. Now, let's see dublogs at work, first in the area of sound.

↊.3 Doubles and Dublogs at Work: Frequency and Sound

[19] *See infra*, Appendix E, at page 96.

[1↊] T. Pendlebury, *TGM: A coherent dozenal metrology based on Tim Gravity & Mass*, available at `http://www.dozenalsociety.org.uk/pdfs/TGMbooklet.pdf`.

Common Dublogs and Associated Ratios

Dub-logs	Note Nos.	Value doz.	Value dec.	Comments	Ratios	Errors p.g.	Errors p.c.
0;0	78	1;0000	1.0000			0	0
0;1	79	1;0869	1.0595	Twelfth root of 2	15/14	-0;50	-0.29
0;2	7↊	1;1577	1.1225	Sixth root of 2	9/8	-0;40	-0.23
0;3	7↋	1;232↋	1.1892	Fourth root of 2	6/5	-1;37	-0.91
0;4	80	1;3152	1.2599	Cube root of 2	5/4	1;18	0.79
0;5	81	1;4027	1.3348		4/3	0;1↋	0.11
0;6	82	1;4↋79	1.4142	Square root of 2	7/5	1;55	1.01
0;7	83	1;5↋91	1.4983		3/2	-0;1↋	-0.11
0;8	84	1;7070	1.5874	Cube root of 4	8/5	-1;18	-0.79
0;9	85	1;8222	1.6818	Fourth root of 8	5/3	1;37	0.90
0;↊	86	1;946↋	1.7818		7/4	2;69	1.78
0;↋	87	1;↊7↊0	1.8877		13/8	0;↋9	0.68
1;0	88	2;0000	2.0000		2/1	0	0

Table 14: Common Dublogs and Associated Ratios

A typical, and extremely useful, example of the utilities of Doubles and dublogs is in the field of frequency, specifically the frequency of sound waves and music. Ever wondered *why* an octave has twelve semitones? This is why.

If we take the twelfth root of 2 in dozenal (1;0869), and then take all of its powers up to zen, we get some very interesting patterns, patterns which don't exist when these same figures are taken in any other base. Taking the logarithm of the resulting figures to base two yields a simple progression of uncias. The entire series is listed in Table 14 on page 63.[1↋] Included in the table is a column giving the corresponding notes on a keyboard; note 78 is the reference note, for reasons which will be become clear shortly.

As can be readily seen from the error columns in Table 14, these ratios are extremely close to accuracy; indeed, they are considerably more accurate than the common rounding of the decimal logarithm of 2 to 0.3.

To explain why these ratios are so amazing and so useful, a common example is needed. The perfect such example is the normal piano keyboard, pictured in Figure ↊ on page 64. Choose an octave on the keyboard; say, the one beginning with Middle C. Press the first note. The sound you hear is the vibration of the air at a given frequency. Now press the next C up, the one that we say is one octave higher than Middle C. The sound that you now hear is *double* the frequency of the first note you played.

Vibration frequencies have to be in a simple ratio to sound good. If you play two notes together which are not in a simple ratio (e.g., 3 : 2 or 2 : 1), you have what is called a *discord*;

[1↋]This is confusing to those who are not mathematically inclined. To simplify matters, think of it this way: $\sqrt[10]{2} = 2^{1/10}$. We then take $(\sqrt[10]{2})^2$, $(\sqrt[10]{2})^3$, and so on. From there, we take the logarithm of the resulting number to base two, which gives us a simple uncial fraction (0;1, 0;2, and so on).

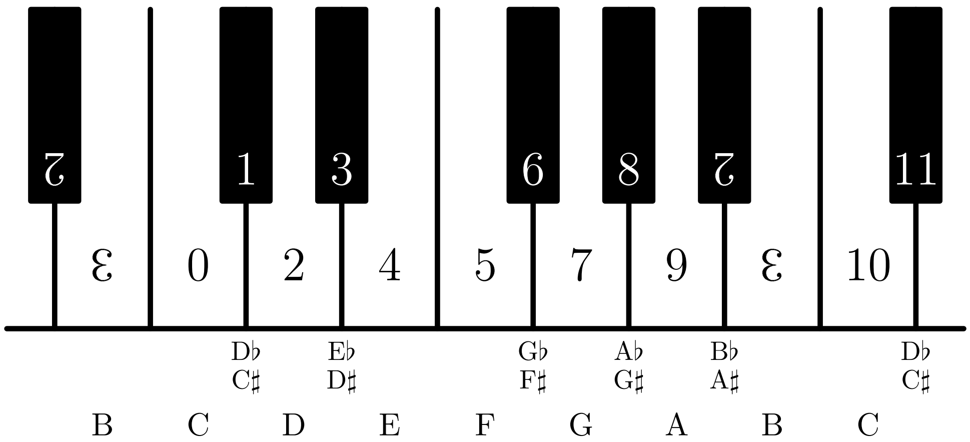

Figure ↊: The keyboard labelled in unciaDoubles.

it doesn't sound right. But when you play two notes together which *are* in a simple ratio, you have what is called a *concord*; it sounds nice, because the vibrations of the two frequencies fall into step every few cycles and thus merge together. We have started with Middle C here, but the starting sound is immaterial; no matter which note is selected, concords and discords will result in the same way, according to where in the octave the two sounds fall.

We'd like, of course, if we could label and describe this perfectly arithmetically. However, it's simply not possible; doing so would require an infinite number of keys in each octave. This is because it requires the successive powers of each ratio, but none of those ratios works out to an exact multiple of two. Fortunately, however, the human ear is imperfect, and we recognize sounds which are not quite in ratio as close enough, provided that they are, in fact, within certain limits of the true ratio.

So we compromise: some number which multiplied repeatedly by itself yields answers very close to the correct ratios and eventually reaches an exact two. Only one number does that: the twelfth root of two.

So a full octave up is one Double of frequency; each individual note is one unciaDouble of frequency. (Incidentally, this provides an excellent definition of a "semitone," something that has eluded musicians for centuries: a semitone is a unciaDouble of frequency.) No more octaves, units of eight which have twelve keys; now there are unquades, units of unqua which have unqua keys. Much more sensible.

Therefore, one can simply number the keys of a piano sequentially, with "Middle C" falling on note 60. Note 50 is C one octave below Middle C, while note 70 is C one octave above it. To find the frequency difference between two notes, take the absolute value of the difference in their note numbers, use the ones place as the exponent of the twelfth root of two expressed in dozenals (or look up the value of the ones place in Table 14 on page 63), and then double that frequency the number of times expressed in the dozens place of the difference. For example, to find the frequency difference between notes 45 and 75:

$|45 - 72| = 29$. So: $(\sqrt[10]{2})^9 = 1;8222$ (or, simply look up the value for 0;9 on the table). Then, double twice, which means $1;8222 \cdot 2 \cdot 2 = 6;8887$.

As mentioned earlier, note 78 is the reference note; this note is A♭ or G♯ in the second octave up from Middle C. In the new nomenclature, of course, this is simply note 78. What does it mean to be the reference note? That is the subject of the next section.

EXERCISES

1. What are the note numbers of the following? (Get units figure from diagram.) (a) B below Middle C. (b) F♯ above Middle C. (c) E♭ below Middle C. (d) E in the second octave down from Middle C. (e) B♭ in the third octave up from Middle C.

↊.4 Frequency

As we've mentioned before, many of these applicatons of Doubles and dublogs are in the field of frequency, which is a property of waves. Sound waves and light waves (though the former are longitudinal and the latter transverse) are the waves we are most familiar with, but there are many other types: X-rays, microwaves, and so on. Every wave has a *frequency*; that is, how many times it goes through its full cycle in a given period of time.

We currently have many different ways of referring to such cycles. Often we use *RPM*, or revolutions per minute; this is essentially a frequency measure, though it is used almost exclusively for circular motion like turning wheels. But the canonical unit in SI metric for frequency is the *hertz* (Hz), which equals a single cycle per second. Naturally, in TGM we time our frequency to the Tim:

Freq (Fq) = 1 cycle per Tim (1 PerTim) = 5.76 Hz

Frequency is what we've been talking about all along, of course; but now we've got a unit for it. This unit is frequently (no pun intended) necessary; to continue with our musical examples, however, it's vital for determination of *pitch*.

Frequency is a funny thing. As long as an instrument's notes sound in these ratios (approximated by the powers of the twelfth root of two), they will sound "in tune"; that is, the songs they play will sound correct, presuming that the musician is sounding them correctly. This is because it is the *ratio* that's important, not the absolute frequencies themselves. However, if two instruments are playing together, and each has its notes in ratio but at different absolute frequencies, they will likely sound quite terrible, because their notes are not in tune with one another. In other words, they are not tuned to the same pitch.

The international standard of pitch, designed to prevent this problem, is đ440 hertz for the A above Middle C (note 69). That note vibrates at 64;48 Freq, not a convenient number for a TGM standard. However, it's important that instruments tuned in TGM units be compatible with those not so tuned, so we would do well to closely approximate the international standard. Fortunately, assuming that note 69 is 64;48 Freq, note 78 is 100;252↊ Freq.

Tuning note 78 to 100 Freq exactly is only 0;252↊ Freq behind every Tim from the international standard. Taking the reciprocal of that gives 4;↋131, which tells us that the two notes must be held for at least ↊ Tim, give or take (twice the reciprocal) for the trained ear to notice the "out-of-tuneness." In other words, an instrument tuned with note 78 at 100 Freq is close enough; so, we have arrived at another auxiliary unit:

TGM Standard Pitch: Note 78 = 100 Freq

This even correspondence means we can define absolute pitch simply by multiplying the figures in Table 14 (on page 63) by 100 for the unquade following note 78, after subtracting 8. E.g., note 81 vibrates at 140;27 Freq (81 - 8 = 75). Other unquades can be treated similarly; subtract 8, find the pitch for the ones column, multiply by 100, and then either double once for every dozen above 7 or half once for every double below 7. For example, note 56:

> 56 − 8 = 4↊. By Table 14, 0;↊ vibrates at 1;946↋; multiply by 100 to get 194;6↋ Freq. 56 is three Doubles down from 78 (one double to 68, one to 58, and 56 is in the double from 48–58), so we halve that frequency thrice: 194;6↋/2/2/2 = 28;0↊46. So our answer is that note 56 vibrates at 28;0↊46 Freq.

This discussion has focused on the piano because it contains many full unquades lined up in a row; however, it is not limited to it. The violin, for example, has four strings, which are tuned to notes 57, 62, 69, and 74. The flute has a *compass* (a range of playable notes) which extends roughly from note 60 to note 90. And so on.

Mr. Pendlebury has also devised an entire system of musical notation based on these revelations regarding the nature of sound, one much simpler than our current one, and which does away with such craziness as sharps and flats entirely. It is beyond the scope of this book; however, in No. 1 of The Dozenal Journal, his article *Music à la Dozen* explains how all these complications, and many others, can be discarded as useless in favor of a simple, coherent notation.[20]

Exercises

2. Find the absolute frequency in Freqs of: (a) Note 86, (b) Note 68, (c) Note 58, (d) Note 48, (e) Note 9↊, (f) Note 49, (g) Note 56.

↊.5 Paper Sizes

Paper sizes in TGM work by ratios, just as they do in the traditional and decimal metric systems. In the latter, paper sizes are designed on the ratio of $1 : \sqrt{2}$, widely regarded as a particularly pleasing ratio (though it is a bit off from "the golden ratio," ϕ). Folding a paper in this ratio in half results in a new sheet with the same ratio.[21]

Metric paper sizes are divided into the *A-series* and the *B-series*. The A-series starts with the A0, which is a sheet of paper with the proportions $1 \times \sqrt{2}$ and which has an area of one square meter. When it is folded in half, it becomes an A1, which has an area of half a square meter but retains the proportions of $1 \times \sqrt{2}$. When an A1 is folded in half, it becomes an A2, with an area of a quarter of a square meter (half the A1) but still with those same proportions. This continues on until at least A↊, which is really too small to be particularly useful.

The B-series, on the other hand, starts with the B0, which is a sheet of paper one meter long, but which also has the proportions of $1 \times \sqrt{2}$. When it is folded in half, it becomes

[20] *See also* Appendix C on page 92, for a place to obtain electronic copies of *Music à la Dozen*.

[21] This section is elaborated based not only on the second edition of the TGM booklet, but also on T. Pendlebury, *Paper Sizes*, 24 The Duodecimal Review 3 (Duodecimal Society of Great Britain, 1182).

the B1, which has a length of half a meter but which retains its proportions of $1 \times \sqrt{2}$. The also continues until at least B↊, which, like the A↊, is probably too small to be useful.

This leaves us with two series of paper sizes, one of which is designed to be a binary division of a square meter in area (the A-series) and the other of which is intended to be a binary division of a meter in length (the B-series). Unusually for metric, this actually produces some fairly convenient sizes; in particular, the A4, which is a fair approximation of American "letter" paper ($8.5'' \times 11''$), has become fairly standard throughout the world.

Curiously enough, TGM produces nearly identical sizes of paper; however, the opposite series correspond to one another. That is, in metric the A-series is based on area, while the B-series is based on length; the TGM length-based series corresponds quite closely to the metric area-based series, while the TGM area-based series corresponds quite closely to the metric length-based series. These correspondences are indeed quite close; so close, indeed, that trimming metric papers to equal TGM sizes is a waste of time. The differences are so small that they can safely be classified as manufacturing error.

The basic TGM length-based series is prefixed "Gf" (pronounced, as by now we all know, "Grafut"); it is followed by a plus or minus sign, depending on whether we are doubling or halving the size (think of "+" sizes as *un*folding and "−" sizes as folding), and then a number to indicate how many times we do either. The basic TGM area-based series is prefixed "Sf" (after, of course, the Surf), and then suffixed in the same way as the length-based series. These are called the Grafut series and the Surf series, and are pronounced (for example) as "Grafut plus one."

Table 15 shows the TGM sizes in Grafut, in millimeters, and then compares them to the exact metric sizes in millimeters and square meters. As noted before, these are not *exactly* equivalent; however, they are so very close as make no real difference. Buying metric A4 paper is the functional equivalent of buying Gf−1 paper; and, in fact, the longer side of an A4 sheet is less than a millimeter and a half away from being a perfect Grafut ruler.

The astute observer will note that the proportions of these papers are not *exactly* $1 : \sqrt{2}$. However, this is a virtue, not a fault; TGM significantly improves on the metric paper sizes precisely because of this fact.

The square root of two (1;4↋79…) is a nonterminating fraction in all bases, by nature; sticking rigidly to it yields extremely clumsy linear dimensions. However, if we round it off to 1;5, we're so close that at normal paper sizes the difference is imperceptible, and it yields much more convenient dimensions. So the TGM paper sizes are this ratio, $1 : 1;5$, so close to $1 : \sqrt{2}$ as to make no real difference, yet a much more manageable number. This takes care of the Grafut series.

For the Surf series, we must find two numbers in the ratio of $1 : \sqrt{2}$ which, when multiplied together, come to 1 unit of area. These two numbers are 0;↊112 and 1;233. If we slightly alter the second number to 1;234 (which divides by two quite neatly, with no remainder, three times), and then divide it by 1;5 (our approximation of $\sqrt{2}$), we get very slightly less than 0;↊1 (0;↊0↋3 precisely) instead. Make those Grafut, and then multiply the two to get a sheet of area equal to 1 Surf. (Strictly speaking, $1;234 \times 0;$↊$1 = 0;$↋↋74, but that's close enough.) Starting from there (a Sf+0 of 1;234 Gf×0;↊1 Gf sheet, with an area of 1 Sf), we come up with a set of paper sizes of easy dimensions which still closely matches the metric B-series.

And so, ironically, metric paper sizes, designed to enshrine the meter and the number ↊

Grafut Series (Length-based)						
TGM Sizes			Decimal Metric Sizes			
TGM	Grafut	Millimeters	Metric	Millimeters	Grafut	Meter^2
Gf+3	2;↊×4;0	837×1183	A0	841×1189	2;↊17×4;0308	1.0
Gf+2	2;0×2;↊	592×837	A1	594×841	2;016×2;↊17	0.5
Gf+1	1;5×2;0	419×592	A2	420×594	1;509×2;016	0.25
Gf+0	1;0×1;5	296×419	A3	297×420	1;009×1;509	0.125
Gf−1	0;86×1;0	209×296	A4	210×297	0;864×1;009	0.0625
Gf−2	0;6×0;86	148×209	A5	148×210	0;604×0;864	0.03125
Gf−3	0;43×0;6	105×148	A6	105×148	0;432×0;604	0.01562
Gf−4	0;3×0;43	74×105	A7	74×105	0;302×0;432	0.00781
Gf−5	0;216×0;3	52×74	A8	52×74	0;217×0;302	0.003906
Gf−6	0;16×0;216	37×52	A9	37×52	0;217×0;302	0.001953
Gf−7	0;109×0;16	26×37	A↊	26×37	0;217×0;302	0.000976

Surf Series (Area-based)						
TGM Sizes				Decimal Metric Sizes		
TGM	Grafut	Surf	Millimeters	Metric	Millimeters	Grafut
Sf+4	3;44×4;914	14	994×1405	B0	1000×1414	3;470×4;948
Sf+3	2;468×3;44	8	703×994	B1	707×1000	2;484×3;470
Sf+2	1;82×2;468	4	497×703	B2	500×707	1;836×2;484
Sf+1	1;234×1;82	2	352×497	B3	353×500	1;242×1;836
Sf+0	0;↊1×1;234	1	249×352	B4	250×353	0;↊19×1;242
Sf−1	0;718×0;↊1	0;6	176×249	B5	176×250	0;721×0;↊19
Sf−2	0;506×0;718	0;3	125×176	B6	125×176	0;50↊×0;721
Sf−3	0;36↊×0;506	0;16	88×125	B7	88×125	0;370×0;50↊
Sf−4	0;263×0;36↊	0;09	62×88	B8	62×88	0;265×0;370
Sf−5	0;195×0;263	0;046	44×62	B9	44×62	0;195×0;262
Sf−6	0;1316×0;195	0;023	31×44	B↊	31×44	0;1312×0;195

Table 15: TGM Paper Sizes

forever, wind up fitting perfectly into the dozenal TGM system.

