# The Dozenal Society of America

# Systems of Numeration

## A Plea for the Duodecimal*

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For many years we have been longing for the *Duodecimal*. Ever since we first came to know of his beauty, his virtues and his power, we have been yearning to see him established in his rightful place, and to see his more successful but less deserving rival, the *Decimal*, driven out. We still remember the backaches, the headaches, the heartaches, and yet other aches, which we have owed, directly or indirectly, to the enthroned tyrant, and we have hoped to see the world, one day, set free from him. Must we ever hope in vain? Efforts have indeed been made, from time to time, to start this revolution; but obstacles, put in the way by those who should have known better, selfish indifference on the part of those who could have accomplished much, but who would not, timidity on the part of those who have never learned to dare, have caused the great work to be left undone. Archimedes is credited with the saying: "Give me a lever long enough and I will move the world." But it is a pretty hard world to move, and we would bespeak a goodly number of levers, long and strong, with plenty of power at the other end.

To drop unprofitable metaphors, we would say, in plain terms, that we deem it high time to throw aside our *decimal* system of numeration and to adopt, in its stead, the *duodecimal*. It is important for our own sakes, but especially, and very much more important for the sake of posterity, that this be done, and that it be done quickly; for, unless it be done soon, it will ere long be too late.

We cannot join battle immediately, however, but must clear the way by a little previous reconnoitering, taking, as it were, a general view of the whole field and of the enemies to be encountered; we must also try to dispose, sweetly if we can but strongly anyhow, of any lesser foes who might be a hindrance to the main action. To do this we will have to recall to the minds of our readers, certain elementary ideas concerning arithmetical notation in general, and a few of the systems of notation in particular.

### Notation.

Notation in general may be defined as the art of expressing numbers by means of written symbols; while a *system* of notation is a particular method of doing this.

### The Unal System.

In order to express a number, say five, we may put down five separate marks of any kind, as 1 1 1 1 1. In that case, each mark would stand for *one* and nothing else. For want of a

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better name we have ventured to coin a word for the time being, and to call this method the *unal* system. Whether any people ever employed it for general business transactions or not, we are unaware. It is however in use even now, though only to a very limited extent, in keeping tally when the things to be recorded come along one by one, as in marking the points in certain games, or the number of barrels, boxes, etc., swung into the hold of a ship. It is very simple in this respect, that only *one* character, figure, symbol, digit (call it what you will) is required. But no people who had much recording to do could have long stuck to such a method. Its grave defect is that, for any but the smallest numbers, the results become unwieldly.

Let any one undertake, for the sake of experiment, to set down *a million* by this method. On an average he could make conveniently about two marks per second. At this rate he would have to work steadily eight hours a day for more than seventeen days; and, allowing, what is pretty liberal, 2500 marks to the page, he would cover 400 pages. When done, he could read it, in the *unal* way, only by repeating *one*, *one*, *one*, etc., till he had reached the million, while the victim who had been forced to listen would not have the faintest scintillation of an idea as to what the whole thing meant.

Or, suppose a professor of astronomy, in the good old, simple, *unal* days, had, by some means or other, come to the knowledge that the sun is distant from the earth ninety-odd millions of miles, and that he wishes to communicate that scrap of information to his pupils. To get it on the blackboard would take, under the conditions stated above, between four and five years, and if his blackboard were four feet wide, it would have to be more than a mile long.

We can hardly conceive of anything more potent than such a system of numbers as *a means of stunting the human mind*. Nothing that we know of could even pretend to compete with it, except perhaps our present, chaotic system of English spelling; and even that, bad as it is, would be left far behind; though, being the sole (soul, sowl, sol, soughl, soal, psoal, psoughl, psol, psoul, psowl ....) competitor, it might well be adjudged a second prize. If the world had been restricted to the *unal* system, it could scarcely have been civilized, at least arithmetically.

## The Binal System.

Let us now pass to the consideration of some other possible systems. The first step we make out of the quagmire of the *unal* system (which can hardly be called a system at all), is into the *binal* system. Here we make use of two marks, or symbols (0 and 1), the former of which (the 0), when standing alone, represents no value whatever, the second of which (the 1), when standing alone, means *one*. But just here, the influence of *system* comes in; for, the value of the symbol 1, when used in conjunction with other 1's or with other 0's, depends on its place in the line; and the influence of the 0 consists in keeping the 1's in their proper places. Now the pith of the *binal* system is in this, that our symbol 1, when moved one space to the left of its primal position, has twice its fundamental value; when moved another space, twice that; and so on, indefinitely. Very poetical, you may say, but somewhat obscure. Let us try to illuminate it a little.

To do so we will take a line of dots, as . . . . which represent nothing except positions. Now, if our symbol 1, is placed on the right-hand dot, thus . . . 1, it means simply *one*. If placed on the next dot ( . . 1 . ) it will have a value of *twice one*, or what *we* call *two*. If placed on the next dot ( . 1 . . ), it will be *twice two*, or what *we* call *four*; and if placed on the next ( 1 . . . ), it will be *eight*; and so on, increasing in a two-fold ratio by every move from the right towards the left. Strictly speaking we have no right to use the word *two*, because that supposes we have a symbol whose *name* is *two*, whereas, the number *two* is not expressed in this system by a symbol, but by a combination, and the name of the combination should be drawn from the components. But it matters little, as we do not intend to set the *binal* system up for use, and our common decimal system even, is not scientifically correct on this point.

The ratio (two), made use of in this system, is the *radix*, the *base* of the system. In what precedes we may notice two things: first, that the dots on the left of the 1 are serving no useful purpose, and may be omitted; second, that the mean, miserable, little dots might be mistaken for fly-specks, or *vice versa*, and that might be the cause of serious errors. So we round each of them out into a full o, big enough to command respect, and to keep the little *ones* in their proper places. In this system therefore, the combination 10 (which should be read, *one-naught*) means not *ten* but *two*. So 1 1 (one-one) means *two* and *one*, or our *three*; 1 0 1 means *five*; 1 1 1 1 means *eight* and *four* and *two* and one, or *fifteen*.

For the sake of comparison, and as a starting-point for those who wish to investigate this matter further, we give, later on, a table showing the way of expressing numbers from zero to a hundred in several systems. We remark in the meantime, however, that the *binal* system possesses the advantage over the *unal* of greater conciseness. To express *a million* in the *unal* system would require, as we have seen, a million marks, while in the *binal* system it is accomplished by the use of only twenty. This is better, but it is still too bulky for convenient handling.

In this system the usual operations of arithmetic would, however, be ideally easy, so much so that it has been suggested that it might be worth the while to translate the numbers from our *decimal* system to the *binal*, perform the required operations, and then translate back again. We are afraid that the easiness of the work would hardly compensate for its extra length. The rules for translating from the decimal system to any other, and *vice versa*, will be given below.

## The Ternal System.

In the *binal* system, just described, we had need of two symbols; in the *ternal* we will need *three*. Let them be 0, 1, 2, and let them have the intrinsic values of *zero*, *one*, *two*, respectively. The essence of the *ternal* system is in this, that the value of 1 or 2 is increased *three*-fold at each removal through one space towards the left; or, what is saying the same thing, that the *base* of the system is *three*. The value of the combination 10 (one-naught) is therefore *three*; of 20 (two-naught), six; of 100 (one-naught-naught), *nine*; of 212 (two-one-two), twenty-three. A million would require thirteen places to be filled with the proper combination of *zeros*, *ones* and *twos*. This is again an improvement, but we hanker after something better yet.

## The Quaternal System, etc.

It ought to be beginning to dawn on us by this time that we may take any whole number whatever as a base, and construct the corresponding system on the lines indicated above. The number of symbols needed in each case will be equal to the number expressed by the *base*; thus, using the symbols with which we are already familiar, we will have, for

The unal system, the symbol 1.
The binal system, 0, 1 (*two* symbols).
The ternal system, 0, 1, 2 (*three symbols*).
The quaternal system, 0, 1, 2, 3 (*four* symbols).
The quinqual system, 0, 1, 2, 3, 4, etc.
The sextal system, 0, 1, 2, 3, 4, 5.
The septimal system, 0, 1, 2, 3, 4, 5, 6.
The octaval system, 0, 1, 2, 3, 4, 5, 6, 7.
The nonal system, 0, 1, 2, 3, 4, 5, 6, 7, 8.
The decimal system, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
The undecimal system, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t.
The duodecimal system, 0, 1. 2, 3, 4, 5, 6, 7, 8, 9, t, l.

Our present usage allows us only *ten* symbols, so that for the undecimal system we need an extra symbol. Its *name* is ten, so that provisionally, until a new sign has been agreed on, we take the initial of *ten*, *i.e.*, t. For the duodecimal system we need yet another; its *name* is *eleven*; so we take the *l*, not the *e*, because that e is too small to stand up in a row of numerical symbols, and besides *l* will stand very well for 'leven. We might go on indefinitely, but since we have set our heart on the duodecimal system, it will be correct to stop *there*.

It is clear that the greater the value of the number used as a base, the more concise the expression, for any number larger than that base, will tend to be. As an illustration of this fact, we will translate *a million* into its equivalent expression in several of the systems. We need:

For the unal, a million separate digits.
For the binal, twenty digits, viz., 11110100001001000000.
For the ternal, thirteen digits, viz., 1212210202001.
For the quaternal, ten digits, viz., 3310021000.
For the quinqual, nine digits, viz., 224000000.
For the sextal, eight digits, viz., 33233344.
For the septimal, eight digits, viz., 11333311.
For the octaval, seven digits, viz., 3641100.
For the nonal, seven digits, viz., 1783661.
For the decimal, seven digits, viz., 1000000.
For the undecimal, six digits, viz., 623351.
For the duodecimal, six digits, viz., 402854.
For the centesimal, four digits, viz., 1000.

In the next table we may see at a glance the relation between the first twelve systems, through all the numbers up to one hundred inclusively. They look formidable enough, but it is only in looks, and need merely to be pondered on, in the light of what we have already seen, in order to be appreciated. There are doubtless many readers of the Quarterly who will just revel among the beauties of these seemingly prosaic columns of figures.

The first vertical row consists merely of our common numbers in regular order, such as we have been fed on from childhood up, or down. They have been placed *there* as a guide to keep us straight, but are found again in their proper place further to the right. Reading horizontally across the page, each number means exactly the same thing, but as a general thing the *names* would be different.