Examples

1. The format of a booklet is four dozen pages, six by eight and a half unciaGrafut (đ148×210 mm). Four pages were printed on each face of a sheet.
   (a) What is the sheet size in Gf (mm)? (2 × 0;6) by (2 × 0;86) (Gf); or (2 × đ148) by (2 × 210) (mm).
   (b) What is its code? A3 = 1 Gf by 1;5 Gf = đ297 mm by 420 mm.
   (c) How many sheets required? 2 faces × 4 pages = 8 per sheet. So number of hseets = 40 / 8 = 6 per booklet.
   (d) The paper substance is 6 $_4$Mz/Sf (đ85 g/m^2). What is the weight of one gross (đ150) booklets? Total area = (6 × 1;5)Sf × 100 = 860 Sf. Weight: 860 × 6 $_4$Mz = 4;3 unciaMaz. (6 × 0.125)m^2 × 150 = 112.5 m^2. Weight: 112.5 |*times* 0.085 kg = 9.5625 kg.

Exercises

3. An A4 sheet is folded in three to go in a long envelope 4;5×8;↊ unciaGrafut (đ109×218 mm). (a) What are the dimensions of the folded sheet in $_1$Gf (mm)? (b) What is its area in $_2$Sf (cm^2)?
4. Another A4 sheet is folded in four to go in an envelope 6;7×4;8 unciaGrafut (đ162×114 mm). (a) Folded dimensiopns in $_1$Gf (mm)? (b) Area in $_2$Sf (cm^2)?

↊.6 The Complex Double: The Euler Number

Population growth is an interesting application of ratios. Over a certain period of time, a population of living things will increase its numbers by a certain amount; over the next period of time, it will do the same. But the second time, it will increase its numbers by a certain ratio of its *increased* population, including the increase from the first period of time. So it's not a linear scale.

The mathematicians say that populations grow proportionally to their size at each passing moment; they grow on a logarithmic scale. Specifically, they grow logarithmically to a certain base. That base is represented by the symbol e, or Euler's number (named after Leonhard Euler, the Swiss mathematician who figured it out). Logarithms to this base are called *natural logarithms*, and they are represented mathematically not by the normal "log," but by "ln". They could be written less succinctly as such: $\log_e$.

Euler's number's value is 2;8752 3606 9821 9Ɛ8. . . . It continues infinitely, but this number is reasonably accurate for practical applications, and can more conveniently by abbreviated as 2;875 (as in decimal it is abbreviated as 2.718). The number appears in many places; for our purposes, it is enough to note that it exists, and to know that it has extensive applications when figuring in ratios.

↊.7 Dub: Dealing with Exponential Increase

Any unit in TGM can be prefixed with "Dub" (abbreviated "D") to facilitate dealing with ratios. "Dub" means "Doubles of." For example, rather than referring to "octaves," we can refer to DubFreq (DFq); 3 DFq is an increase of 3 octaves. Power is sometimes also measured with logarithmic units; TGM can accomodate this, as well. 3 DubPov (DPv) is three doublings of power. So if we start with 8 Pov and we see an increase of 3 DPv, we have $8 \cdot 2 \cdot 2 \cdot 2 = 54$ Pv at the end.

Chapter Ɛ: Looking at Light

FUNDAMENTAL REALITY: THE LIGHT-GIVING SUN

WE'VE ALREADY TALKED about electromagnetic radiation, and we've already talked about frequency; this is really all we need to know about electromagnetic radiation *per se*, and light is just one small segment of the vast range of electromagnetic frequencies (specifically, that segment with wavelengths from between 4000 $_{9}$Gf ($\not\partial$3960 Å) and 7800 $_{9}$Gf ($\not\partial$7591 Å)).

However, light is by far the most evident part of the electromagnetic spectrum to us, the only part that is visible, and we experience the world largely through it, so it is to be expected that there are a number of units and concepts unique to it.

Ɛ.1 Light as Light

Not all light is created equal. Different wavelengths of light within the visible spectrum give rise to different colors of light; lower wavelengths are red, higher are violet. But the main concepts to be aware of are *luminous flux* (also called light power), *light density*, and *luminous intensity*.

Luminous flux is the measure of the amount of visible light emitted by a source; in SI metric, this is measured in *lumens* (lm), which is one candela per steradian. In TGM, luminous flux is the primary unit; it is measured in Lypov (Lp), and maintains the $1:1$ correspondence of TGM units:

$$\textbf{Lypov (Lp)} = 1\ {}_{5}\text{Pv at } 5730\ {}_{9}\text{Gf} = 1.179597 \text{ lm}$$

This wavelength was chosen because of its visibility factor, which is higher than other wavelengths. Light at this wavelength is yellow-green, and is easier to see than other wavelengths. For this yellow-green wavelength, the *visibility factor* is thus 1 Lypov per pentciaPov. Lower wavelengths, like orange and medium green, are 0;9; deeper red and greens can range from 0;6 to 0;2, while higher-frequency light like blues and violets are 0;1 and 0;2.

Light density is luminous flux per unit area; in TGM, this is measured in Lypov per Surf, or Lyde:

$$\textbf{Lyde (Ld)} = 13.4921485 \text{ lum/m}^2 = 1.2534616 \text{ lum/ft}^2$$

Finally, *luminous intensity*, or *light intensity*, is the intensity of light of a given frequency from a power source of a given strength; in SI metric it is measured in candelas (cd), which is the luminous intensity of light at 540×10^{12} Hz and a radiant intensity (power, roughly) of $\frac{1}{683.0}$ watts per steradian. As light radiates away from its source, its density decreases in proportion to the square of the distance; therefore, by multiplying light density (Lyde) by the square of the distance, one can find the luminous intensity of the source, which is measured in QuaraLydes (QLd):

QuaraLyde (QLd) = 1 Lypov/steradian = 1.179597 cd

All these are properties of light as light; however, we must also consider light as we see it.

Ɛ.2 Vision and Photography

A camera is more or less a mechanical eye, and film is just a permanently-impressible retina; therefore, most of what we discuss in this section will be applicable to both fields. The younger among us will be unfamiliar with film itself; however, even digital cameras operate on the same principles, merely replacing chemistry with electronics, so the concepts should be familiar to all.

To take a picture with a camera, or to see anything, light has to strike upon the film or the retina after entering the camera or eye through the aperture or pupil. In a camera, the light causes a chemical reaction on the film (or an electrical one on sensors); in an eye, the light induces chemical reactions which send signals to the brain. Bright light will cause the pupil to get smaller; in a camera, unless one contracts the aperture, the photograph will be *washed out*; that is, too much light will be present to form a clear image. Adjusting the aperture or pupil in this case ensures that the *exposure*—the amount of light permitted to strike the operative surface—is appropriate to the *sensitivity* of that surface.

How do we know the appropriate size for the aperture? The question seems the height of stupidity; we know the appropriate size for the aperture of a camera by adjusting it until the picture looks right, and we don't need to know the appropriate size for our pupils because our brains take care of that without us knowing. All true; however, there is a mathematical principle which our brains follow, and which we unconsciously follow ourselves when we adjust our cameras, when we're adjusting our various apertures. That principle is that the aperture's value is proportional to the focal length of the lens. In photography, this is called the *f-stop*.

F-stop is listed in simple ratios, such as "f/16" and "f/2." This means simply that the diameter of the aperture is the focal length of the lens divided by the denominator (the bottom part of the fraction). Twice the diameter of the aperture covers four times as much area and admits four times as much light. These are, the astute reader will have already noticed, Doubles; each f-stop is the square root of two times the next stop.

For the eye, the focal length of the lens is a little less than 1 $_1$Gf (a unciaGrafut, about the diameter of an average eyeball), while the aperture diameter runs from around 3 $_2$Gf (three biciaGrafut) to 1 $_2$Gf (one biciaGrafut). This works out to an f-stop of from f/4 to f/Ɛ. The amount of exposure that comes through the pupil is the luminous flux multiplied by the time that the light is allowed in; true to form, TGM provides a specific unit for this:

Lyqua (Lq) = 1 LpTm = 0.2047911 lm·s

This measures the amount of light that is allowed into the aperture. It's important to ensure that this amount conforms to the receptive medium (film, retina, sensors, or whatever); in other words, each medium has a *correct exposure*. The higher the correct exposure is, the lower the *sensitivity* of the medium is. TGM provides a specific unit for sensitivity, as well.

$$\textbf{Senz (Sz)} = 1/\text{LdTm} = \text{Sf}/\text{Lq} = 0.426915 \text{ m}^2/\text{lum}\cdot\text{s}$$

In older systems, sensitivity was customarily measured with a logarithmic unit, while TGM here provides a simpler arithmetic unit. As discussed above,[22] however, a logarithmic unit can easily be constructed for Senz with the Double prefix: the DubSenz (DSz). 0 DSz is equal to ASA 16, 13 DIN, 24 Sch°, and 23 BS°.

Exercises

1. Write down the dublogs of: (a) f/8, (b) f/14, (c) f/Ɛ, (d) f/1↊, (e) f/5;7, (f) 28 Ld, (g) ↊8 Ld, (h) 0;6 Tm (1/↊ s), (i) 0.09 Tm (1/đ100 s).
2. A lamp has an intensity of ↊0 QuaraLyde (đ140 candela). What is the illumination in Lydes (lum/m^2) at 6 Gf (2 m)?

Ɛ.3 Stars and the Sun

Of course, our sun *is* a star, unremarkable except that it is our own; but it's significant enough to us that it deserves a special mention.

The brightness of stars have traditionally been measured as *apparent magnitudes*. This is a fundamentally subjective system and works something like as follows, having been derived from the ancient Greek system of classifying stars into six categories based on their brightness. The dimmest stars visible to the average, unaided human eye are of magnitude 6. These increase logarithmically; that is, by Doubles, with magitude 5 being twice as bright as magnitude 6, magnitude 4 being twice the magnitude of magnitude 5, and so on. The system is more scientific now; the star Vega is the standard star, measured at magnitude 0; the brightest star in the sky, Sirius, is magnitude -1.4. There are now more than six magnitudes. Most importantly, they are not Doubles anymore; each level of magnitude is a multiplication by 2.512. This number is the fifth root of a hundred ($\sqrt[5]{100.0}$), and was chosen to ensure that subtracting 5 magnitudes would mean an increase in brightness of one hundred times.[23]

These magnitudes are called *apparent* because they don't really reflect the actual brightness of the stars; they only reflect how bright the stars look to us on Earth. This appearance is affected by distance, light color, intermediate objects, and of course the *actual magnitude* of the star, which is how much light the star actually emits. This is measured by pretending that all the stars are some uniform distance away—typically ten parsecs, which is 5;7898 ^{14}Gf (unquadquaGrafut)—and determining what magnitudes they would have were they actually there.

But this is TGM. Such slavish worship of the ten-god and silly messing about with parsecs and fives is totally unnecessary here. Brightness can easily be measured on a normal logarithmic scale, specifically a scale of Doubles. Each DubBrite (DBt) is twice the brightness of the last. 0 DBt is the dimmest star visible to the naked eye. If the DubBrite number

[22] *See supra*, Section ↊.7, at 69.

[23] In this scale, the full moon has a magnitude of -12.74, the sun -26.74.

is negative, it means that the star is *not* visible to the naked eye, but rather requires the assistance of binoculars, a telescope, or some similar aid. And so goes the scale of apparent magnitude.

Assuming that 0 DBt is equal to the old system's magnitude 6 (which in theory it ought to be, as both represent the threshhold of visibility), Sol's magnitude would be ↋;676 DBt. Our sun, Sol, is such an important part of everything on our planet that it makes sense to give it prominence with regard to absolute magnitude. If apparent magnitude is *brightness*, then we can call absolute magnitude *brilliance*, and say that the brightness of our sun at the distance of one lightyear is 10 DubBril (DBl).

Both of these are logarithmic units. So ↋ DBl is half the sun's brilliance, and 11 DBl is twice it. 0 DBt is 6.24 traditional magnitude, only barely visible to the naked eye. At the distance of 1 Astru (the average distance of the earth to the sun (about 8;207↊ $^{↊}$Gf (decquaGrafut)), the sun's brightness is 37;↋ DBt.

And now we have come full circle. We began our discussion of the TGM system with a conversation about time, and how our perception of time depends upon the cycles of the sun. We now end it defining units to describe the brightness and brilliance of the sun. And so the last reality of TGM is truly its first, and the great harmony of creation can be clearly seen.

Exercises

3. Find the dublog of the fifth root of a hundred (nearest unciaDouble).
   To convert Mags to dubBrites subtract 6.24, dozenize, and multiply by the dublog you found in 3. Change the sign + to - or vice versa.
4. (a) Sirius is mag. -1.46. What is its brightness in dubBrites?
   (b) How many times brighter is the Sun? (Sun = 37;↋ DBt). (Abg of DBt difference)
   (c) Sirius is 9 lightyears away. What is its brilliance in DBls? (DBt + 2Dlg Lys).
   (d) How many times more brilliant than the Sun is it in reality? (Sun = 10 DBl).

Part 3

Getting to Grips

Chapter 10: Using Dozenals

THE BIGGEST DIFFICULTY in using TGM isn't really the TGM system itself, which is quite simple and logical compared to both SI and the imperial and customary systems. The biggest difficulty is the dozenal system itself. Not that the dozenal system is more difficult than decimal; quite the contrary, it is considerably easier. But it's not what most people are used to, and so the biggest hurdle in using TGM is getting accustomed to the superior dozenal system.

10.1 Thinking in Dozenals

It's helpful to simply watch numbers throughout our daily lives and try to apply dozenals to them; this is the biggest and simplest way to get accustomed to this superior system. When measuring out wood for a construction project—say, a son's Boy Scout birdhouse, or a compost bin for the backyard—using feet and inches, write them in dozenals rather than decimals and see the simplicity emerge. For example, to find the floor area of a birdhouse which will have walls six inches wide and eight inches long, don't write the equations as follows:

$$6'' \times 8'' = 48''$$

How many square feet and inches is d̸48 square inches? Write them out instead as dozenal fractions:

$$0;6 \times 0;8 = 0;4$$

0;4 square feet; it's as simple as that.

It's also instructive to try reckoning months and years in dozenals, as they are naturally applicable to that even to us accustomed to thinking in decimals. For children under two years old, we commonly refer to ages in months; so when you remark on the age of such a child, or when someone remarks on it to you, practice thinking dozenals by rephrasing such ages.

"*He's eighteen months old.*" Well, we're really saying that he's a year and a half old, aren't we? So he's 1;6 years old, or 16 months old. Easy. "*He's twenty months old.*" Well, not as simple there, but still quite straightforward. He's 1;8 years old, or 18 months old. Those of us who work in criminal justice will frequently hear long sentences expressed in months; e.g., a murderer might be sentenced to "*three hundred and sixty months*." Well, let's dozenize first; d̸360 is equal to 260. How many years is that? Once we have the number of months in dozenal, the number of years is a simple matter of moving the uncial point. 260 months is 26 years.

This sort of practice can continue *ad infinitum.* Think of ages in unquenniums, not in decades; if you're just entering your fifth decade, you may be beginning to feel old, but you can take heart that you're only a third of the way through your fourth unquennium. If you're entering your *fourth* decade, then you're still just a baby, only halfway through your third unquennium. Think of the years in terms of unquenniums (unquaYears) and biquenniums (biquaYears), rather than decades and centuries. The Soviet Union didn't collapse at the end of the 1980s; it collapsed at the end of the 1190s (1199, to be more precise). The wild and crazy decade wasn't the '60s; it was the '70s (1170s), and into the 80s, too. And the era of nationalistic colonial empires wasn't the nineteenth century; it was the triqua-aughts (1000–1100, or 1728–1872, though really it continued to 113Ƨ, or even 1161; these arbitrary time periods aren't any more descriptive of real history in dozenal than in decimal).

Think dozenal, don't just say it; it does little good to be forever converting dozenals to decimals in your head in order to "understand" them. One must get used to thinking of numbers in a different language, a task that really takes surprisingly little time.

Most helpful will probably be simply doing a lot of math problems, particularly the multiplication tables. These have been drilled so thoroughly into our heads in decimal form that banishing them, or at least making them sit aside for a moment, in favor of dozenal equivalents can take some practice. So just do some problems, for four or five minutes a day; the regularities of the dozenal multiplication tables are such that within a week or two they'll be coming to you just as easily as decimal ones do today.