Equivalent Expression for One Hundred Numbers
in Various Systems

| | Unal. | Binal. | Ternal. | Quaternal. | Quinqual. | Sextal. | Septimal. | Octaval. | Nonal. | Decimal. | Undecimal. | Duodecimal. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (0) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| (1) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| (2) | 11 | 10 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| (3) | 111 | 11 | 10 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| (4) | 1111 | 100 | 11 | 10 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| (5) | 11111 | 101 | 12 | 11 | 10 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| (6) | 111111 | 110 | 20 | 12 | 11 | 10 | 6 | 6 | 6 | 6 | 6 | 6 |
| (7) | etc. | 111 | 21 | 13 | 12 | 11 | 10 | 7 | 7 | 7 | 7 | 7 |
| (8) | etc. | 1000 | 22 | 20 | 13 | 12 | 11 | 10 | 8 | 8 | 8 | 8 |
| (9) | ” | 1001 | 100 | 21 | 14 | 13 | 12 | 11 | 10 | 9 | 9 | 9 |
| (10) | | 1010 | 101 | 22 | 20 | 14 | 13 | 12 | 11 | 10 | t | t |
| (11) | | 1011 | 102 | 23 | 21 | 15 | 14 | 13 | 12 | 11 | 10 | l |
| (12) | | 1100 | 110 | 30 | 22 | 20 | 15 | 14 | 13 | 12 | 11 | 10 |
| (13) | | 1101 | 111 | 31 | 23 | 21 | 16 | 15 | 14 | 13 | 12 | 11 |
| (14) | | 1110 | 112 | 32 | 24 | 22 | 20 | 16 | 15 | 14 | 13 | 12 |
| (15) | | 1111 | 120 | 33 | 30 | 23 | 21 | 17 | 16 | 15 | 14 | 13 |
| (16) | | 10000 | 121 | 100 | 31 | 24 | 22 | 20 | 17 | 16 | 15 | 14 |
| (17) | | 10001 | 122 | 101 | 32 | 25 | 23 | 21 | 18 | 17 | 16 | 15 |
| (18) | | 10010 | 200 | 102 | 33 | 30 | 24 | 22 | 20 | 18 | 17 | 16 |
| (19) | | 10011 | 201 | 103 | 34 | 31 | 25 | 23 | 21 | 19 | 18 | 17 |
| (20) | | 10100 | 202 | 110 | 40 | 32 | 26 | 24 | 22 | 20 | 19 | 18 |
| (21) | | 10101 | 210 | 111 | 41 | 33 | 30 | 25 | 23 | 21 | 1t | 19 |
| (22) | | 10110 | 211 | 112 | 42 | 34 | 31 | 26 | 24 | 22 | 20 | 1t |
| (23) | | 10111 | 212 | 113 | 43 | 35 | 32 | 27 | 25 | 23 | 21 | 1l |
| (24) | | 11000 | 220 | 120 | 44 | 40 | 33 | 30 | 26 | 24 | 22 | 20 |

Equivalent Expression—continued

| | Unal. | Binal. | Ternal. | Quaternal. | Quinqual. | Sextal. | Septimal. | Octaval. | Nonal. | Decimal. | Undecimal. | Duodecimal. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (25) | | 11001 | 221 | 121 | 100 | 41 | 34 | 31 | 27 | 25 | 23 | 21 |
| (26) | | 11010 | 222 | 122 | 101 | 42 | 35 | 32 | 28 | 26 | 24 | 22 |
| (27) | | 11011 | 1000 | 123 | 102 | 43 | 36 | 33 | 30 | 27 | 25 | 23 |
| (28) | | 11100 | 1001 | 130 | 103 | 44 | 40 | 34 | 31 | 28 | 26 | 24 |
| (29) | | 11101 | 1002 | 131 | 104 | 45 | 41 | 35 | 32 | 29 | 27 | 25 |
| (30) | | 11110 | 1010 | 132 | 110 | 50 | 42 | 36 | 33 | 30 | 28 | 26 |
| (31) | | 11111 | 1011 | 133 | 111 | 51 | 43 | 37 | 34 | 31 | 29 | 27 |
| (32) | | 100000 | 1012 | 200 | 112 | 52 | 44 | 40 | 35 | 32 | 2t | 28 |
| (33) | | 100001 | 1020 | 201 | 113 | 53 | 45 | 41 | 36 | 33 | 30 | 29 |
| (34) | | 100010 | 1021 | 202 | 114 | 54 | 46 | 42 | 37 | 34 | 31 | 2t |
| (35) | | 100011 | 1022 | 203 | 120 | 55 | 50 | 43 | 38 | 35 | 32 | 2l |
| (36) | | 100100 | 1100 | 210 | 121 | 100 | 51 | 44 | 40 | 36 | 33 | 30 |
| (37) | | 100101 | 1101 | 211 | 122 | 101 | 52 | 45 | 41 | 37 | 34 | 31 |
| (38) | | 100110 | 1102 | 212 | 123 | 102 | 53 | 46 | 42 | 38 | 35 | 32 |
| (39) | | 100111 | 1110 | 213 | 124 | 103 | 54 | 47 | 43 | 39 | 36 | 33 |
| (40) | | 101000 | 1111 | 220 | 130 | 104 | 55 | 50 | 44 | 40 | 37 | 34 |
| (41) | | 101001 | 1112 | 221 | 131 | 105 | 56 | 51 | 45 | 41 | 38 | 35 |
| (42) | | 101010 | 1120 | 222 | 132 | 110 | 60 | 52 | 46 | 42 | 39 | 36 |
| (43) | | 101011 | 1121 | 223 | 133 | 111 | 61 | 53 | 47 | 43 | 3t | 37 |
| (44) | | 101100 | 1122 | 230 | 134 | 112 | 62 | 54 | 48 | 44 | 40 | 38 |
| (45) | | 101101 | 1200 | 231 | 140 | 113 | 63 | 55 | 50 | 45 | 41 | 39 |
| (46) | | 101110 | 1201 | 232 | 141 | 114 | 64 | 56 | 51 | 46 | 42 | 3t |
| (47) | | 101111 | 1202 | 233 | 142 | 115 | 65 | 57 | 52 | 47 | 43 | 3l |
| (48) | | 110000 | 1210 | 300 | 143 | 120 | 66 | 60 | 53 | 48 | 44 | 40 |
| (49) | | 110001 | 1211 | 301 | 144 | 121 | 100 | 61 | 54 | 49 | 45 | 41 |
| (50) | | 110010 | 1212 | 302 | 200 | 122 | 101 | 62 | 55 | 50 | 46 | 42 |
| (51) | | 110011 | 1220 | 303 | 201 | 123 | 102 | 63 | 56 | 51 | 47 | 43 |
| (52) | | 110100 | 1221 | 310 | 202 | 124 | 103 | 64 | 57 | 52 | 48 | 44 |

Equivalent Expression—continued

| Unal. | Binal. | Ternal. | Quaternal. | Quinqual. | Sextal. | Septimal. | Octaval. | Nonal. | Decimal. | Undecimal. | Duodecimal. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (53) | 110101 | 1222 | 311 | 203 | 125 | 104 | 65 | 58 | 53 | 49 | 45 |
| (54) | 110110 | 2000 | 312 | 204 | 130 | 105 | 66 | 60 | 54 | 4t | 46 |
| (55) | 110111 | 2001 | 313 | 210 | 131 | 106 | 67 | 61 | 55 | 50 | 47 |
| (56) | 111000 | 2002 | 320 | 211 | 132 | 110 | 70 | 62 | 56 | 51 | 48 |
| (57) | 111001 | 2010 | 321 | 212 | 133 | 111 | 71 | 63 | 57 | 52 | 49 |
| (58) | 111010 | 2011 | 322 | 213 | 134 | 112 | 72 | 64 | 58 | 53 | 4t |
| (59) | 111011 | 2012 | 323 | 214 | 135 | 113 | 73 | 65 | 59 | 54 | 4l |
| (60) | 111100 | 2020 | 330 | 220 | 140 | 114 | 74 | 66 | 60 | 55 | 50 |
| (61) | 111101 | 2021 | 331 | 221 | 141 | 115 | 75 | 67 | 61 | 56 | 51 |
| (62) | 111110 | 2022 | 332 | 222 | 142 | 116 | 76 | 68 | 62 | 57 | 52 |
| (63) | 111111 | 2100 | 333 | 223 | 143 | 120 | 77 | 70 | 63 | 58 | 53 |
| (64) | 1000000 | 2101 | 1000 | 224 | 144 | 121 | 100 | 71 | 64 | 59 | 54 |
| (65) | 1000001 | 2102 | 1001 | 230 | 145 | 122 | 101 | 72 | 65 | 5t | 55 |
| (66) | 1000010 | 2110 | 1002 | 231 | 150 | 123 | 102 | 73 | 66 | 60 | 56 |
| (67) | 1000011 | 2111 | 1003 | 232 | 151 | 124 | 103 | 74 | 67 | 61 | 57 |
| (68) | 1000100 | 2112 | 1010 | 233 | 152 | 125 | 104 | 75 | 68 | 62 | 58 |
| (69) | 1000101 | 2120 | 1011 | 234 | 153 | 126 | 105 | 76 | 69 | 63 | 59 |
| (70) | 1000110 | 2121 | 1012 | 240 | 154 | 130 | 106 | 77 | 70 | 64 | 5t |
| (71) | 1000111 | 2122 | 1013 | 241 | 155 | 131 | 107 | 78 | 71 | 65 | 5l |
| (72) | 1001000 | 2200 | 1020 | 242 | 200 | 132 | 110 | 80 | 72 | 66 | 60 |
| (73) | 1001001 | 2201 | 1021 | 243 | 201 | 133 | 111 | 81 | 73 | 67 | 61 |
| (74) | 1001010 | 2202 | 1022 | 244 | 202 | 134 | 112 | 82 | 74 | 68 | 62 |
| (75) | 1001011 | 2210 | 1023 | 300 | 203 | 135 | 113 | 83 | 75 | 69 | 63 |
| (76) | 1001100 | 2211 | 1030 | 301 | 204 | 136 | 114 | 84 | 76 | 6t | 64 |
| (77) | 1001101 | 2212 | 1031 | 302 | 205 | 140 | 115 | 85 | 77 | 70 | 65 |
| (78) | 1001110 | 2220 | 1032 | 303 | 210 | 141 | 116 | 86 | 78 | 71 | 66 |
| (79) | 1001111 | 2221 | 1033 | 304 | 211 | 142 | 117 | 87 | 79 | 72 | 67 |
| (80) | 1010000 | 2222 | 1100 | 310 | 212 | 143 | 120 | 88 | 80 | 73 | 68 |