There are utilities to help with this. One excellent utility, producing random single-digit multiplication problems throughout the table (excluding the ones and zens tables, since they are so easy as to require no practice) can be found on the Internet:

http://www.dozprog.webs.com/dozmult/dozmult.html

As said, four or five minutes a day for a week or two should be enough to get one thinking in dozenals with some facility.

There are, of course, other ways to help accustom one to this superior number system; the Dozenal Society of America[24] and the Dozenal Society of Great Britain[25] have many such resources available.

10.2 Calculating in Dozenals

Beyond simple mental calculations, however, there are also many tools available to help us work with dozenals, just as there are tools for helping us deal with decimals. These range from the simple abacus to the powerful and complex computerized calculator. The most simple of such tasks is converting numbers from decimal to dozenal vice versa.

Of course, all of these conversions can be done fairly easily, if labor-intensively, by hand.[26] But failing that, there are a number of software converters available for doing this task easily and accurately, and many of them work without any download necessary. The most complete of these is probably Alen Peacock's, which bills itself as "The World's First Online Dozenal/Decimal Converter Calculator."[27] Although the converter uses the unusual characters * and # for Ƨ and Ɛ, it is otherwise extremely simple. Simply enter the dozenal or decimal number one wants to convert into the appropriate field on the page, and the converted number will appear in the other field as you type. Unfortunately, though it does work with fractional numbers, it does *not* work with exponential notation (so-called "scientific notation," such as 1.492×10^3).

The dozenal suite of programs[28] offers a downloadable, command-line solution. Among

[24] http://www.dozenal.org

[25] http://www.dozenalsociety.org.uk/

[26] *See, e.g.*, Gene Zirkel et al., *Decimal-Dozenal Conversion Rules* (Dozenal Society of America, 11Ɛ1), available at http://www.dozenal.org/articles/articles.html.

[27] http://flud.org/dozenal-calc.html

[28] http://dozenal.sourceforge.net

other things, it offers two programs, `dec` and `doz`, which convert dozenal to decimal and decimal to dozenal respectively. Both programs accept scientific notation in the form common on digital calculators; e.g., 1.492e3, and are accurate to approximately 13 fractional places (this is by far the most common number; the actual number is dependent upon individual computer hardware). They will also output results in exponential notation if requested by a special "flag," in this case `-e`. Both programs also allow specification of precision by another special flag, `-k`, followed by the number of places of precision that are desired. The default is four places; but if the result is a whole number, no places will be displayed. Precision numbers are specified in dozenal, of course, for `doz`, which converts decimal numbers to dozenal; and decimal for `dec`, which converts dozenal numbers to decimal.

Despite being apparently complex in description, the programs are quite easy to use. To convert d̸1492 to dozenal, for example, simply type the following:

```
doz 1492
```

The answer, ↊44, will appear on the screen. If you want to add a fraction, then simply add one:

```
doz 1492.333333333333333333
```

And the answer, ↊44;4000, will also appear. (You may also get the answer ↊44;3↋↋↋; this is an unavoidable artifact of computer floating-point calculations. One will note that these numbers are virtually the same.) If you want the answer in exponential notation, provide the program the `-e` flag:

```
doz -e 1492.33333333333333333
```

And you'll get the answer: ↊;443↋e2. But due to the default precision value (4), it's no longer clear that this is really ↊44;4, so you ask `doz` to give you a few more digits of precision:

```
doz -e -k 8 1492.3333333333333333
```

And the answer, ↊;443↋↋↋↋↋e2 is quickly produced. You're clearly dealing with ↊44;4 now.

`dec` works in much the same way, the only difference being that precision values are entered in decimal, not dozenal, notation. (In other words, entering "T" for precision will work in `doz`, meaning "↊," but not in `dec`, which only knows decimal.)

Both programs, when reading dozenal numbers, understand a variety of common notations for ↊ and ↋. For ↊, they understand "X," "x," "T," "t," "A," and "a"; for ↋, they understand "E," "B," and "b." These may be mixed in any way desired; entering `dec xXtTaAbBE` will correctiong produce the decimal value of ↊ ↊↊↊↊ ↊↋↋↋ (4 690 709 567). When outputting, they produce only "X" for ↊ and "E" for ↋, following the use of F. Emerson Andrews in his initial dozenal masterpiece, *An Excursion in Numbers*.[29]

All this about converting numbers from base to base is all well and good; but what about more complex calculations?

Abaci are always available, and for four-function calculations are arguably faster than digital calculators; these can easily be adjusted to work with dozenals rather than decimals,

[29] F. Emerson Andrews, *An Excursion in Numbers*, available at `http://www.dozenal.org/articles/articles.html`.

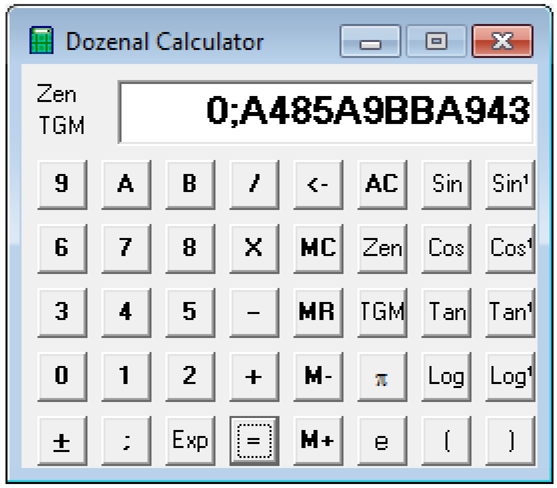

Figure ↋: ZCalculator.exe, running in Windows 7.

often without any changes to the device itself.[2↊] But for those of a more modern bent, there are other excellent tools available.

Written specifically for Windows, but also runnable in GNU/Linux and other systems capable of running wine,[2↋] is ZCalculator.exe by Michael Punter. ZCalc, as this author has come to call it for short, is a fine little program which produces a visual calculator on the desktop. It can operate equally well in decimal and dozenal modes; switch between them by hitting the the "Zen" button (which will turn to "Dec" for switching back). The calculator has trigonometric functions, logarithms, and the reciprocals of each, and operates in the infix notation we're all familiar with. It can work with exponential notation (the "Exp" key), has a couple of key constants available (π and e), and has some rudimentary memory functions. It uses "A" for ten and "B" for elv, a common shortcut in digital environments when real dozenal characters (like ↊ and ↋) aren't available. ZCalc also uses the Humphrey point (";") for uncials, and it has options for angle measurements in a single 12-segment circle ("1c"), unciaPis ("TGM," as shown in Figure ↋), in đ360 degrees, and in 260 degrees, giving a great deal of flexibility. It can also function as a simple converter; entering the number to convert and then striking the "Zen" (or "Dec") button will convert the number from one base to the other.

The dozenal suite of computer programs[30] also includes a complete postfix-notation ("Reverse Polish Notation," or RPN) calculator, which also works at the command line, called `dozdc`. While it can work in one-command units on the command line, the easiest way to experiment is to enter the program interactively by simply typing `dozdc`. Explaining RPN is beyond the scope of this little work, though it's an easy concept to understand and provides an *unambiguous* notation for mathematical expressions (unlike our normal infix notation, which requires lots of parentheses and rules concerning priority to ensure that operations

[2↊] *See, e.g.*, Robert Edelen, *A Duodecimal Abacus* in 2↊ The Duodecimal Bulletin ↋ (1175), available at http://www.dozenal.org/archive/dbpict02.html.

[2↋] http://www.winehq.org/

[30] http://dozenal.sourceforge.net

are done in the correct order). It used to be quite common not long ago on calculators produced by HP and Sinclair. Without explaining the concept of RPN, however, an example of `dozdc`'s operation is as follows:

```
8 1 3 / ^ 2 * =
```

This takes the cube root of eight (`8 1 3 / ^`, meaning "take eight; divide one by three; and then raise eight to the power of the result"), and then multiplies it by two (`2 *`), yielding 4.

`dozdc` will work with equations of arbitrary length, accepts all the same characters for ↊ and ↋ that `doz` and `dec` do, and even allows for some simple programming and variable handling.

Now that we have the tools we need to get used to working with dozenals, what can we do to get used to working in TGM itself?

Chapter 11: Using TGM

Getting used to using TGM is best done by simply *using TGM*. This sounds like a platitude, but it really needs to be stated. Too often enthusiasts of a new measuring system are content to simply study and pronounce its virtues without ever actually making use of the qualities they praise so highly. Some ways of using TGM are actually quite simple and can be done daily; let's consider some as good examples.

11.1 Thinking in TGM

Consider heights as an example. In America, at least, heights are routinely given in feet and inches, such that even without saying the units it's perfectly clear what is being meant. "She's five-two," or "he's six-one," are common phrases devoid of ambiguity. Feet and inches are so conveniently sized and easy for this type of length measurement that it's unsurprising that they persist quite tenaciously, even in many countries which have metricated. But the Grafut is only a little less than a foot; why not start thinking about heights and similar lenths in Grafut, instead?

Your author, for example, is almost exactly six feet tall. (About an eighth of an inch less, but close enough.) Rather than thinking of myself as six feet tall, however, I can think of myself as 6;2 Grafut tall. It's just as easy to say, too; "I'm six dit two." "She's five dit eight." Because feet are already in dozenal, one can simply transform the inches part into dozenal and convert this number from feet and inches to Grafut, using any number of convenient tools[31] or simply by hand or head (add one unciaGrafut per two and a half feet; this will give approximately the right figure, if you're working with short distances like heights). For example, if someone is "five foot ten," a longish way of saying that he's five feet, ten inches tall, simply convert that into the dozenal number 5;↊ and convert it. The one-unciaGrafut rule given above yields about six Grafut; the exact figure is 6;01↊9 and some change, which is certainly close enough for government work, a difference of less than two biciaGrafut (for

[31] *See infra*, Section 11.3, at 7↋.

comparison, of slightly less than two millimeters). A person this height would undoubtedly refer to himself as "six Grafut even," or "six Grafut tall."

We've already seen that in the printing and publishing field, converting to TGM will be a non-issue. The current standard printer's point, which is quite similar to the Postscript "big point," is almost identical to two triciaGrafut; so setting the TGM point equal to two triciaGrafut and the TGM pica (twelve points) to two biciaGrafut will result in almost no change in actual practice.[32]

Turn to the kitchen for further examples. We often measure our kitchen ingredients in volume rather than mass; almost always, in fact. The standard TGM volume unit, the Volm, while a quite convenient size for shipping and the like, is too large for ordinary kitchen use. (It is about six and three quarters American gallons.) However, the Volm offers subsidiary and supplemental units which are quite convenient for daily home use.

When you're cooking, translate your recipes into TGM. When the recipe calls for a teaspoon (approximately 5 milliliters) of something, consider it 4 quadciaVolm. Better yet, consider it a third of a triciaVolm, and that will remind you that a tablespoon is about equal to a triciaVolm. A cup is nearly one and a third biciaVolm, but don't think of it that way; think of it as nearly half a Tumblol, which is a big pint for Americans and a slightly small pint for the British. Two Tumblols are a Quartol; four Quartols are a Galvol; these are all quite close to our quarts and gallons, and will be just as convenient for the kitchen.

11.2 Tools for TGM

To use TGM, of course, one needs tools. These necessary tools can be divided into two main groups: one, the physical measuring tools necessary for measuring physical objects; and two, tools to facilitate conversion of TGM units into units of the older systems.

Regarding the first, oftentimes these can be jerry-rigged from existing equipment, sometimes without any actual change to equipment beyond relabelling. In the kitchen, for example, simply relabel one's teaspoon as 4 $_4$Vm, and one's tablespoon as 1 $_3$Vm. Relabel one's cup as 14 $_2$Vm, and relabel the quarter-cups on the side as 4 $_2$Vm. There are sixteen tablespoons in a cup, of course, and there are onezen-four (14) $_3$Vm in 14 $_2$Vm. This may seem opaque at first, but once one is accustomed to thinking in twelves rather than tens, it's truly just as transparent as cents in a dollar.

The Grafut is likewise easy; being so close to the foot, one can adjust to its use with little trouble. Rulers scaled in inches and centimeters are readily available; simply mark with a black marker off at 29.5 centimeters, or (if your ruler doesn't have centimeters) at 11.64 inches; then use another ruler to mark off the unciaGrafut at 0.97 inches each, and you have a Grafut-scaled ruler. Alternatively, one can simply download a paper with a perfectly Grafut-sized ruler on it.[33] This may require some modification based on the printer being used; but once done, one can simply wrap an existing ruler in this paper, cover it with tape, and use it as appropriate. It's not pretty, but it's functional, and will serve well until TGM is popular enough to inspire ruler manufacturers to produce them for us.

[32] *See supra*, at 1↊.

[33] Inquire about this in the "TGM Ruler" thread at the DozensOnline forum, `http://z13.invisionfree.com/DozensOnline/index.php`.

The next trip you plan, plan in quadquaGrafut, triquaGrafut, or Gravmiles, not miles. The mile is a remarkably convenient unit, but the quadquaGrafut, for example, given its relationship to the other TGM units, is even more so. Equal to about $3\frac{3}{4}$ miles, or a bit under $6\frac{1}{6}$ kilometers, the quadquaGrafut (doubtlessly it will come to be called simply the "quadqua" when referring to long distances) is a very convenient size for discussing long distances. Not only is this about the distance that a healthy, unloaded person can walk in an hour, but it also provides us with much lower numbers than miles do, when we're typically dealing with large numbers of them. San Francisco to New York City, for example, is đ2,909 miles roadwise (less, of course, as the crow flies); but it is only 537;6 quadquaGrafut. Perhaps even more demonstrative, New York City to Miami is đ1,290 miles, but only 242;7 quadquaGrafut.

Using them is easy; as a rough estimate, simply divide the number of miles by four. For more accurate but still rough and mental division, divide the number of miles by 3;9, or $\frac{13}{4}$. This is at least as easy as the multiplication by $\frac{5}{8}$ urged upon us to convert miles to kilometers.

Weights are more difficult, as finding scales calibrated in TGM units is, at this point, impossible, and there is no easy way to modify most existing scales to read out Maz. (We may as well work in mass rather than weight, as amount of matter, rather than gravitational pull thereon, is what we're really talking about, and the 1 : 1 correspondence between the two at a standard Gee is one of TGM's many strengths.) If one has an analog scale (the kind without a digital readout; that is, with a wheel inside it with the scale in pounds or kilograms printed on it), one can simply remove the scale and recalibrate it with a marker or pen: one Maz is just under 49 pounds, and little bit more under 22 kilograms, while a unciaMaz is about 4 pounds and unqua ounces (just a hair under $4\frac{2}{3}$ pounds), and just a little heavier hair under $2\frac{1}{6}$ kilograms. Most likely, it would be easiest to subdivide these by twos—halves and quarters—then to divide them into full uncias, since we're probably getting to very small distances on the wheel here. However, since halves and quarters would be even biciaMaz, this is not a problem. A biciaMaz is just under $6\frac{1}{3}$ ounces, or 0;48↋↊ pounds (0;21↊2 kilograms, or just under đ180 grams); finer gradations than this on a common household scale are likely unnecessary. Or, of course, one can scale in Poundoids or Kilgs, which should proceed in the same way.

These analog scales are few and far between in our digital age, however, and the author has not yet concluded on a satisfactory substitute for it. For the time being, then, we may be required to weigh in pounds or kilograms and convert to Maz.

Still, analog kitchen scales, scaled in grams, are still often available, and doing something like what is outlined above, in the appropriately smaller subdivisions, is still quite within reach.

If you work customarily with much larger quantities, the septquaMaz is a fair approximation of the metric megaton; the metric ton is about a quarter of a biquaMaz (0;3282). For imperial ("long") tons, a biquaMaz is very slightly less than $3\frac{2}{3}$ (3;7↋90); for the American ("short") ton, use $4\frac{1}{10}$ (4;130↋).

11.3 Calculating TGM

Because TGM is a relatively new and unknown system, there is a shortage of tools available for its use. However, there are two excellent utilities which provide for easy and very accurate (to within 8–↊ uncial or decimal places at least) conversions for these units, to metric or imperial/customary units, or even to other lesser-known systems.

One of these is web-based only, written by Takashi Suga, titled simply "A Converter." Its interface is complex, and consequently powerful; the only feature missing is the ability to select a level of precision (precision is instead calculated automatically from the number of digits input; but there are no options for selecting methods of rounding to that number, either). It is available at:

http://hosi.org/cgi-bin/conv.cgi

The converter works effectively with units from the imperial/customary systems; SI metric; TGM; and Suga's own system. It accepts input in either decimal or dozenal, with options to specify which is intended.

Another web-based system, based on another system we'll examine later, is the "TGM Unit Converter." It is available at:

http://gorpub.freeshell.org/dozenal/blosxom.cgi/tgmconv.html

This converter accepts its input only in dozenal, and outputs it only in dozenal. It allows one to specify exponential notation if desired, and also to specify the number of digits of precision one desires (the default is 4). It contains links to a page specifying the acceptable units, which is (almost) all of the TGM units, plus all the common metric and imperial/customary units.