EQUIVALENT EXPRESSION—CONTINUED

| Unal. | | Binal. | Ternal. | Quaternal. | Quinqual. | Sextal. | Septimal. | Octaval. | Nonal. | Decimal. | Undecimal. | Duodecimal. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (81) | | 1010001 | 10000 | 1101 | 311 | 213 | 144 | 121 | 100 | 81 | 74 | 69 |
| (82) | | 1010010 | 10001 | 1102 | 312 | 214 | 145 | 122 | 101 | 82 | 75 | 6t |
| (83) | | 1010011 | 10002 | 1103 | 313 | 215 | 146 | 123 | 102 | 83 | 76 | 6l |
| (84) | | 1010100 | 10010 | 1110 | 314 | 220 | 150 | 124 | 103 | 84 | 77 | 70 |
| (85) | | 1010101 | 10011 | 1111 | 320 | 221 | 151 | 125 | 104 | 85 | 78 | 71 |
| (86) | | 1010110 | 10012 | 1112 | 321 | 222 | 152 | 126 | 105 | 86 | 79 | 72 |
| (87) | | 1010111 | 10020 | 1113 | 322 | 223 | 153 | 127 | 106 | 87 | 7t | 73 |
| (88) | | 1011000 | 10021 | 1120 | 323 | 224 | 154 | 130 | 107 | 88 | 80 | 74 |
| (89) | | 1011001 | 10022 | 1121 | 324 | 225 | 155 | 131 | 108 | 89 | 81 | 75 |
| (90) | | 1011010 | 10100 | 1122 | 330 | 230 | 156 | 132 | 110 | 90 | 82 | 76 |
| (91) | | 1011011 | 10101 | 1123 | 331 | 231 | 160 | 133 | 111 | 91 | 83 | 77 |
| (92) | | 1011100 | 10102 | 1130 | 332 | 232 | 161 | 134 | 112 | 92 | 84 | 78 |
| (93) | | 1011101 | 10110 | 1131 | 333 | 233 | 162 | 135 | 113 | 93 | 85 | 79 |
| (94) | | 1011110 | 10111 | 1132 | 334 | 234 | 163 | 136 | 114 | 94 | 86 | 7t |
| (95) | | 1011111 | 10112 | 1133 | 340 | 235 | 164 | 137 | 115 | 95 | 87 | 7l |
| (96) | | 1100000 | 10120 | 1200 | 341 | 240 | 165 | 140 | 116 | 96 | 88 | 80 |
| (97) | | 1100001 | 10121 | 1201 | 342 | 241 | 166 | 141 | 117 | 97 | 89 | 81 |
| (98) | | 1100010 | 10122 | 1202 | 343 | 242 | 200 | 142 | 118 | 98 | 8t | 82 |
| (99) | | 1100011 | 10200 | 1203 | 344 | 243 | 201 | 143 | 120 | 99 | 90 | 83 |
| (100) | | 1100100 | 10201 | 1210 | 400 | 244 | 202 | 144 | 121 | 100 | 91 | 84 |

The table might, of course, be indefinitely extended, but we have given enough, we think, to show how tables of any length and width might be constructed. A few remarks, however, will help to make matters still clearer.

We begin all the systems from *zero*, or *nothing*, because that is the law for the beginning of all created things, and also because it makes the grouping more symmetrical. The natural turning-point in the grouping is where the number of symbols of the combination changes, as we have indicated by a dash at the end of each group. It will be noticed also that each symbol, taken singly, has the same value in all the systems in which it may be used that it has in our decimal system. Thus, the intrinsic value of the symbol 7 is *seven* always; but its *systematic* value depends, as we have seen, on its place in the line. To understand more clearly how the foregoing table was built up. and how it might be extended to any dimensions, let us take one of the systems and follow it through. Any one you please. The *septimal*, you say? Well, here goes.

In this system we have *seven* symbols (0 to 6 inclusively). We write them down, as in the table, in their regular order, till we have used them up. We can go no further with single symbols, for we have no more; hence, we must resort to combinations. We begin over again, therefore, by taking the first symbol which has a value of its own, viz., the 1, and move it one space to the left, and keep it there by means of our stop-gap, the 0, thus getting the combination 10 (one-naught), which is the first *systematic* number in this system. Its value is, of course, *seven.* We then go on by replacing the 0 by each of the other symbols in regular order, and thus get 10, 11, 12, etc., to 16, which last combination means *seven* and *six*, *i.e.*, what *we* call *thirteen.* Here again we are stopped for want of symbols, and so we resort to other combinations by taking the next symbol, the 2, and putting it in the second rank, just as we did with the 1. This gives us the combination 20, meaning evidently $(2 \times 7) + 0 =$ *fourteen* in our parlance. Then 21, which is $(2 \times 7) + 1$, will be *fifteen*; 23 = seventeen; 26 = twenty. Another start will give us 30 = twenty-one, etc., to 36 = *twenty-seven.* When we have reached 66, *i.e.*, $(6 \times 7) + 6 =$ *forty-eight*, we are at the end of a group, and can do no more with only two symbols. So we will start afresh with our 1, and move it a second time to the left, thus getting 100. It is evident that this means $(1 \times 7 \times 7) + 0 =$ *forty-nine.* Then all is plain sailing again (as 101 = *fifty*; 106 = *fifty-five*) till we reach 666, *i.e.*, $(6 \times 7 \times 7) + (6 \times 7) + 6 =$ *three hundred and forty-two.* Then our 1 makes another move, and we have 1000 or $(1 \times 7 \times 7 \times 7) + 0 =$ *three hundred and forty-three.* Enough. The same method is applicable to any and every system, the only difference between them being in the *base*, and in the consequent number of symbols employed.

But we are so accustomed to the use of the *decimal* system, that, at first, it is difficult to *think* in any other; and, just as a person who is not quite familiar with a foreign language, finds it necessary to think in his own and then translate, so in dealing with these unfamiliar systems, it is necessary to know how to translate. To do this we might go back to the beginning (0) and build up the system to the desired spot. This would, in the case of large numbers, be very laborious, but we may arrive at the result, by a shorter cut, in using the following Rules:

1. To translate from the *decimal* to any other system.

*Divide the given number by the base of the other system, and, on a line with the quotient, set down the remainder (even if it be a 0)—divide the quotient so obtained by the base again, for a new quotient and remainder, and so continue, until the last quotient is less than the base.* This last quotient, with the several remainders in their backward order, will be the number required.

*Example.*—Translate 237,985 of our *decimal* system into the equivalent expression of the *ternal* system.

| | | |
|---|---|---|
| 3 | 237985 | |
| 3 | 79328 | 1 |
| 3 | 26442 | 2 |
| 3 | 8814 | 0 |
| 3 | 2938 | 0 |
| 3 | 979 | 1 |

3|326|1
3|108|2
3|36|0
3|12|0
3|4|0
3|1|1 The required expression is therefore 110002110021.

2. To translate from any other system into the *decimal.*

*Multiply the left-hand figure of the given number by the base of that other system, and add in the next figure at the right. Multiply the sum so obtained by the base again, and add in, as before, the next right-hand figure. Continue the successive multiplications until the last right-hand figure has been added.* The last sum will be the given number expressed in the decimal system.

*Example.*—Translate 43021 of the *quinqual* system into its equivalent in the decimal.

$$
\begin{array}{l}
43021 \\
\underline{5\phantom{000}} \\
20 + 3 = 23 \\
\phantom{20 + 3 = }\underline{\phantom{2}5} \\
\phantom{20 + 3 = }115 + 0 = 115 \\
\phantom{20 + 3 = 115 + 0 = }\underline{\phantom{11}5} \\
\phantom{20 + 3 = 115 + 0 = }575 + 2 = 577 \\
\phantom{20 + 3 = 115 + 0 = 575 + 2 = }\underline{\phantom{57}5} \\
\phantom{20 + 3 = 115 + 0 = 575 + 2 = }2885 + 1 = 2886 \quad \text{Answer.}
\end{array}
$$

## Arithmetical Operations.

All the operations of arithmetic can be readily performed in any of the systems, and a little practice, with a clear head, would soon render one fairly an adept. We will give a few examples, say in the *quaternal* system.

### *Addition.*

| |
|---:|
| *123* |
| *311* |
| *232* |
| *312* |
| + *131* |
| *3101* |

This, at first sight, looks crooked enough, but when we remember that one remove towards the left means a fourfold (*not* a *tenfold*) increase, it straightens itself out immediately. The 312 sum of the first column (at the right) is *nine*, and that is (2 × 4) + 1, or *two* to be carried to the column of the *fours*, and one left for the column of units. The second column, including the “carried” *two*, adds up to *twelve*, that is, *three* to be carried to the column of *sixteens*, and *zero* left over for the column of *fours*. Finally, the “carried” *three* being included, the third column foots up to *thirteen*, = (3 × 4) + 1, that is, *three* for the fourth rank, and *one* left over for the third; so we have 3101 as a result.

*(Note.—In practice the adding will be done mentally.) It can be demonstrated that these Rules are exact for whole numbers, but the demonstration would require more space than we can afford. Fractions require a somewhat different treatment.

*Subtraction.*

```
  331032
− 120233
  210133
```

Here again a little of the *quaternal* spirit is required to help us see how 3 from 2 leaves 3. The secret, a very open one, is, that the one (1) which we "borrow" from the next left-hand rank is really *four* (not ten), which being added to the 2 makes *six*, from which we take 3, and so have 3 for a remainder. The same idea is to be kept in mind every time we "borrow" or "carry."

*Multiplication.*

```
   3012
    302
  12030
 21102
2122230
```

Don't forget that every *four*, in one rank, is to be carried as a unit to the next, and all will be well.

*Division.*

```
130)121110)312
    1122
      231
      132
        330
        330
```

If a *decimal* boy were to bring *that* as an example of division, we should suspect a leak somewhere in the brain; if a *quaternal* boy did not bring that result, we would mildly inquire "Why not?" What we have to say about fractions will come a little later.

## Numeration as it was Among the Ancients.

Now that we have glanced at a few of the possible systems of notation, the question that may suggest itself is: What is the use of it all? Why lead us through a labyrinth of *possible* systems when we have an *actual* one in daily use? Kind, patient reader, please allow us to answer your question by another. Suppose you had just emigrated to some other planet inhabited by rational beings like ourselves, but who as yet were totally unacquainted with any arithmetic, though just beginning to feel the need of the science of numbers. Suppose moreover you had been called thither for the express purpose of being their teacher in that branch. Knowing all you know now, which of the possible systems would you adopt in starting the arithmetic of your new world? A wrong choice here would entail on you many a left-handed blessing for all time to come. Reflect now. Were you to adopt the *unal* system, the æons of time would be too short to figure up your market bills. The *binal*, as we shall see, would have to be condemned on account of its interminable fractions. The *ternal* and *nonal* would be even worse in this respect, and the *quaternal* and *octaval* very little better, although the *expressions* would be more compact. The *quinqual*, *septimal* and *undecimal* are simply horrid in this matter of interminate fractions. What then? Why, try the next, the *decimal* system. Unhappily for us our ancestors did so, or rather they stopped, unhappily, without looking farther. How came they to do so? It is hard to tell, but it is believed that our decimal system owes its origin to the *ten* fingers of the human hands. We have all counted on our fingers more or less, and the very word "digit," used both for "finger" and for "numerical symbol," seems to confirm this belief. However this may be, it is about certain that the forms of our symbols are derived from the Sanskrit, although other derivations have been suggested, and that the *decimal* system, as such, came from the Hindoos to the Arabs, by whom it was introduced into Europe not earlier than the eleventh century.