For stand-alone converters, which can be run even when an Internet connection is unavailable, this author is unaware of any options but for `tgmconv`. Part of the dozenal package mentioned above, which includes `doz`, `dec`, and `dozdc`, `tgmconv` is a command-line program which allows the conversion of TGM units to customary/imperial and SI metric units, and vice-versa, along with the conversion of imperial/customary and SI metric units to each other (though this is a side effect, not its intended purpose).

`tgmconv` carries the same options as its sister programs of the dozenal suite; namely, `-k` for specifying precision, and `-e` for specifying that the output should be in exponential notation. However, it also allows the specification of two other options, `-i` and `-o`. The first indicates the unit that the input will be in; the second indicates the unit that the output should be in. `tgmconv` does not check that these values make sense; if you specify input in feet and output in joules, it will obediently apply the conversion factors and give you an answer, which will be worth about as much as one would expect. It expects units to be given to it according to a fairly particular but also fairly intuitive syntax, which can be found explicated in abbreviated form on the Internet[34] and in extended form in the manual.[35]

Queries to `tgmconv` are thus fairly intuitive, and like its sister programs, they can be made from the command line, from a file, or to the program acting interactively. For a further explanation of this, please see the manual; for example purposes here, we will use command-line queries. So, for example, to convert a trip of 78.4 miles to quadquaGrafut, execute this command:

[34] http://gorpub.freeshell.org/dozenal/blosxom.cgi/tgmconv.html

[35] Available in the normal place for man pages on your operating system, or at http://dozenal.sourceforge.net/tgmconv_man.html.

```
tgmconv -imi -o4^Gf "66;4"
```

("66;4" is, of course, 78.4 in dozenal; if you're not yet facile at converting such things mentally, one can chain the dozenal programs together; try `doz -k1 78.4 | tgmconv -imi -o4^Gf`.) This will obediently produce the correct answer of 18;6894. This is, of course, more precision than you really have any right to, having given `tgmconv` only three significant digits; so if that's important to you, tell `tgmconv` to be careful about limiting its precision:

```
tgmconv -k1 -imi -o4^Gf "66;4"
```

Precision, in the dozenal suite, isn't precisely significant digits; but one can use it to that effect most of the time, and in this case `tgmconv` will produce the answer to three significant digits, 18;6.

`tgmconv`, especially in conjunction with her sister programs in the dozenal suite, is a very powerful program for using the TGM system, and a full perusal of its manual is worth the read.

Part 4

Appendices

Appendix A: Answers to the Exercises

Note that the decimal answers are *analagous*, not equivalent, to the TGM values. This is to avoid introducing unnecessary traps of arithmetic into the answers when the goal is simply to illustrate the TGM system.

Chapter 1: Digits and Bases

1. (a) 34, (b) 23, (c) 18, (d) 47, (e) 130, (f) 143.
2. (a) BB, (b) AA, (c) 3B, (d) 16, (e) 1B, (f) 2A.

Chapter 2: Spelling in Dozens

Answers include, but are not limited to:

1. *Four unqua elv* (*fourqua elv*); *four dit elv unqua* (^{1}4;↋).
2. *Five biqua nine unqua two*; *five biqua nine two*; *five dit nine two biqua* (^{2}5;92).
3. *Lev triqua ten biqua elv unqua ten*; *lev triqua ten elv ten*; *lev dit ten elv ten triqua* (^{3}↋;↊↋↊).
4. *Six septqua, nine hexqua, seven pentqua, eight quadqua, four triqua, five biqua, nine unqua, seven dit four five nine eight*; *six septqua nine seven eight four five nine seven dit four five nine eight*; *six dit nine seven eight four five nine seven four five nine eight septqua* (^{7}6;9784 5974 598).
5. *Zero dit zero nine eight four five eight seven six*; *nine dit eight four five eight seven six bicia* ($_2$9;845876); *nine bicia eight four five eight seven six.*

Chapter 3: Time

1. (a) 7;9 Hr (b) 8;↊ Hr (c) 12;1 Hr (d) 1↊;8 Hr (e) 5;46 Hr
2. (a) 2 a.m.(Calif.) is 2;0 hr. + 14 = 16;0 Hr (Hong Kong). (b) 9;6 Hr, + 14 = 21;6 i.e. 1;6 Hr the next day in Hong Kong. (c) 10;0 Hr, 24;0 = 4;0 Hr on the next day. (d) 15;9 Hr, 9;9 Hr next day. (e) 1↋;4 Hr, 13;4 next day.
3. 60 + 5 + ;4 = (a) 65;4 Hr, or (b) 65 4000 Tm

Chapter 4: Space

1. (a) 16 cu.Gf (0.625 m^3) or 16 Vm (625 liters). (b) 16 ^{2}Tm or 0;16 Hr ($\not{d}$625 s or 10 min 25 sec).
2. (a) $\frac{14-6\text{Vl}}{20\text{Tm}} = 0;5\text{Vl}/\text{Tm}$. $\frac{60-24}{4} = 9\text{mph/s}$. $\frac{100-40}{4} = 15\text{kmh}^{-1}\text{s}^{-1}$.
   (b) 0;5 Gf/Tm^2. $9 \times \frac{5280}{3600} = 13.2\text{ft/s}^2$. $15 \times \frac{1000}{3600} = 4.16\text{m} \cdot \text{s}^{-2}$.
   (c) 0;5 G. $13.2/32.1741 = 0.41026\text{g}$. $4.16/9.80665 = 0.4249\text{g}$.

Chapter 5: Matter and Force

1. (a) 0;16 Vm (đ741 cc). (b) 0;08 Sf or 8 $_2$Sf (9.876 sq.cm).
2. **TGM:** (5 - 3)Mg / 8 Mz = 0;3 G.
   SI: (đ125 - 75) kgf / 200 kg = 0.25 g = 2.45 m/s^2.
3. **TGM:** 3;26 Mg / 0;7 Sf = 5;6 Pm.
   SI: đ83 kgf / 0.051 ×9.80665 m/s^2 = 15.960 N/m^2 or pascal
4. **TGM:** 14 Pm
   SI: đ4700 kgf/m^2 × 9.80665 m/s^2 = 46.091 N/m^2, say 46 pascal.
5. **TGM:** 3 ^{2}Mg / 7 $_3$Sf = 5;19 ^{4}Pm
   Cust: 10.5 / 0.6 = 17.5 ton/in^2 = 39200 lb/in^2.

Chapter 6: Work, Energy, Heat, and Power

1. (a) (16 - ↊) Gf × 16 Mg = 100 Wg. (b) 16 Mg × 16 Gf = 230 Wg.
   (c) Since falling starts at zero velocity, the velocity at any instant is proportional to time elapsed. In TGM v in Vlos is numerically equal to t in Tim. In metric $v = 9.8t\text{m} \cdot \text{s}^{-2}$. Let t = time to reach the ground.

Average velocity	$16/t$	Gf/Tm	$3.25/t$	m·s^{-2}
Final velocity	$30/t$	Vlos	$6.5/t$	m·s^{-2}
which also equals	t	Vlos	$9.8t$	
So final vel.	$\sqrt{30}$	6 Vl	$\sqrt{6.5 \times 9.8}$	7.98 m·s^{-1}

   (d) kinetic energy = $\frac{1}{2}mv^2$; 16 Mz × 30 Vv = 230 Wg; 450 kg × (6.5 × 9.8)m^2·s^{-2} = 28665 J. (e) 8 Mg × 20 Gf = 140 Wg. 2kN × 8 m = đ16 kJ. (f) Friction, i.e. heat generated at the touching surface of box and ground. (g) (3.25 - 2.5)m × 450 kgf × 9.8 N/kgf = 3307.5 N·m or J. (450 × 9.8)N × 3.25 m = 14332.5 J.
   In TGM, due to G = 1, bodies falling from a rest hit the ground at a velocity equal to the *square root of twice the height*, and with kinetic energy equal to *twice the height multiplied by the mass.*
2. TGM on the left, SI metric on the right:

T_1	1700 + 130	=	1830 ^{2}Cg	đ273 + 18	=	291 K
T_2	1700 + 1300	=	2↊00 ^{2}Cg	đ273 + 216	=	489 K
P_2	$P_1 \times \frac{T_2}{T_1}$					
	2↊00 1830 /	=	1;82	1 Atm × 489/291	=	1.68 Atm
	1;82 × 30	=	50;6	1.68 × 101.325 kPa	=	170 kPa
	50;6 - 1;8	=	4↊;↊ Pm			
	4↊;↊/ 2↋	=	1;82 Atz			

Chapter 7: Angles, Rotation, Radiation, and Perspective

1.

45	15	10	5	20	25	65	75	80	22.5	2.5	7°30'	1°15'	120
^{v}3	^{v}1	^{v}08	^{v}04	^{v}14	^{v}18	^{v}44	^{v}5	^{v}58	^{v}16	^{v}02	^{v}06	^{v}01	^{v}8

2.

Angle	^{v}3	^{v}5	^{v}4	^{v}16	^{v}04	^{v}18	^{v}6	^{v}8	1^{v}4	
Complement	^{v}3	^{v}1	^{v}2	^{v}46	^{v}58	^{v}44	0	$-^{v}$2	$-^{v}$2	$+\theta =$ ^{v}6
Supplement	^{v}9	^{v}7	^{v}8	^{v}↊6	^{v}↋8	^{v}↊4	^{v}6	^{v}4	$-^{v}$4	$+\theta = 1$ Pi
Opposite	1^{v}3	1^{v}5	1^{v}4	1^{v}16	1^{v}04	1^{v}18	1^{v}6	1^{v}8	^{v}4	$\theta \pm 1$
Negative	1^{v}9	1^{v}7	1^{v}8	1^{v}↊6	1^{v}↋8	1^{v}↊4	1^{v}6	1^{v}4	^{v}8	$+\theta = 2$ Pi

3. (a) ^{v}6 (b) ^{v}3 (c) ^{v}4 (d) ^{v}4 (e) ^{v}82 (f) ^{v}18. Sum of angles always = 1^{v}0.

4. (a) 6^{v}0 (b) 5^{v}0 (c) 2^{v}0 (d) 2^{v}0 (e) 3^{v}0 (f) ^{v}9 (g) ^{v}8 (h) ^{v}86↊35 (i) ^{v}94 (j) 135°, 120°, 128°34'17", 140°.

5. (a) 10 rGf (10 rad) (b) 7;4 rVl (44 rad/s) (c) 1;2 rev/Tm (7 rev/s) (d) 120 rev/$_2$Hr (420 rpm).

6. (a) **TGM**: 7 RMg/6QMz = 1;2 rG. **Cust.**: 400 lb·ft/(340/32.2) lb·ft^2 = 37.88 rad/s^2. (b) **TGM**: 7 RMg $\times 4\pi$ rGf = 73;↋7 Wg. **Cust.**: 400 lb·ft $\times 4\pi$ rad = 5026.54 ft·lb.

7. (a) 56;84 unoctquaSurf (2.25 $\times 10^{22}$ m^2) (b) 0;3499 Penz (1391 W/m^2).

8. (a) 3 and 2;6 $_2$rGf (0.02 and 0.1666... rad) (b) 7;6 $_4$qSf (0.00033... sr).

9. 1;1 ^{5}Au. 1 parsec = 206265 AU. Parallax ".76 is 206265 / .76 = 271$\times 10^3$ AU (= ^{5}1;1).

Chapter 8: Electromagnetism

1. (a) 6 Kr (3 A), (b) 1;8 Pv (720 W), (c) 1;8 ^{3}Wg (216 kJ), (d) 1;8 ^{4}Wg (720 kWh)
2. (a) **TGM**: C = $_7 1 \times 6 \times 0;24/_4 7 = 2$ $_4$Kp
   SI: $8.8 \times 10^{-12} \times 6 \times .019/.0001 = .01\mu$F
   (b) **TGM**: RC = $6 \times {}_4 2 = 1$ $_3$Tm
   SI: $10^4 \times .01 \times 10^{-6} = .1$ ms
   (c) **TGM**: $I_{max} = {}_3 16$ / 6 = 3 $_3$Kr
   SI: 9 / 10^4 = .9 mA
   (d) **TGM**: i = $_3 3(;77) = 1;$↊9 $_3$Kr
   SI: .9 mA (.632) = .5688 mA
3. Current = $_2$2 / $_3$20 = 1 Kr (.5 A). Time = 64 KrHr / 1 Kr = 64 Hr (đ76 hr).
4.

TGM	**SI**
B = 12 $_6$Fm / 8 $_3$Sf = 1;9 $_3$Fz	7×10^{-4} / (4×10^{-4}) = 1.75 Wb/m^2
Kurns for the air gap:	
$(_3 1;9/_9 2\pi)(_2 1)$ = 2414 Kurns	1.75 / $(4\pi\times 10^{-7})(2\times 10^{-3})$ = 2785 At
From graph:	
H for B = $_3$1;9 is 23 ^{2}Kn/Gf	H for 1.79 Wb/m^2 is 6500 At/m
Multiply by 0;9 Gf = 1830 Kn	Multiply by .22 m = 1430 At
Total Kn = 3414 + 1830 = 5044	Total 2785 + 1430 = 4215 At
Divide by current: 5044 / ;6 = ↊088 Turns	4215 / .25 = 16860 turns

5. (a) 100 / 1400 = ;09; đ100 / 1600 = .0625. (b) $_3$340 $\times$ 0;09 = 26 $_3$Pl; đ240 $\times$ .0625 = 15 V. (c) $_3$340 $\times$ 1;5 = 488 $_3$Pl; đ240 $\times$ 1.41 = 339 V. $_3$26 $\times$ 1;5 = 36;6 $_3$Pl; 15 $\times$ 1.41 = 21.2 V. (d) 4 $\times$;09 = ;3 Kr; 2 $\times$ .0625 = .125 A.

Chapter 9: Counting Particles

1. (a) 2×10 + 6 + 14 = 3↊ m_u. (b) 3↊ mMz. (c) 3↊× ${}_{23}$8;↊ = 2;9↊4 ${}_{21}$Mz. (d) 46 m_u × $1.7\times10^{-27} = 7.82 \times 10^{-26}$ kg.
2. (a) 3↊ mMz × 2M = 78 Mz. (b) 46 m_u × $2N_o$ = 92 g = 0.92 kg.
3. (a) Electrons per molecule: 2 × 6 + 6 + 8 = 22 (đ26). Per Molz: 22M. (b) 22 × ^{22}1;44 = 5;148 bitriqua. (c) 26 × 6 × $10^{23} = 1.56\times10^{25}$.
4.

TGM	SI
8;8 ^{3}Vm is ;8 Avolz	1.5×10^{-2} m^3/2.24×10^{-2} m^3 = .681$\overline{8}$
;8 Avz × 1;6 Atz = 1 Mlz	.681$\overline{8}$ × 1.5 Atm = 1.023 mol

5. (a)

-2°C × 10 = -20 = -18 d° Add 1700 and × 2 = 3188 ^{2}Wg/CgMlz	RT = 8.3 × 273.15 = 2267.145 J/mol

(b)

Divide by 2↋ Pm = 10↋30 Vm, the Avolz.	Divide by 101325 N/m^2 = .02237 m^3; i.e., V_o

(c)

Divide by 1;6 (=× 0;8) = 8754 Vm, the actual volume.	Divide by 1.5 = 0.0149 m^3, the actual volume.

6. See 5(a) decimal example above.
7. Molecular mass of NaCl = 1↋ + 2↋ = 4↊. So:

4↊ ${}_{4}$Mz = 1 ${}_{4}$Mlz	58 g = 1 mol NaCl
Molvity = ${}_{4}1/{}_{1}3$ = 4 ${}_{4}$Mlv	Molarity = 1 / 5 = .2M solution

8. **TGM**: It = 10 × ^{4}1 = 1 pentquaQuel; a/v = 44 / 6 = 8;8
   SI: 6 A × 3600 s = 21600 coulomb = 52 / 6 = 8.$\overline{6}$
   (a) **TGM**: Mass deposited = ^{5}8;8 / ^{9}5;7499 = 1;664 ${}_{4}$Mz
       SI: 21600 × 8.$\overline{6}$ / (9.6487×10^4) = 1.94 g
   (b) Thickness = Mass/(density × area)
       TGM: 0;0001664 / (7;2 × 0;35) = 9;0↋ ${}_{5}$Gf
       SI: 0.00194 kg / (7200 × 0.25) = 1.0$\overline{7}$ × 10^{-5}.
9. (a) 5;87↊ zH. (b) 6;21 dH. (c) 3.17 pH.

Chapter ↊: Doubles and Dublogs

1. (a) 5↋, (b) 66, (c) 53, (d) 44, (e) 8↊.
2. (a) 194;6↋, (b) 60, (c) 30, (d) 16, (e) 45↊;64, (f) 17;0↊2, (g) 28;0↊46.
3. (a) 4 × 8;6 ${}_{1}$Gf (đ99 × 210 mm). (b) 2↊ biciaSurf (207.9 cm^2).
4. (a) 6 × 4;3 unciaGrafut (đ148 × 105 mm). (b) 21;6 biciaSurf (155.4 cm^2).