Among the ancient Hebrews and Greeks the decimal system was indeed in use, but in a very imperfect form, and what knowledge they had of it, was probably acquired through their intercourse with India. For symbols they relied on the letters of their alphabets; but as the Hebrew alphabet contained only twenty-two letters, five more symbols were invented, in order to make three groups of nine each. For the same reason the Greeks added three new symbols. The Hebrews put these new symbols at the end of their alphabet; the Greeks put one at the sixth place, one at the eighteenth, and the third at the end. Given now these twenty-seven symbols, the method of using them was the same with both nations. The first nine letters were used for our 1, 2, 3, 4, 5, 6, 7, 8, 9; the second nine for our 10, 20, 30, 40, 50, 60, 70, 80, 90; the third nine for 100, 200, 300, 400, 500, 600, 700, 800, 900. In Greek usage an accent was placed over a letter when used as a numeral; when placed below the letter it increased its value a thousand times. M, used as a prefix, increased the value of a numeral ten-thousand times. Combinations were formed by placing these numeral letters in juxtaposition. For example, to indicate two-hundred and seventy-nine, a Greek would write $\sigma\grave{o}\theta$, as if we were to write 200, 70, 9. Fractions were written, clumsily enough, by setting the *numerator* apart from the integer to which it belonged, and then the denominator a little higher up, as we write an exponent. They had nothing analogous to our decimal fractions.

The Romans seem to have got badly mixed. It is true that the idea of *ten* and its submultiple, *five*, runs through their notation, but the idea of position to determine the value of a symbol never worked its way into their brains. They used letters as numerals, all of which (except the unit, of course) are multiples of ten or five: thus, I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, and M = 1000; but the value is not based upon alphabetical order. Many attempts have been made to account for these symbols, but no completely satisfactory solution has been reached; at best, we have only plausible guesses. The I to represent 1 is natural enough, being a single stroke. The V has been supposed to be the half of an X; but the X comes from where? Echo answers, "Where?" This X has been very aptly called the *crux*, the *cross*, of the Roman notation. We have either read somewhere, or dreamed, that the V as a numeral was derived from the appearance of the human hand held up, in which the *five* fingers point out radially, like a bunch of carrots, the thumb and little finger making an angle with each other of about 45°. Now, leave out the three middle fingers, for the sake of brevity, and the outline left is a perfect V. After that, the X is simply two V's placed point to point. The L may be the half of the old square form of C (⊏), and C was probably adopted as being the initial of *Centum*, a hundred; and M, in like manner, because it is the initial of *Mille*, a thousand. The ancient rounded form of M was like this: CIƆ; and one-half of this IƆ later on became D, or 500.

However all this may be, the Romans contrived, by means of repetitions and combinations of these numeral letters, to express whatever ideas they had of numbers. To do so, they not only employed their symbols *additatively*, but, strangely enough, in the case of I and X, *subtractively* also. When either I or X stood at the left of a number larger than itself, it was to be subtracted, as V = 5, but IV = 4; X = 10, but IX = 9; and XL= 50 − 10 = 40; but, VI = 6, XI = 11, LX = 60, etc. Another element of confusion was introduced by such forms as IIX = 8, XIIX = 18, XXC = 80, etc.

The Roman schoolboy must have had a hard tussle in his arithmetical work with such an insane notation, and was doubtless glad when the time came to exchange his stylus and tablet for a sword and shield.

Such was, in outline, numeration among the ancients.

## Numeration as it is among Ourselves.

The so-called Arabic, or decimal, system of numeration, which found its way into Europe six or seven hundred years ago, was certainly a vast improvement on the Hebrew, Greek, and Roman systems. It had, indeed, so much to recommend it, that it won its way through nearly the whole civilized world. Besides this, during the past hundred years a strong effort has been continuously applied, in certain quarters to make it the basis of all measurements, *i.e.*, to make all our measures and weights, whether of solids, liquids, or gases, start from one common standard, and to have all the multiples and submultiples of that standard arranged according to the decimal scale. The French *metric* system is the realization of this idea, and receives its name from the chosen unit of length, the *metre*. This system has been adopted and made obligatory, in France, Prussia, Italy, Spain, and in some minor countries; but, although rendered legal in Great Britain in 1864, and in the United States in 1866, it has never taken root among English-speaking nations. That it has not, shows conclusively that there is something radically wrong with the decimal system itself. Just in what that wrongness consists will presently appear. We are not sorry, then, that the *decimo-metric* system has failed to establish itself; not, indeed, that we are in love with our own barbarous standards of weights and measures, which are so bad that human ingenuity could hardly have invented worse, but because we are convinced that infinitely better can be done; and *that better* will consist in throwing over the whole concern, the *decimal* system of numbers included, and in putting in place thereof, the *duodecimal* system. We can then, *with very slight*, and *surprisingly few changes*, bring all our weights and measures of every kind, with their multiples and subdivisions, into strict correspondence with the base of our system of notation. The saving in time and labor with a *duodecimo-metric* system would be simply incalculable, and men and nations yet unborn would look back and bless us for having delivered them from the thraldom of the *decimal*.