Chapter ↋: Looking at Light

1. (a) -3, (b) -4, (c) -3;6, (d) -4;6, (e) -2;6, (f) 5, (g) 7, (h) -1, (i) -4.
2. E = ↊0 QLd / 6^2Q = 3;4 Ld (E = đ140 cd / 2^2m^2 = 35 lum/m^2). (Q = (Gf-radial)^2, remember?)
3. 2.512 = 2;619, Dlg of which is 1;3↋43, say 1;4.

4. (a) -1.46 - 6.24 = -7.7 = -7;85. Divided by 0;9 = Ƨ;3 DBt. (b) 37;Ɛ- Ƨ;3 = 29;8 + 9,,28;3- = 9,1;5 Abg = (without tables) is twice 4/3 = 2;8 neen times. (c) Dlg 9 = 3;2; 3;2 + 3;2 + Ƨ;3 = 14;7 DBl (answer). (d) 14;7 - 10 = 4;7 Abg = half as much again as 2^4. Sirius is 20 times the brilliance of Sun.

Appendix B: The Uncial System

Throughout this book we have used a system of referring to dozenal numbers that we have called the "Uncial" system. This is, in fact, a system devised by a group of dozenalists collaborating via the Internet, and its full name is *Systematic Dozenal Nomenclature*. It is a system designed to be easily learned by new dozenalists, while still being robust enough to cover all possible needs for referring to numbers with words. As such, it is a system which is powerful in its simplicity, yet still remarkably versatile. We have treated it thus far as a simple system of prefixes; this appendix gives a fuller explanation of the system, with many more examples.

At the base of the SDN is a simple set of roots, one for each number in the dozenal system. The roots selected for zero through nine are identical to those chosen by IUPAC; these prefixes are internationally known and recognized due to their adoption by IUPAC, and consequently engender little controversy among those who care about such things.

However, since SDN is intended for use with the dozenal system, an extension to the IUPAC roots was necessary; namely, to provide roots for Ƨ and Ɛ. The roots chosen were *dec* for ten, which is universally recognized as such, and *lev* for eleven, which is easily enough derived.

To make these roots useful for *counting*, however, rather than merely for naming elements (the chief purpose to which IUPAC puts them), SDN requires a further system. In brief, these roots can be used in two primary ways:

1. As multiplier prefixes; that is, as simply indicating that the word to which they are attached is to be multiplied by that number. An example of such a usage is the well-known English prefix *tri-*, as in "*tri*cycle" or "*tri*angle"; the root simply indicates that the "cycle" is in fact three cycles, and the "angle" is in fact three angles. The SDN roots can all be used for such purposes; we will see an example of their use as such in polygon and polyhedron names shortly. Sometimes, however, simply using the roots will be ambiguous; in this case, clearly related multiplier forms must be used. See Table 16 on page 8Ƨ.
2. As exponential prefixes; that is, as indicating that the word to which they are attached is to be multiplied by twelve to the power of that number. We have used them in this way consistently throughout this book; for example, a *biquennium* indicates that the year ("ennium") is to be multiplied by twelve ("qu-") to the power of two ("bi"). The prefixes may indicate either a positive or a negative power of twelve.

SDN must also provide an easy way to tell in which way the roots are being used; confusing their use as multipliers with their use as powers would be intolerably ambiguous. There is a big difference between a *biquennium* (100 years) and a *biennium* (2 years). A full table of this system can be found in Table 16 on page 8Ƨ.

			Powers	
Value	Root	Multiplier	Positive	Negative
0	nil	nili	nilqua	nilcia
1	un	uni	unqua	uncia
2	bi	bina	biqua	bicia
3	tri	trina	triqua	tricia
4	quad	quadra	quadqua	quadcia
5	pent	penta	pentqua	pentcia
6	hex	hexa	hexqua	hexcia
7	sept	septa	septqua	septcia
8	oct	octa	octqua	octcia
9	enn	ennea	ennqua	enncia
↊	dec	deca	decqua	deccia
↋	lev	leva	levqua	levcia

Table 16: The full Systematic Dozenal Nomenclature system.

Very simply, there are a mere twelve new words that must be learned by the beginner; these are the words in the "Root" column in Table 16. Most of these words will already be quite familiar to most learners, like the *bi-* from *bi*cycle and the *oct-* from *oct*opus.

When we write numbers in digits, we simply line them up according to place value notation; so, e.g., 1 biqua, seven unqua, and four is "174." These roots are combined in precisely the same way. To form prefixes that refer to numbers higher than eleven, we simply combine these roots so that they label each successive digit in the desired number. E.g., to describe a mythical animal with unqua-two (decimal "fourteen") feet, one might use the word "unbipedal" ("un" for the digit "1," "bi" for the digit "2" [completing the desired number, "12"], and "pedal" to show that we're talking about feet). This can extend as far as one cares to extend it; if one were to actually count the legs on an odd (not to say nonexistent) species of millipede, for example, one might find that it was *hexquadoctapedal* (that is, that it has 648 legs); more likely, one would find that it is trinilipedal (has 30 legs).

The astute reader will have noticed, though, that we are not *always* simply cramming the roots together to form these words. Sometimes we take the form of the word from the "Multiplier" column of Table 16 instead of the word from the "Root" column. Take, for example, "hexquadoctapedal"; why is it not simply "hexquadoctpedal?" There are two possible reasons for doing this: first, for euphony, or more simply because *not* doing it makes the resulting word hard to say; and second, to make our meaning clear. We will see how using the root, rather than the multiplier, might create confusion later.

In most cases, however, using the root is perfectly acceptable; this is particularly common for the roots for "two" and "three." Take, for example, "bicycle." "Bicycle" is a perfectly regular use of SDN (indeed, one of SDN's primary strengths is that it leaves such common words untouched while still regularizing our system for speaking about numbers). We don't need to use the multiplier prefix to make this easier to say, and there's no question that when we say "bicycle" we mean a two-wheeled vehicle. Similarly, consider "bipedal," a normal English word that is also perfectly correct SDN; the multiplier prefix is not needed.

However, consider a creature with four legs as a contrary example. In current, standard English we call such creatures "quadrupeds," and say that they are "quadrupedal." "Quadpedal" is very difficult to say, and SDN does not require us to abandon the "-ru-" and use the simple root "quad." So SDN notes that, when using "quad-" to form words, it is sometimes necessary to use the multiplier form rather than the simple root to make the word more easily pronounceable. (Indeed, as we said before, sometimes it's necessary to makes the meaning clear, as well.)

An incredibly wide field is opened up by a regular system of roots and prefixes of this sort; a few examples have already been provided, but they go on nearly forever. Presently, for another example, we have bicycles, tricycles, "four-wheelers," and "eighteen-wheelers." With SDN there is no question how to form appropriate compounds to cover these cases; in addition to bicycles and tricycles, we have quadracycles and unhexacycles ("eighteen" being spelled "16" in dozenal).

Again, presently we have centennials and bicentennials, but we're consistently confused about what to call d̸150th anniversaries and d̸350th birthdays of things. And what do we call a fourtieth wedding anniversary again? SDN makes these questions easy; for dozenal analogs we have unpentnilennials and tripentnilennials, and that fourtieth wedding anniversary (well, four dozenth, anyway) is the quadnilennial (or quadrununquennial; but *not* quadunquennial, which would mean the 10^{40} anniversary, not at all what is meant).

As for the power prefixes, we've been using them consistently throughout this entire book, so their use should be fairly clear by now. Table 4 on page ↊ gives a clear overview of what each one is. However, the principle is simple: to form a positive power prefix, simply add "-qua-" to the normal root prefix; to form a negative power prefix, add "-cia-".

Just as multiplier prefixes may be strung together in place-value form, as for our mythical "hexquadoctapedal" creature, so power prefixes can be so strung. Indeed, Table 4 itself contains an example of such: "bibiqua," meaning 10^{22}. One simply forms the power as normal; in this case, "bibi" for twoqua two (22). Then, one adds the exponential suffix, "-qua," to finish it off. "Bibiqua" can only possibly mean 10^{22}; there is no ambiguity about it. So we have wonderful words like "biquennium" for 100 years and "triquennium" for 1000; perfectly regular, easy to pronounce, and easy to form.

Here, however, we come to a possible source of confusion. How do we form a word for a period of 200 years? One way, the most straightforward, is simply to string together the digits: "binilnilennium." There's certainly nothing wrong with this approach. On the other hand, we might want to say "twice a biquennium," and in this case we must be a bit more careful. We can't simply say "bibiquennium"; as we've already seen, "bibiqua" means 10^{22}, so "bibiquennium" means a period 10^{22} years, quite a bit longer than the period we're trying to express. Here, it's *necessary* that we use the multiplier word rather than the simple root; "binabiquennium" is what we want, an expression which clearly refers to 200 years. (Note that this problem only arises when mixing multiplier and power prefixes; when such mixing is not done, there is no issue.)

Similarly, "binabibiqua" means "twice bibiqua," a very long period indeed. Another example is 10^{63}, the (very) approximate number of atoms in the observable universe. This number is hextriqua: "hex" for the six, "tri" for the three, and "qua" to show that it's

SDN Applied to Polygons and Polyhedra					
	Polygons			Polyhedrons	
Sides/	Systematic Name			Systematic Name	
Faces	Multiplier	Power	Traditional	Multiplier	Power
3	*tri*gon		triangle		
4	*quadra*gon		rectangle	*quadra*hedron	
5	*penta*gon		pentagon	*penta*hedron	
6	*hexa*gon		hexagon	*hexa*hedron	
8	*octa*gon		octagon	*octa*hedron	
↊	*deca*gon		decagon	*deca*hedron	
10	*unnili*gon	*unqua*gon	dodecagon	*unnili*hedron	*unqua*hedron
18	*unocta*gon		icosagon	*unocta*hedron	
20	*binili*gon	*binunqua*gon	tetracosagon	*binili*hedron	*binunqua*hedron

Table 17: Systematic Dozenal Nomenclature applied to polygons and polyhedra.

an exponent. Hex*a*triqua, on the other hand, would be six ("hexa") times triqua (1000), meaning 6000.

When using the multiplier forms results in two vowels being stuck together (e.g., "pentaunqual"), an optional *-n-* can be inserted to prevent these vowels from being elided together. This gives "pentanunqual" rather than simply "pentaunqual." Either is acceptable provided that the vowels are pronounced clearly and separately.

One application of SDN can be found in Table 17 on page 90. The naming of polygons and polyhedra is one area that can be immensely simplified by SDN. Polygon names in particular are extremely irregular in English; we have the tri*angle* and the rect*angle* (also called the quadri*lateral*, and which when regular is called a *square*), but the penta*gon* further up. Furthermore, higher-number polygons become irregular; we have "icosa-" as a prefix for "twenty," a prefix that is rarely used anywhere else. SDN uses the same prefixes that it always does, simply affixing "-gon" for polygons and "-hedron" for polyhedra. Polyhedra in particular are much more complex than this; this table only covers regular polyhedra, specifically the Platonic solids. However, the simplification of the system is clear. (Note that the power-prefix form of a 20-sided polygon, "binunquagon," requires the use of the multiplier form, "bin(a)," and not simply "bi." "Biunquagon" would be a polygon with 10^{21} sides, not at all what is intended.)

Another area in which SDN substantially regularizes things is that of base names. Our current system, such as it is, gives us no regular way of forming base names, and in fact gives us another prefix meaning "twenty" entirely different from that used for polygons (see Table 17). Not to mention that we get a "sen-" prefix meaning "six" and a "non-" prefix meaning "nine" that we rarely see anywhere else. SDN fixes all this, producing a logical system without any such irregularities or inconsistent roots.

Finally, let us briefly consider polynomials; or rather, the names that we apply to polynomials. Some of these names are quite confusing. Consider, for example, the *quadratic*

Radix	SDN Name Multiplier	Power	Traditional Name
2	binal		binary
3	trinal		ternary
4	quadral		quaternary
5	pental		quinary
6	hexal		senary
7	septal		septary
8	octal		octal
9	ennal		nonary
↊	decal		decimal
↋	leval		
10	unnilimal	unqual	duodecimal, dozenal
14	unquadral		hexadecimal
16	unhexal		
18	unoctal		vigesimal
20	binilimal	binunqual	quadravigesimal
50	pentnilimal	pentaunqual	sexagesimal

Table 18: SYSTEMATIC DOZENAL NOMENCLATURE applied to number bases.

equation, of the form $ax^2 + bx + c$. The term "quadratic" comes from the Latin *quadrus*, meaning "square," and such equations are so called because the highest-order exponent in them is a square. However, when we square algebraically rather than geometrically, we no longer have *four* sides (making the name "quadratic" logical), but rather a superscripted *two*, which makes the term "quadratic" confusing. (Many beginners expect it to have a four in it somewhere, not realizing that it really only refers to a square.) SDN solves this problem by using the suffix "-ate," similar to that in quadr*atic*; however, it attaches this to the root expressing the highest-order term in the polynomial. So a quadratic equation, in SDN is called a *binate equation*, because the highest-order exponent is, in fact, a two. This can be extended; e.g., the example of a binate equation given above has only one variable, making it a *univariate* binate equation; one with two variables is instead a *binavariate* binate equation, and one with two variables and a fifth power as its highest-order exponent is a *binavariate pentate* equation. Regularize, simplify; these are the watchwords of SDN.

Thus far the SDN system permits the formation of words for any *integer*, an impressive enough feat on its own. However, SDN can also produce words even for *fractional* values. It can do this in two ways: by simple place notation, and by the use of the fractional affix.

The first of these is the simplest. We have already observed the formation of roots by means of *place notation*; e.g., to form a word for "247," we simply concatenate the appropriate roots: "biquadsepta." By using the Humphrey point (pronounced, in the dozenal world, "dit"), we can extend this to include any other value. We've seen that once every two years can be expressed as "biennially"; now we can say that twice every year (or, rather, once every half-year, or "semiannually") can be expressed as "dithexennially" (since one half, in dozenal, is 0;6, "dit six"). Similarly, publishing once every two weeks (twice a month, twenty-four times a year; also known as "biweekly") could be expressed as "ditnilhexennially" (0;06 of a year) though clearly "biweekly" in this case is the better choice.

This method works wonderfully when the fractional value is *rational*; that is, when it terminates in some limited, and preferably few, number of digits. But what about long or repeating fractions, such as one seventh (in dozenal, $0;\overline{186\text{↊}35}$)? "Ditunocthexdectripentennially" is clumsy enough; the fact that this only covers a single period of the repeating digits makes the option clearly unacceptable.

For this purpose, SDN offers a *fractional affix*, "-per-", which rather than expressing uncials (like "dit") expresses *division*, allowing the formation of arbitrary fractions. So, for example, our problem expressing $^{1}/_{7}$ with the simple "dit" root disappears, for we can simply say "persepta" rather than "septa," which means "one seventh" rather than "seven." Any root to the left of the "per" becomes a numerator, while anything to the right becomes a denominator. E.g., "bipersepta" means "$^{2}/_{7}$," and "quadperpentnil" means "$^{4}/_{50}$."

"Per" can also be used with exponential words. Pergrossages will, of course, replace percents in the dozenal world; these can easily be referred to with "perbiqua." E.g., "$^{37}/_{100}$" can be "threequa seven perbiqua." If one wished to make it entirely one word, one could say that it was "triseptaperbiqua," perhaps to describe the results of a poll (e.g., "Support for the measure was merely triseptaperbiqual"). Which form to use is, of course, up to the speaker.

It should be noted that "per" cannot span multiplier prefixes; it is limited to the roots which are adjacent to it. In other word, *division* (by "per") is *higher precedence* than *multiplication* (by the multiplier forms of SDN roots). Without this rule, expressing a complex unit quantity like "$^{37}/_{89}$ trinaHours" (*tri* (number root) + *na* (multiplier) + *Hours* (unit) is regular SDN for "three-hour period") would be difficult to do unambiguously. "Triseptaperoctennatrina Hours" might mean "$^{37}/_{893}$ Hours" instead, because there's no way to determine where the denominator of the fraction ends. *With* this rule, on the other hand, "triseptaperoctennatrinaHours" is perfectly regular and unambiguous; to express "$^{37}/_{893}$ Hours," we would instead say "triseptaperoctenntrinaHours."

Clearly, the necessity for such constructions will be quite unusual. Nevertheless, SDN must provide for the creation of words describing *all* numbers, and so the constructions are available. We have, then, a system for creating words for any number, integral or fractional, using only twelve roots, two power particles ("qua" and "cia"), a fractional point particle ("dit"), and a reciprocation particle ("per").

SDN is, all in all, an admirably simple system which nevertheless meets all the most complex requirements of a modern and scientific civilization. Its use is encouraged for all.

Appendix C: Further Reading

C.1 On Dozenalism

Andrews, F. Emerson. *An Excursion in Numbers.* THE ATLANTIC MONTHLY 459 (1152), available at http://www.dozenal.org/articles/Excursion.pdf.

_________. *My Love Affair with Dozens.* XI MICHIGAN QUARTERLY REVIEW 2 (1184), available at http://www.dozenal.org/articles/DSA-MyLoveAffair.pdf.

AUCTORES MULTI. 1–50 THE DUODECIMAL BULLETIN. Dozenal Society of America: St. Louis, MO, 1161–11Ɛ7. Available digitally at http://www.dozenal.org/archive/archive.html.

Beard, Ralph. *Why Change?* (1164), available at http://www.dozenal.org/articles/db043r2.pdf.

de Vlieger, Michael. *Symbology Overview* (11Ɛ6), available at http://www.dozenal.org/articles/db4a211.pdf. The definitive work on symbology.

Goodman III, Donald P. *A Primer on Dozenalism* (11Ɛ5), available at http://dgoodmaniii.files.wordpress.com/2009/08/dozprim.pdf.

Malone, James. *Eggsactly a Dozen: A Simple Approach to Duodecimal Counting* (1191), available at http://www.dozenal.org/articles/db2610b.pdf. Probably the best single introduction to dozenal counting; and only a page long.

Schiffman, Jay. *Fundamental Operations in the Duodecimal System* (1192), available at http://www.dozenal.org/articles/db31315.pdf.

SEELY, F. HOWARD. MANUAL OF THE DOZEN SYSTEM. Dozenal Society of America: St. Louis, MO, 1174. Available digitally at http://www.dozenal.org/archive/archive.html.

Zirkel, Gene. *A Brief Introduction to Dozenal Counting* (Dozenal Society of America, 11Ɛ3), available at http://dozenal.org/articles/db38206.pdf.

_________ et al. *Decimal-Dozeal Conversion Rules* (Dozenal Society of America, 11Ɛ1), available at http://www.dozenal.org/articles/DSA-ConversionRules.pdf.

C.2 On TGM

Pendlebury, Tom. *A case for charge, continued.* 1 THE DOZENAL JOURNAL 36 (DSA and DSGB, Winter 1191).

_________. *Music à la Dozen.* 1 THE DOZENAL JOURNAL 36 (DSA and DSGB, Winter 1191). See also *Music à la Dozen* at http://gorpub.freeshell.org/dozenal/blosxom.cgi/dozapp.html#music.

———. TGM. First Edition. Dozenal Society of Great Britain: Denmead, Hants, U.K., available at http://dozenalsociety.org.uk/pdfs/TGMFirstEd.pdf.

———. TGM: A coherent dozenal metrology based on Time Gravity & Mass. Second Edition. Dozenal Society of Great Britain: Denmead, Hants, U.K., 1195, available at http://www.dozenalsociety.org.uk/pdfs/TGMbooklet.pdf; the units appendix is available separately at http://www.dozenalsociety.org.uk/pdfs/TGMUnits.pdf.

Appendix D: Some Common Physical Constants

TGM Values				Metric Values	
Sym.	Description	Value	Units	Value	Units
g	Accel. of Gravity	1	G	9.810049	m/s^2
M	Avogadro's No. (TGM)	1;439↊9197	^{22}Mlz^{-1}	$6.02214129 \times 10^{23}$	mol^{-1}
m_u	Atomic mass unit (mMz)	8;9↊82↋6996	${}_{23}$Mz	$1.660538921 \times 10^{-27}$	kg
Atz	Atmos. Pressure Std.	2↋	Pm	1.015204×10^{5}	N/m^2
R	Gas Const.	1;↋↊7818↊2	PmVm/Cg	8.3144621	J/mol·K
V_m	Normal Gas Volume	1;0↋79410	^{4}Vm/Mlz	2.2413968×10^{-2}	m^3/mol
c	Light Speed (Vacuum)	4↊↋4 9923;07↋↋	Vl	299,792,458	m/s
ly	Lightyear	2;0↋061605	^{13}Gf	9.4605284×10^{15}	m
μ_0	Permeability of Space	2π	${}_{9}$Gn/Gf	$4\pi \times 10^{-7}$	H/m
ϵ_0	Permittivity of Space	↋;490614979	${}_{8}$Kp/Gf	$8.854187817 \times 10^{-12}$	F/m
G	Gravitational Const.	↊;46681	${}_{9}$QMg/Mz^2	6.67384×10^{-11}	N·m^2/kg^2
h	Planck's Const.	1;8↊658↋3↊	${}_{28}$WgTm	$6.62606957 \times 10^{-34}$	J·s
c_1	First Radiation Const.	2;0246↊47↋	${}_{16}$Pv/Sf	$3.74177153 \times 10^{-16}$	W/m^2
c_2	Second Radiation Const.	5;↊0↊0372	^{1}GfCg	1.4387770×10^{-2}	m·K
R_∞	Rydberg's Const.	1;1058↋4↊↋42177	^{6}Gf^{-1}	$1.0973731568539 \times 10^{7}$	m^{-1}
k	Boltzmann's Const.	1;569458↋	${}_{22}$Wg/Cg	$1.3806488 \times 10^{-23}$	J/K
e	Electron's Charge	4;169090 8↊7	${}_{15}$Ql	$1.602176565 \times 10^{-19}$	C
m_e	Electron's Rest Mass	8;44662383	${}_{26}$Mz	$9.10938291 \times 10^{-31}$	kg
$m_e c^2$	Electron's Rest Energy	1;49↊4↋001	${}_{12}$Wg	$8.18710506 \times 10^{-14}$	J
e/m_e	E.'s Mass/Energy Quot.	-5;↋1273829↋	^{10}Ql/Mz	$-1.758820088 \times 10^{11}$	C/kg
r_e	Electron's Radius	1;029↋↋92158	${}_{11}$Gf	$2.8179403267 \times 10^{-15}$	m
ePl	electron-Pel	4;1690908↊7↊↋6	${}_{15}$Wg	$8.712607996978 \times 10^{2}$	eV
Me	Emelectron	5;749799	^{9}Ql	2.585036×10^{4}	faraday
m_p	Proton's Rest Mass	8;↊7↋24↊280	${}_{23}$Mz	$1.672621777 \times 10^{-27}$	kg
$m_p c^2$	Proton's Rest Energy	1;5↊603611↋	${}_{↋}$Wg	$1.503277484 \times 10^{-10}$	J
e/m_p	Pr.'s Mass/Energy Quot.	5;6↊↋65325	^{9}Ql/Mz	9.57883358×10^{7}	C/kg
m_n	Neutron's Rest Mass	8;↊98456520	${}_{23}$Mz	$1.674927351 \times 10^{-27}$	kg
$m_n c^2$	Neutron's Rest Energy	1;5↊96↊5232	${}_{↋}$Wg	$1.505349631 \times 10^{-10}$	J
μ_B	Bohr magneton	9;↊16↊65↋8	${}_{19}$Wg/Fz	$9.27400968 \times 10^{-24}$	J/T
Ci	Curie	1;2↋330766	^{9}Tm^{-1}	3.7×10^{10}	s^{-1}
σ	Stefan-Boltzmann Const.	1;582709	${}_{1↊}$Pv/SfCg^4	5.670373×10^{-8}	W/m^{2}K^4

Table 19: Some Common Physical Constants, in TGM and SI metric

Appendix E: Table of Units

TGM			SI and other equivalents		Conversion				Common Equivalents
Ser. No., Unit	Abbrev.				Dec. Units	Dozenal		Dublog	
Time (symbol t)									1,1 Hour Hr = 1 ^{4}Tm
1. Tim	Tm	$\frac{Day}{200,000}$	0.17361	second	second	5;915343↊1	Tm	2;6390	1,2 Day Dy = 2 ^{5}Tm
					5 min	1	^{3}Tm	3,0;0000	Thirty days = 5 ^{6}Tm
triciaTim	$_3$Tm		0.1004693929	ms	ms	9;↋53326↊4	$_3$Tm	$\bar{3}$,3;3947	1,5 Year = 265 Dy = 5;0↊ ^{7}Tm
									1,5a Leapyear = 5;1 ^{7}Tm
Acceleration (symbol f)									
2. Gee	G	Earth's	9.8100494007	m/s^2	m/s^2	0;12819015	G	$\bar{1}$,0;35↊4	System requirements produced
		gravity	32.18520142	ft/s^2	ft/s^2	0;04583309	G	$\bar{2}$,2;1↋33	a G slightly different to
									the SI g = 9.80665
Length (symbol L or l)									Accel. of grav. = 1 Gf/Tm^2
3. Grafut	Gf	GTm^2	0.2956829126	m	m	3;4701219↊	Gf	1;9117	
			0.9700882959	ft	ft	1;045345↊0	Gf	0;0638	Gf a small foot
			11.6410595508	in	in	0;1045345↊	Gf	$\bar{1}$,0;0638	$_1$Gf a small inch
biciaGrafut	$_2$Gf		2.0533535596	mm	mm	0;5↊16727↋	$_2$Gf	$\bar{1}$,2;6692	$_2$Gf close to 2 mm
hexciaGrafut	$_6$Gf		0.9902360916x10^{-7}	m	μ	↊;12247357	$_6$Gf	$\bar{6}$,3;4049	$_6$Gf close to -1 micron
quadquaGrafut	^{4}Gf		6.1312808754	km	km	1↋5↊;00233↊8	Gf	3,0;↋760	
			3.8098013075	stat. mile	mi	3;1969786	^{3}Gf	3,1;7↊43	
Velocity (symbol v)									
4. Vlos	Vl	$\frac{\text{vel. light}}{4↊↋49923}$	1.7031335765	m/s	m/s	0;7067255↊	Vl	$\bar{1}$,2;9974	Precision root of system =
			5.5877085843	ft/s	ft/s	2;1930041↋	$_1$Vl	$\bar{1}$,1;1295	A comfort. walking pace
			For mph and km/h see quadquaGrafut, e.g. 1 Vl = 6.131..km/h, 1 km/h = 0;1↋5↊ Vl						
Area (symbol A or a)									
5. Surf	Sf	Gf^2	8.7428384796x10^{-2}	m^2	m^2	↋;5308↋881	Sf	3;6233	Sf a small sq ft
			0.9410713018	ft^2	ft^2	1;09025668	Sf	0;1075	^{1}Sf a large sq metre
quadquaSurf	^{4}Sf		0.4479810495	acre	acre	2;29538098	^{4}Sf	4,1;1↊9↊	
			0.1812914987	hectare	ha	5;62373↋49	^{4}Sf	4,2;5692	
hexquaSurf	^{6}Sf		0.2610597581	km^2	km^2	3;9↋72107↋	^{6}Sf	6,1;↋301	
			0.1007957361	mile^2	mile^2	9;↋076↋↊30	^{6}Sf	6,3;3886	ten ^{6}Sf to 1 sq mile
quadciaSurf	$_4$Sf		4.2162608409	mm^2	mm^2	2;↊1↊127↊9	$_5$Sf	$\bar{5}$,1;6136	
Volume or Capacity (symbol V)									
6. Volm	Vm	Gf^3	2.5851079459x10^{-2}	m^3[36]	m^3	3;28244↊22	^{1}Vm	1,1;8320	
			25.8503556494	litre	litre	5;6↊1↊4457	$_2$Vm	$\bar{2}$,2;5898	Vm approx 25 litre

[36]In October 1983 the General Conference on Weights and Measures redefined the metre to derive from the velocity of light. c is now = 299 792 458 m/s exactly. This is a matter of precision only and does not affect the general value. For the same reasons TGM is now precisionized on the

TGM			SI and other equivalents		Conversion				Common Equivalents
Ser. No., Unit	Abbrev.				Dec. Units	Dozenal		Dublog	
			0.9129222555	ft^3	ft^3	1;1189↊583	Vm	1;16↋3	Vm a small cubic foot
			0.033811935989	yd^3	yd^3	2;56↊↊2696	^{1}Vm	1,1;3749	2 cu yd is about 50 Vm
			5.6864459111	Imp. gal.	Imp. gal.	2;13↊69515	$_1$Vm	$\overline{1}$,1;0↋1↊	2 $_1$Vm about mean Imp./US gal.
			6.8290818133	US gal.	US gal.	1;91051389	$_1$Vm	$\overline{1}$,0;9914	
biciaVolm	$_2$Vm		0.3159136617	Imp. pint	Imp. pint	3;1↋9↊22↋1	$_2$Vm	$\overline{2}$,1;7↋47	3 $_2$Vm about av. Imp./US pints
			0.3793934341	US pint	US pint	2;77677↋72	$_2$Vm	$\overline{2}$,1;4942	
quadciaVolm	$_4$Vm		1.2466413797	ml	ml	9;7615↊↊↋↊	$_5$Vm	$\overline{5}$,3;3253	5 ml = 4;017 quadciaVolm
Mass (symbol m)									
7. Maz	Mz	VmDz	25.8503556494	kg	kg	5;6↊1↊4457	$_2$Mz	$\overline{2}$,2;5898	4 Mz is a little over 100 kg.
			56.9902828681	lb	lb	2;63↊2748↊	$_2$Mz	$\overline{2}$,1;406↊	1 lb over half cwt
biquaMaz	^{2}Mz		3.7224512135	tonne	tonne	3;28263384	^{1}Mz	1,1;8321	100 000 lb = 1;02 ^{3}Mz
triciaMaz	$_3$Mz		0.5276878043	oz	oz	1;↊8↊7↋668	$_3$Mz	$\overline{3}$,0;↋098	$_3$Mz just over half ounce
quadciaMaz	$_4$Mz		1.2466413797	gm	gm	9;7615↊↊↋↊	$_5$Mz	$\overline{5}$,3;3253	5 gm is virtually 4 $_4$Mz
Density (symbol d)									
8. Denz	Dz	Max. dens. of water at 2↋ Prem	999.9720008	kg/m^3	kg/dm^3	1;00005↋73	Dz	0;0000	
Velocity squared (symbol v^2)									
9. Vlov	Vv	Vl^2	2.9006639794	m^2/s^2	m^2/s^2	0;417885↋7	Vv	$\overline{1}$,2;06↋↋	
Momentum (symbol mv)									
↊. Mav	Mv	MzVl	44.0266086711	kg m/s	kg m/s	3;32↋↊3114	$_2$Mv	$\overline{2}$,1;8623	
			318.4450928058	lb ft/s	lb ft/s	5;5149196↋	$_3$Mv	$\overline{3}$,2;5344	
Force (symbol F)									
↋. Mag	Mg	MzG	253.5932659458	newton	N	6;9928453↊	$_3$Mg	$\overline{3}$,2;9280	4 Mag is 1.0144 kilonewton.
			25.8593164787	kgf	kgf	5;69↊64071	$_2$Mg	$\overline{2}$,2;588↊	Figures differ from those
			57.0100381234	lbf	lbf	2;63885635	$_2$Mg	$\overline{2}$,1;405↋	for mass due to variance in standards of g.
Tension (symbol F_{tens})									
10. Tenz	Tz	Mg/Gf	857.652759595	N/m	N/m	2;0216↋099	$_3$Tz	$\overline{3}$,1;0165	
quadciaTenz	$_4$Tz		41.360569039	dyn/cm	dyn/cm	3;5941↋896	$_6$Tz	$\overline{6}$,1;971↋	
Pressure, *Stress* (symbol p)									
11. Prem	Pm	Mg/Sf	2900.582763	N/m^2	N/m^2	7;19539173	$_4$Pm	$\overline{4}$,2;↊077	
			0.4206939918	lb/in^2	lb/in^2	2;4635↊97↊	Pm	1;2↋↊7	
			21.76	mmHg	mmHg	6;75	$_2$Pm	$\overline{2}$,2;8871	Based on density Hg = 11;71
			0.85669	inHg	inHg	1;2011	Pm	0;2817	(13.59)

velocity of light, see Vlos. Present standards also allow the word "litre" to be used as a general synonym of "cubic decimetre", but the kilogramme has remained unaltered. 1 kg of pure water at maximum density still occupies 1.000028 dm^3. Tihs discrepancy is precluded from the TGM Denz and related units, and accounts for apparent discrepancies between mass and non-litre volume figures. The Maz is defined as the mass of 1 Volm of pure water at maximum density.

TGM			SI and other equivalents		Conversion				Common Equivalents
Ser. No., Unit	Abbrev.				Dec. Units	Dozenal		Dublog	
quadquaPrem	^{4}Pm		60.1464841756x10^6	N/m^2	MN/m^2	2;4891242↊	^{2}Pm	2,1;3145	
			3.8944243812	ton/in^2	ton/in^2	3;0↋86506↋6	^{3}Pm	3,1;7596	
11,1 Atmoz	Atz	2↋ Pm	1.015204x10^5	N/m^2	Atm.	0;↋↋88	Atz	3;6↋↊0	Standard atmos.: Atz TGM. Atm SI = 1.01325x10^5 N/m^2
Work or Energy, *Heat*, etc. (symbol W) (H, e or E not recommended)									
12. Werg	Wg	MgGf	74.9831954874	J	J	1;↋0660616	$_2$Wg	$\overline{2}$,0;↋369	Wg is 75 J. J is 2 $_2$Wg.
			55.3047707318	ft-lbf	ft-lbf	2;72↋35262	$_2$Wg	$\overline{2}$,1;4698	
			17.9149911570	cal 15°	cal 15°	8;05571↊↊↊	$_2$Wg	$\overline{2}$,3;00↋↊	
			0.0711051221	BTU	BTU	12;09206882	Wg	1,0;2817	
pentquaWerg	^{5}Wg		5.1828384721	kWh	kWh	2;394↊9107	^{4}Wg	4,1;0650	
12,1 electron-Pel	ePl		1.3959244694x10^{-16}	J	Jx10^{-15}	7;1↋6↋↊569	ePl	2;↊109	See also page 48.
Power (symbol P)									
13. Pov	Pv	Wg/Tm	431.9032060077	J/s or W	W	4;00167091	$_3$Pv	$\overline{3}$,2;0007	The biciaPov is 3 watts
			318.5554794154	ft-lbf/s	ft-lbf/s	5;51161↊↊3	$_3$Pv	$\overline{3}$,2;5335	
			0.5789587212	HP	HP	1;88880371	Pv	0;9566	
Viscosity (symbols, dynamic ?, kinematic v)									
14. Viscod	Vsd	MgTm/Sf	503.5733963715	N s/m^2	N s/m^2	3;52171036	$_3$Vsd	$\overline{3}$,1;941↊	
15. Viskin	Vsn	Sf/Tm	0.50358749643	m^2/s	m^2/s	1;↋9↋46851	Vsn	0;↋↊62	
Note: MgTm/Sf = (Gf^{3}DzGf/Tm^2) x Tm/Gf^2 = SfDz/Tm, so 1 Viscod = 1 Viskin x 1 Denz									
Plane Angle (symbol θ)									
16.a Radian	rad	Arc =							
or radiFut	rGf	radius	57°.2957795131	degree	degree	2;61↊↋303↊	$_2$rGf	$\overline{2}$,1;3↋56	
16.b Pi	Pi	π radians	180° or semicircle		15°	1 unciaPi			
biciaPi	$_2$Pi		1°15'		1°	0;97249725	$_2$Pi	$\overline{3}$,3;31↊6	4 $_2$Pi = 5°
triciaPi	$_3$Pi		6'15"		1'	1;↋0591534	$_4$Pi	$\overline{4}$,0;↋363	4 $_3$Pi = 25'
quadciaPi	$_4$Pi		31".25		1"	4;73675↊33	$_6$Pi	$\overline{6}$,2;2549	4 $_4$Pi = 125"
pentciaPi	$_5$Pi		2".60416						
Solid Angle									
17. Steradian	Sr	$\frac{Surf}{Gfrad.^2}$	$\frac{m^2}{mradius^2}$						Total solid angle of sphere is 4π qSf
quariSurf	qSf								
Angular Velocity (symbol ω)									
18. radiVlos	rVl	rGf/Tm	5.76	rad/s	rad/s	2;1	$_1$rVl	$\overline{1}$,1;0859	unciaradiVlos = Vl(cricumf.)/Gf(rad.)/zen
Angular Acceleration (symbol α)									
19. radiGee	rG	rGf/Tm^2	33.1776	rad/s^2	rad/s^2	4;41	$_2$rG	$\overline{2}$,2;14↋6	Accel. of frictionless windlass (rad 1Gf) loaded then released
Frequency or Revolution (symbol f)									
1↊. *Freq	Fq	2Pi/Tm	5.76	c/s or Hz	Hz	2;1	$_1$Fq	$\overline{1}$,1;0859	50Hz=8;82Fq; 144Hz=21Fq
*Cim in earlier edition			345.6	RPM	RPM	5	$_3$Fq	$\overline{3}$,2;3↊44	60Hz=↊;5Fq; 300Hz=44;1Fq
triquaFreq	^{3}Fq		9.95328	kHz	10 kHz	1;00814	^{3}Fq	3,0;00↋8	100Hz=15;44Fq; 1kHz= 125;74Fq; 1curie=1;3$_3$Fq

TGM			SI and other equivalents		Conversion				Common Equivalents
Ser. No., Unit	Abbrev.				Dec. Units	Dozenal		Dublog	
Angular Momentum, *Moment of Momentum* (symbol L)									
1Ɛ. radaMav	RMv	Mv x Gf(rad)	13.0179158832	kg m^2/s	kg m^2/s	Ɛ;08↊6Ɛ6Ɛ6	$_2$RMv	$\overline{2}$,3;573↊	
Torque, *Turning Moment*, *Moment of Force* (symbol T)									
20. radaMag	RMg	Mg x Gf(rad)	74.9831954874	Nm	Nm	1;Ɛ0660616	$_2$RMg	$\overline{2}$,0;Ɛ369	
Moment of Inertia, *Mass in rotation* (symbol I)									
21. quaraMaz	QMz	RMg/rG	2.2600548408	kg m^2	kg m^2	5;386ƐƐ970	$_1$QMz	$\overline{1}$,2;4↊↊1	
Work or Energy in rotation (symbol W)— RMg x rGf = MgGf = Werg (see unit 12)									
Centripetal Acceleration = v^2/r, i.e., radiVlov, = G x rGf, = G for radion (rGf) = l. (see unit 2)									
Centripetal or Centrifugal Force—Mag (x radian). (see unit Ɛ)									
Electric Current (symol I, or i for instantaneous value)									
22. Kur	Kr	$\sqrt{\text{Mg/Mb}}$	0.4957220687	amp	A	2;02597↊834	Kr	1;0195	Two to the amp
pentciaKur	$_5$Kr		1.9921958137	μA	μA	6;03474748	$_6$Kr	$\overline{6}$,2;7107	pentciaKur is two microamps
quadquaKur	^4Kr		10.2792928167	kA	kA	1;2013142↊	^{3}Kr	3,0;281↊	Ten and a bit kA to the ^{4}Kr
Magneto-Motive Force (symbol NI, N = No. of turns. If electron path is straight magnetic flux is circular, and vice versa)									
23. Kurn	Kn	KrN	0.4957220687	AT	AT	2;02597↊834	Kn	1;0195	
Electro-Motive Force, *Potential* (symbol E, or e for instantaneous value)									Kr x Pl = Pv
24. Pel	Pl	Vl$\sqrt{\text{MgMb}}$	871.2607996978	volt	kV	1;1933Ɛ↊↊1	Pl	0;2477	7 kV is 8 Pl
unquaPel	^{1}Pl		10.4551295964	kV	10 kV	0;Ɛ5893Ɛ05	^{1}Pl	3;62Ɛ↊	unquaPel is just over ten kV
biquaPel	^{2}Pl		125.4615551565	kV	MV	7;Ɛ79187Ɛ2	^{2}Pl	2,2;ƐƐ2↊	biquaPels 8 to the megavolt
unciaPel	$_1$Pl		72.6050666415	V	100V	1;463ƐƐ↊73	$_1$Pl	$\overline{1}$,0;5661	240V is virtually 340 $_3$Pl
biciaPel	$_2$Pl		6.0504222201	V	10V	1;79ƐƐƐ↊3Ɛ	$_2$Pl	$\overline{2}$,0;8847	110V is virtually 160 $_3$Pl
triciaPel	$_3$Pl		0.5042018517	V	V	1;Ɛ9724771	$_3$Pl	$\overline{3}$,0;Ɛ↊32	Two to the volt
hexciaPel	$_6$Pl		0.2917834790	mV	mV	3;516248↊9	$_6$Pl	$\overline{6}$,1;93↊8	
octciaPel	$_8$Pl		2.0262741596	μV	μV	0;5Ɛ096893	$_8$Pl	$\overline{9}$,2;6963	Microvolts two to the octciaPel
The same figures apply for conversion of the energy units electron-Pel and electron-volt									
Resistance (R), *Reactance* (X), *Impedance* (Z)									Og x Kp = Tm
25. Og	Og	Pl/Kr	1757.55903297	Ohm	kilohm	6;9Ɛ222121	$_1$Og	$\overline{1}$,2;9310	4 Og is 7 kilohm
triquaOg	^{3}Og		3.0370620090	Megohm	Megohm	3;Ɛ4Ɛ79992↊	^{2}Og	2,1;Ɛ955	Megohm is 4 biquaOg
triciaOg	$_3$Og		1.0171059219	Ohm	Ohm	0;Ɛ96Ɛ33105	$_3$Og	$\overline{4}$,3;6886	The ohm is a triciaOg
Conductance (G), *Susceptance* (B), *Admittance* (Y)									
26. Go	Go	Kr/Pl	568.9709314098	μMho	mMho	1;91108Ɛ1↊	Go	0;991↊	
triquaGo	^{3}Go		0.9831817695	Mho	Mho	1;02568601	^{3}Go	3,0;0363	The mho is a triquaGo
Quantity or Charge (symbol Q)									
27. Quel	Ql	KrTm	86.0628591502	mCoulomb	mC	1;80Ɛ34748	$_2$Ql	$\overline{2}$,0;8↊Ɛ3	
unquaQuel	^{1}Ql		1.0327543098	Coulomb	C	0;Ɛ7524207	^{1}Ql	3;6566	The coulomb is a unquaQuel
Capacity (symbol C)									
28. Kap	Kp	Ql/Pl	98.7796755920	μFarad	Farad	5;↊3765921	^{3}Kp	3,2;6734	The Kap is virtually 100 μF
unciaKap	$_1$Kp		8.2316396327	μF	μF	1;55Ɛ088↊Ɛ	$_2$Kp	$\overline{2}$,0;6638	
pentciaKap	$_5$Kp		0.3969733619	nF	nF	2;628Ɛ2↊Ɛ↊	$_5$Kp	$\overline{5}$,1;3ƐƐ3	The pentciaKap is 400 picofarad
octciaKap	$_8$Kp		0.2297299548	pF	pF	4;4297↊5ƐƐ8	$_8$Kp	$\overline{8}$,2;156↊	
Permittivity (symbol ϵ)									

Ser. No., Unit	Abbrev. (TGM)		SI and other equivalents		Dec. Units	Dozenal		Dublog	Common Equivalents
29. Mit	Mt	1/MbVv	334.0729930165	μF/m	μF/m	5;20↊15093	$_3$Mt	$\bar{3}$,2;454↊	
Magnetic Flux (symbol Φ, or ϕ for instantaneous value)									
2↊. Flum	Fm	MbKrGf	151.2605555031	weber	Wb	0;↋5107↋98	$_2$Fm	$\bar{3}$,3;6202	Weber just under the biciaFlum
hexciaFlum	$_6$Fm		50.6568539895	μWb	100μWb	1;↋8322↋0↋	$_6$Fm	$\bar{6}$,0;↋936	
Magnetic Flux Density (symbol B)									
2↋. Flenz	Fz	Fm/Sf	1730.1080862416	Tesla	T(=Wb/m^2)	0;↋↋9↊8980	$_3$Fz	$\bar{4}$,3;6↋↋9	The tesla is a triciaFlenz
triciaFlenz	$_3$Fz		1.0012199573	T	mT	1;88641939	$_6$Fz	$\bar{6}$,0;9546	
octciaFlenz	$_8$Fz		4.0236784550	μT	μT	2;↋955↋053	$_9$Fz	$\bar{9}$,1;6↋01	$_8$Fz is 4 μT, μFz is 3 $_9$Fz
Inductance, Self- (L) and *Mutual-* (M), generation of emf									
30. Gen	Gn	MbGf	305.1317765583	henry	H	5;7↋5↊7271	$_3$Gn	$\bar{3}$,2;6029	
hexciaGen	$_6$Gn		0.1021880146	mH	mH	9;95200↋47	$_6$Gn	$\bar{6}$,3;35↊4	hexciaGens ten to the millihenry
Reluctance (symbol S)									
31. Lukt	Lk	Kn/Fm	0.0032772725649	AT/Wb	AT/Wb	2;1516↋863	^{2}Lk	2,1;1001	
Permeability (symbol μ)									Basis of electrical units.
32. Meab	Mb	Ennua($\frac{\mu_0}{2\pi}$)	1031.9560704exact = 2x10^{-7}/*10^{-9}	H/m	H/m	1;811628exact =10^{-9}/2x10^{-7}	$_3$Mb	$\bar{3}$,0;8↋12	
Power Density, Intensity (symbol I)									
33. Penz	Pz	Pv/Sf	4.9400798953	kW/m^2	kW/m^2	2;51960515	$_1$Pz	$\bar{1}$,1;2447	The Penz is 5 kW/m^2
Radiant Power (point source)									
34. QuaraPenz	QPz	Pv/qSf	431.9032060077	W/Sr	W/Sr	4;00167091	$_3$Pr	$\bar{3}$,2;0007	For other factors see Unit 13 Called "Prad" in earlier edition
Light Power, *Luminous Flux* (symbol F)									
35. Lypov	Lp	See page 6↊	1.1795967512	lumen	lm	↊;20↊↊7↋81	$_1$Lp	$\bar{1}$,3;41↋1	1 pentciaPov for vis.factor 1
Illumination (E), *Luminance or Brightness* (L or B)									Ld = 1.25 lum/ft^2
36. Lyde	Ld	Lp/Sf	13.4921485051	lm/m^2	lm/m^2	↊;80↊88694	$_2$Ld	$\bar{2}$,3;4↋↊8	$_1$Ld = 1.124 lum/m^2; See p 34
Light Sensitivity									Supersedes the logarithmic
37. Senz	Sz	1/LdTm	0.4269149571	m^2/lm s	m^2/lm s	2;41378995	Sz	1;289↋	unit in earlier issue.
Luminous Intensity (symbol I)									
38. QuaraLyde	QLd	Lp/qSf	1.1795967512	candela or lm/Sr	cd, lm/Sr	↊;20↊↊7↋81	$_1$Lr	$\bar{1}$,3;41↋1	Called "Luprad" in earlier edition.
Temperature (symbol T, θ)									
39. Calg	Cg								
biquaCalg			0.1 exactly	kelvin	K	↊	^{2}Cg		
The following are supplementary to the earlier edition. Gaps in serial nos. are to allow for classified expansion.									
Heat Capacity (symbol C), *Entropy* (symbol S)									
3↊. Calkap	Ck	Wg/Cg	107.9758015019	kJ/K	kJ/K	1;40062430	$_2$Ck	$\bar{2}$,0;4↋99	kJ/K are 9 to the unciaCalkap
pentciaCalkap	$_5$Ck		0.4339305294	J/K	J/K	2;37↊254↋5	$_5$Ck	$\bar{5}$,1;2554	
Specific Heat Capacity (symbols C_P, C_V), *Specific Entropy*									
3↋. Calsp	Csp	Wg/CgMz	4.1769561304	kJ/K kg	kJ/K kg	2;↊5846983	$_1$Csp	$\bar{1}$,1;632↊	Min Sp.H.water = 4.1779 kJ/K kg
quadciaCalsp	$_4$Csp		0.2014349986	J/K kg	J/K kg	4;↋6↊54908	$_4$Csp	$\bar{4}$,2;38↊6	5 to the quadciaCalsp
Thermal Conductivity (symbol k)									

TGM			SI and other equivalents		Conversion				Common Equivalents
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42. Caldu	Cdu	Wg/Gf/SfCg	2.1034039851	MW/m K	kW/m K	9;↊3719↋98	$_4$Cdu	$\overline{4}$,3;3749	
quadciaCaldu	$_4$Cdu		101.4373063783	W/m K	W/m K	1;50508↋14	$_6$Cdu	$\overline{2}$,0;6096	
Temperature Gradient (symbol dθ/dx)									
49. Temgra	Tgr	Cg/Gf	2.3486120262	mK/m	K/m	2;↋5949856	^{2}Tgr	2,1;6928	
biquaTemgra	^{2}Tgr		0.3382001318	K/m	K/cm	2;0782407↊6	^{4}Tgr	4,1;0558	
quadquaTemgra	^{4}Tgr		48.7008189752	K/m	K/mm	1;8649↋489	^{5}Tgr	5,0;9371	
Specific Energy, *Specific Latent Heat* (symbol L)									
4↋. Wesp	Wsp	Wg/Mz	2.9006639794	J/kg	J/kg	4;17885↋66	$_1$Wsp	$\overline{1}$,2;06↋↋	1 Roentgen=4;↋↊↊$_3$Wsp
quadquaWesp	^{4}Wsp		60.1481682772	kJ/kg	kJ/kg	2;488↋980↊	^{2}Wsp	2,1;3144	
hexquaWesp	^{6}Wsp		8.6613362319	kJ/g	kJ/g	1;476107↊1	^{5}Wsp	5,0;578↊	
Flow									
53. Flo	Fl	Vm/Tm	0.1489022177	m^3/s	m^3/s	6;870↋2126	Flo	2;8↋79	
			148.89804854	litre/s	li/s	↋;731↊7139	$_3$Flo	$\overline{3}$,3;6535	biciaFlo is a litre per sec.
			5.2584321917	ft^3/s	ft^3/s	0;2347469↊	Flo	$\overline{1}$,1;234↊	
			8.9341330612	m^3/min	m^3/min	1;414↋↊02↋	$_1$Flo	$\overline{1}$,0;5136	
biciaFlo	$_2$Flo		2.1910134132	ft^3/min	ft^3/min	5;58814451	$_3$Flo	$\overline{3}$,2;5535	
			13.6474701869	Impg./min	Impg./min	↊;6749↋590	$_4$Flo	$\overline{4}$,3;4970	
			16.3897963520	USgal/min	USgal/min	8;95216679	$_4$Flo	$\overline{4}$,3;1757	
Sound Intensity, *Loudness*									
56. Zond	Zd	[37]	4.095558148x10^{-10}	(N/m^2)^2	(N/m^{2}x10^{-5})^2	2;↋1↋06795	$_1$Zd	$\overline{1}$,1;6740	
Specific Volume									
58. Vosp	Vsp	Vm/Mz	1.000028x10^{-3}	m^3/kg	m^3/kg	6;↋3↋7↋74↋	^{2}Vsp	2,2;9672	Conversion figures are simply the SI litre/m^3 discrepancy
Activity									
5↊. Ag	Ag	Mv/Gf	13.0179158832	kg m^2/s	kg m^2/s	↋;08↊6↋6↋6	$_2$Ag	$\overline{2}$,3;573↊	Ag=MvGf=WgTm=MzSf/Tm
Electric Dipole Moment (symbol P$_e$									
60. radaQuel	RQl	QlxGf(rad)	2.5447316859x10^{-2}	Cm	Cm	3;3368↋↋86	^{1}RQl	1,1;8653	
Resistivity (symbol ρ)									
61. Rezy	Ry	OgSf/Gf	519.6801739150	ωm^2/m	ωm^2/m	3;3↊998926	$_3$Ry	$\overline{3}$,1;8974	Though arithmetically Sf/Gf=Gf
quadciaRezy	$_4$Ry		2.5061736782	ωcm^2/cm	ωcm^2/cm	4;955↋7418	$_5$Ry	$\overline{5}$,2;3144	here it means cross sect/length
Conductivity (symbol σ)									
62. Eldu	Edu	GoGf/Sf	0.00192426044	mho m/m^2	mho m/m^2	3;7381↋410	^{2}Edu	2,1;1276	
quadquaEldu	^{4}Edu		0.3990146448	mho cm/cm^2	mho cm/cm^2	2;60↊80261	^{4}Edu	4,1;3↊↊5	
Ionic Mobility (symbol u$_i$									
64. Imo	Im	Vl/Egr	0.000577998570	m^2/V s	m^2/V s	1;00213693	^{3}Im	3,0;0030	triquaImo is sq m per volt-sec

[37]The loudness of a sound of frequency 100 Freq (829.4 Hz) and root mean square pressure of 3 octciaPrem (2 x 10^{-5} N/m^2). Called the Threshold of Hearing, it is the softest sound just audible to those of average hearing. Vibration intensity is proportional to the square of the pressure. Sound intensity or loudness varies also with frequency, but hardly at all from about 800 Hz to 1400 Hz which spans both TGM reference (=829.44 Hz) and the SI reference 1000 Hz.

TGM			SI and other equivalents		Conversion				Common Equivalents
Ser. No., Unit	Abbrev.				Dec. Units	Dozenal		Dublog	
			5.7799857012	cm^2/V s	cm^2/V s	2;0↊↋6760↋	$_1$Im	$\bar{1}$,1;0792	
Electric Flux Density (symbol D)									
66. Quenz	Qz	Ql/Sf	0.9843812093	C/m^2	C/m^2	1;02350153	Qz	0;0333	Quenz is a coulomb per sq m
Electrochemical Equivalence (symbol z)									
68. Depoz	Dp	Mz/Ql	300.365987195	kg/C	kg/C	5;90519↋05	$_3$Dp	$\bar{3}$,2;6360	
hexciaDepoz	$_6$Dp		0.10059196138	g/C	dg/C	0;↋↋19↋819	$_6$Dp	$\bar{7}$,3;6↋01	hexciaDepoz is a gram/coulomb
Potential Gradient (symbol dE/dz)									
69. Elgra	Egr	PlGf	2.9466051727	kV/m	kV/m	4;07530174	$_1$Egr	$\bar{1}$,2;0389	
quadciaElgra	$_4$Egr		0.1421009489	V/m	V/m	7;05445283	$_4$Egr	$\bar{4}$,2;9944	
Magnetic Moment (symbol T)									
70. radaFlum	RFm	FmxGf(rad)	44.7251616108	Wb m	Wb m	3;2776↋581	$_2$RFm	$\bar{2}$,1;82↋0	
Magnetic Field Strength or Gradient (symbol H)									
79. Magra	Mgr	Kn/Gf	1.6765326896	AT/m	AT/m	7;1↊847356	$_1$Mgr	$\bar{1}$,2;↊0↊7	
Light Quantity									
81. Lyqua	Lq	LpTm	0.2047911026	lum s	lum s	4;↊71↊4954	Lq	2;3553	
Wave Number (symbol v), *Lens Power* (f^{-2})									
83. Perfut	PGf	1/Gf	3.3820013177	diopt.,m^{-1}	diopt.,m^{-1}	3;66↋34543	$_1$PGf	$\bar{1}$,1;9↋12	
Light Efficiency									
85. Lytef	Lf	Lp/Pv	2.7311599794x10^{-3}	lum/W	lum/W	2;6618↊247	^{2}Lf	2,1;41↊6	
pentquaLytef	^{5}Lf		679.6	lum/W					See page 34
Specific Optical Rotation (symbol α)									
8↋. Orosp	Osp	rGf/GfDz	0.0033820960138	rad m^2/kg	rad m^2/kg	2;07811925	^{2}Osp	2,1;0557	Metric m^2/kg = /m(kg/m^3)
			0.33820960138	rad dm^2kg	rad dm^2/kg	2;↋5931131	Osp	1;6927	*dm^2/kg=/dm(g/cm^3)*
Amount of Substance									
90. Molz	Mlz	M items	25.8503556494	kilomoles	kmol	5;6↊71↊4457	$_2$Mlz	$\bar{2}$,2;5898	
quadciaMolz	$_4$Mlz		1.2466413797	mol	mol	9;76152↊↊↋	$_5$Mlz	$\bar{5}$,3;3253	
Molzar Extinction or Absorption (symbol ϵ)									
91. Surfolz	Slz	Vlz/Gf	3.3820960138	mm^2/mol	mm^2/mol	3;66↋13886	$_1$Slz	$\bar{1}$,1;9↋12	
biquaSurfolz	^{2}Slz		4.8702182598	cm^2/mol	cm^2/mol	2;56986↋07	^{1}Slz	1,1;3741	
quadquaSurfolz	^{4}Slz		0.7013114294	dm^3/mol cm	dm^3/mol cm	1;513↋5688	^{4}Slz	4,0;6186	
hexquaSurfolz	^{6}Slz		10.0988845836	m^2/mol	m^2/mol	1;23136773	^{5}Slz	5,0;2↋↊0	
Molzar Volume, *Molzar Refraction* (symbols V_m, R_m)									
92. Volmolz	Vlz	Vm/Mlz	1.000028	cm^3/mol	cm^3/mol	0;↋↋↋↋5049	Vlz	$\bar{1}$,3;7029	
hexquaVolmolz	^{6}Vlz		2.9860676076	m^3/mol	m^3/mol	4;0283	^{5}Vlz	5,2;0017	
Molvity (Molarity) (Recommended symbol M_v)									
93. Molv	Mlv	Mlz/Vm	999.9720007840	mol/dm^3	mol/dm^3	1;88↊0989↋	$_3$Mlv	$\bar{3}$,0;9578	
Molmity (Molality) (Recommended symbol M_m)									
94. Molm	Mlm	Mlz/Mz	1000.0	mol/kg	mol/kg	1;889↋9843	$_3$Mlm	$\bar{3}$,0;9577	
Molzar Enthalpy (symbol H)									
95. Wergolz	Wlz	Wg/Mlz	2.9006639794	J/kmol	J/kmol	4;17885↋67	$_1$Wlz	$\bar{1}$,2;06↋↋	
quadquaWergolz	^{4}Wlz		60.1481682772	J/mol	J/mol	2;4888↋980↊	^{2}Wlz	2,1;3144	

TGM			SI and other equivalents		Conversion				Common Equivalents
Ser. No., Unit	Abbrev.				Dec. Units	Dozenal		Dublog	
hexquaWergolz	^{6}Wlz		8.6613362319	kJ/mol	kJ/mol	1;476107↊1	^{5}Wlz	5,0;578↊	
Molzar Conductivity (symbol Λ)									
96. Eldulz	Eul	GoSf/Mlz	0.01924314319	S cm^2/kmol	S cm^2/kmol	4;3↋722778	^{1}Eul	1,2;145↋	
quadquaEldulz	^{4}Eul		0.3990258172	S cm^2/mol	S cm^2/mol	2;60↊6688↋	^{4}Eul	4,1;3↊↊4	
octquaEldulz	^{8}Eul		0.8274199345	S m^2/mol	S m^2/mol	1;26050507	^{8}Eul	8,0;3343	
Molzar Entropy (symbol Sm)									
97. Calgolz	Clz	Wg/CgMlz	4.1769561304	J/K mol	J/K mol	2;↊5846983	$_{1}$Clz	$\bar{1}$,1;632↋	
Molzar Optical Rotation (symbol *alpha*m)									
9↋. Orolz	Olz	rGfVlz/Gf	3.3820960138	rad mm^2/mol	rad mm^2/mol	3;66↋13886	$_{1}$Olz	$\bar{1}$,1;9↋12	
hexquaOrolz	^{6}Olz		10.0988845836	rad m^2/mol	rad m^2/mol	1;23136773	^{5}Olz	5,0;2↋↊0	

Auxiliary Units

These are of kinds already covered by the main system but deviate by factors to make them "handy" or suitable for special applications. They are given on a take or leave basis. A serial number implies the unit is deemed essential (usually due to some inescapable natural reality).

	Hour (Hr)	=	1 0000	Tim			
	Block	=	1000	Tim			
	Bictic	=	100	Tim			
	Unctic	=	10	Tim			
1,1	*Day* (Dy)	=	20	hours	=	2	pentquaTim
	Week	=	7	Dy	=	1;2	hexquaTim
	Lunar month	=	24	Dy	=	4;8	hexquaTim
	Minor month	=	26	Dy	=	5	hexquaTim
	Major month	=	27	Dy	=	5;2	hexquaTim
	unciaYear ($_{1}$Yr)	=	26;52↊↊629	Dy	=	5;0↊599057	hexquaTim
1,2	*Astronomical Year* (Yr)	=	265;2↊↊629	Dy	=	5;0↊599057456	septquaTim
		=	365.242199	Dy	=	31556925.9747	seconds
	(Minor) year	=	265	Dy	=	5;0↊	septquaTim
	Leap year	=	266	Dy	=	5;1	septquaTim
	Quadrennium[38]	=	↊19	Dy	=	1;836	octquaTim
	Unquennium[39]	=	2;653	triquaDay	=	5;0↊6	octquaTim
	unquaYear (^{1}Yr)	=	2;652↊↊629	triquaDay	=	5;0↊599057	akaTim
	(Major) century[3↊]	=	8;4	unquennium	=	3;6336	ennquaTim

[38] The quadrennium is a unit consisting of three normal years plus one leap year.

[39] The unquennium is a unit consisting of three quadrennia. Note: The unquennium is always the same length no matter at what point of time the counting starts, unlike the decade which contains sometimes two, sometimes three leap years. Exception: when it spans a dropped leap day at the turn of the century.

[3↊] A major century also equals 21 quadrennium and 1;9179 quadquaDays.

(Minor) century[3Ɛ]	=	1;9178	quadquaDay	=	3;6335ƐƧ	ennquaTim
biquaYear ^{2}Yr	=	2;652ƧƧ629	quadquaDay	=	5;0Ƨ599057	ennquaTim

A handy way to express date and time as an absolute measure, is to follow the normal way of counting anything else, putting larger units first, and counting only completed units, e.g. 9-45 p.m. on June 25th 1984 is 17c6Ɛy5m20d19;9h—meaning onezen-seven centuries + sixzen-elv years + 5 months + twozen days + 19;9Hr.

Gravinch or Unch	=	1	$_1$Gf	=	0;Ɛ783	in
Gravpalm	=	3	$_1$Gf	=	2;Ƨ3Ɛ0	in
Gravhand	=	4	$_1$Gf	=	3;Ƨ693	in
Gravyard or Trifut (TGM yard)	=	3	Gf	=	0;Ɛ783	yd
unquaTrifut	=	30	Gf	=	10.6446	m
triquaTrifut or Gravmile (TGM mile)	=	3000	Gf	=	1676.2	yd[40]
binatriquaGrafut or Gravklick	=	2	^{3}Gf	=	1;0319	km
Link	=	0;6	Gf	=	5.82	in[41]
Chenz (TGM chain)	=	100	Links[42]	=	23.282	yd[43]
Nafut (Nf) (navigation)	=	0;Ɛ4205939[44]	Gf	=	0.91734	ft[45]

4 septquaNafut = 1 Equator. 2 hexquaNafut = 1 unciaPi (15°) of longitude at Equator.

3,1	Astru (Au)	Astronomical Unit, = mean distance sun-Earth	
		= 8;207Ƨ420 decquaGrafut ($1.4959787\text{x}10^{11}$ m)	
	Parallax	e.g. of 1 hexciaRadian based on 1 Astru = 1 hexciaAstru	
	Traditional parsec	= 9Ɛ449 Astru (206 265, i.e. no. of secs in a radian)	
3,2	Lightyear (Ly)	(distance traveled by light in 1 year)	
		= 2;0Ɛ061606 untriquaGrafut ($9.460528\text{x}10^{15}$ m)	
	Lighttim (Ltm)	=	4ƧƐ49923 Grafut exactly
	Navlos	= 1 Nf/Tm	= 0;Ɛ4205939 Vl (5.797893518 km/hr = 3.130612 knot(int)
4,1	Lightvlos (c)	(velocity of light)	= 4ƧƐ49923 Vlos exactly
			= 299792458 m/s exactly

3Ɛ Minor centuries are identical to the major centuries minus 1 day. We do have to live with centuries pro tem. The dropping of the leap day falls due astronomically every Ƨ8 (128) years, not at the century.

40 The trinaTrifut also equals 1.53282 kilometres.

41 Not 7.92.

42 Equaling 60 Gf.

43 Not 22 yd.

44 Equator/40000000 i.e. 3 akiEquators.

45 This also equals 0.279615204 m.

Sipvol	(TGM tsp)	=	4	$_4$Vm	=	0.8424	Imp.Tsp.	=	1.0117	US Tsp.	=	0.0050	L
Supvol	(TGM tbsp)	=	1	$_3$Vm	=	0.8424	Imp.Tbs.	=	1.0117	US Tbs.	=	0.0149	L
Cupvol	(TGM cup)	=	16	$_3$Vm	=	0.9447	Imp.Cup	=	1.1382	US Cup	=	0.2692	L
Tumblol	(TGM pint)	=	3	$_2$Vm	=	0.94774	Imp.Pint	=	1.13818	US Pint	=	0.538549	litre
Quartol	(TGM quart)	=	6	$_2$Vm	=	1.89548	Imp.Pint	=	2.27636	US Pint	=	1.077098	litre
Galvol	(TGM gallon)	=	2	$_1$Vm	=	7.58192	Imp.Pint	=	9.10544	US Pint	=	4.308392	litre

Teaspoonful: the 5 millilitre teaspoon is 4 quadciaVolm

6,1 *Avolz* (Avz) (Avogadro volume for TGM) = Volume of one Molz of a perfect gas at Ice Point (0 d°) and one Atmoz, = 1;0Ɛ41↊quadciaVolm. See page 57.

7,1	emiMaz	=	Maz divided by Avogadro's (TGM) number	=	8;9↊865	bitriciaMaz
		=	gram divided by Avogadro's (gram) number	=	1.66057x10^{-27}	kg
		=	unifed atomic mass unit mu			
	Oumz (TGM ounce)	=	2 triciaMaz	=	1.0553756	oz(avoir)
	14 oumz	=	28 triciaMaz	=	1.0553756	lb(avoir)
	30 oumz	=	6 biciaMaz	=	1.077098	kg
	160 oumz	=	3 unciaMaz	=	1.01768	stone
	1000 oumz	=	2 Maz	=	1.01768	cwt

9,1 *Lightvlov* (c^2) (square of the velocity of light, = $1/\mu_o\epsilon_o$
= 20;17145 unbiquaVlov = 8.987552x10^{16} m^2/s^2

11,1 *Atmoz* (Atz) (TGM standard atmosphere) = 2Ɛ Prem = 1.015204x10^5 Pa or N/m^2
= 1.001928411 decimal Standard Atmospheres
= 29.978 inches or 761.465 mm of mercury

12,1 *electron-Pel* (ePl) = electron's charge x Pel = 4;169150 unpentciaWerg
= 871.2608 electron-volts = 1.3959244694x10^{-16} J
triqua-electron-Pel (^{3}ePl) = 1.505538662 MeV = 2.4121574831x10−13 J
1 MeV = 7;Ɛ79187Ɛ2 ^{2}ePl = 1;47225897 levciaWerg

27,1 *electron* (e) = the charge of one electron = 4;169150 unpentciaWerg
= 1.602189x10^{-19} coulomb

27,2 *Emelectron* (Me) = the charge of 1 Mole of electrons = 25860.36 faraday
= 5;749690 ennquaQuel = 2.4941802635×10^9 coulomb

36,1 Brite (Bt) = the brightness of the Sun at a distance of 2^6 i.e. 54 (64) lightyears.

38,1 Bril (Bl) = luminous intensity of the Sun = 1 Brite at 54 Lightyears.[46]

39,1 decigree (d°) = one tenth of a degree Celsius. See page 31.
18°C = 180 d° = *130 d°

[46]Note: Brils measure intensity of a source, Brites measure apparent brightness at point of observation, normally the Earth. A star emitting four times the light of the Sun is 4 Bril. If its distance happens to be 54 Lightyear its apparent brightness is 4 Brite. If ↊8 Ly, only 1 Bt (in accordance with the inverse square law).

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