## The Plea.

### *Numeration as it Should Be.*

The *duodecimal* system of numeration should be adopted to the exclusion of the *decimal* and every other. This is our thesis; and we will now indicate briefly some of the more important proofs that may be used to sustain it. Of course, we claim no originality, and seek no patent either for the idea or the proofs; but we would wish to see this matter taken up for concerted action by enough gallant soldiers, enlisted under the banner of the Duodecimal, to carry him on to victory, and crown him king in the great land of Arithmos.

An article which appeared in the *Educational Review* for November, 1891, from the pen of Professor William B. Smith, of the University of Missouri, treats this matter pretty fully. We have taken the liberty of borrowing from it a good deal of what follows, and also of modifying some minor details of the proposed scheme; but we feel confident that the Professor will not take this amiss, as we will not take it amiss should some one see fit to modify for the better what we have written. Be this as it may, the grand central idea, the complete triumph of the duodecimal system must come; it is on the way:

> "Then let us pray that come it may,
> As come it will for a' that,
> That youth at school no more shall *grieve*
> O'er 'rithmetic and a' that."

The battle is on now between the only great rivals, the *decimal* and the *duodecimal*, the former of which has the great advantage of being *in possession*; the latter has only its own intrinsic excellence on which to rely.

The requisites for a good system of numeration are principally the following:

1. It must be thoroughly *systematic*, down to the very marrow of its bones. This implies that it must admit of no irregularities, no exceptional cases, no chance of doubt as to meaning in any case, no confusion; and that it should be so clear that, once started on the track, you can go on to any number without fear of failing or faltering. This first requisite can be fulfilled only by making the value of a symbol depend on its position, according to a geometrical series. On this score any of the systems, from the *binal* upwards, could claim admittance; but the Hebrew, Greek, and Roman notations will have to be irrevocably excluded, and we will have none of them.

2. Whatever the system adopted, the expression for large numbers should be reasonably concise. This condition throws out immediately all the systems described above from the *unal* to the *octaval* at least, as may be easily seen by reference to the table already given. The realization of this condition depends upon the value of the *base* chosen; the larger the base, the fewer will be the digits required to represent a given number, and the higher the number to be represented, the more this virtue of the base will assert itself. Thus to represent ten in the *decimal* system requires two digits (*i.e.*, 10), to represent a million, seven digits are necessary (*i.e.*, 1,000,000); but, if our *base* were one-hundred (*centestimal* system), *ten* would be represented by *one* digit and a *million* would be represented by *four* (*i.e.*, 1000). But we must not exaggerate here, as the larger the base the greater the number of separate symbols to be invented, and kept perfectly distinct from each other under every condition of careless writing, and bad penmanship, and poor mnemonics. This consideration prohibits us from adopting as a base, though otherwise excellent, such a number as *sixty* or even *twenty-four*, while any base between *twelve* and *twenty-four* must be rejected for reasons given below.

In regard to conciseness the decimal system is fairly good, but the duodecimal is measurably better. Thus, all numbers below 144 (decimal system) are expressed in the duodecimal by *two* figures at most; all below 1728, by *three* figures; all below 20,736, by *four*; all below 248,832, by *five*; and so on.

3. In the third place, the perfection of a system of numeration depends on the facility which it offers for the handling of its fractions. Two points of view are possible here. We may consider the merits of a system merely as a system of numbers, as such, or as a system to be applied to the various needs of business, trades, arts, and manufactures. From either

point of view the duodecimal system is far in advance of the decimal. Vulgar fractions may, it is true, be expressed with perfect accuracy in any of the regular systems. Thus $\frac{1}{111111111111}$, $\frac{1}{1100}$, $\frac{1}{110}$, $\frac{1}{30}$, $\frac{1}{22}$, $\frac{1}{20}$, $\frac{1}{15}$, $\frac{1}{14}$, $\frac{1}{13}$, $\frac{1}{12}$, $\frac{1}{11}$, $\frac{1}{10}$, are a dozen equally accurate ways of expressing, each in its own system, the value *one-twelfth*. But vulgar fractions are time-consuming, brain-benumbing devices. In order to *add* together several fractions, you will usually have to *divide*, *multiply*, again *divide* and *multiply*, then *add*, and *divide* again. To *subtract*, the same number of operations are required. To *multiply* or *divide* fractions is slightly simpler, only two multiplications and one division being necessary in each case. To raise to powers and extract roots requires at least twice as much labor as in whole numbers, and when we have fractions and whole numbers together, it is worse yet. The well-known lines of an unknown genius (only one word having been altered) are not inappropriate:

“Multiplication is vexation,
Division is as bad;
The Rule of Three, it puzzles me,
And *fractions* set me mad.”

Now, when fractions are written as sub-powers of the *base* of your system, and when they come out even, all this waste and confusion is avoided; as when we write 0.5 for $\frac{1}{2}$ or 0.1875 for $\frac{3}{16}$. Then addition is addition and nothing more, subtraction is only subtraction, and so of other operations.

But as there is never any great loss without some slight gain, so we suppose there can be no great gain without some slight loss, and in this connection we come across the snag of what are called *interminate*, or *circulating*, or *recurring* fractions, *i.e.*, fractions which cannot be expressed *accurately* with any finite number of digits. An example of this is the fraction *one-third*, which, *decimally*, becomes 0.3333 . . . . etc. to no end. Now, operations performed on these recurring fractions as such, can never give us exact results, but only approximations, and this is a serious defect. And although we can *calculate* their exact value, still these endless tail-ends are great nuisances. No system can be entirely free from them, but the fewer there are of them in any given system, the more perfect is that system. Now of the first eleven natural divisions of unity (viz. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$) *eight* become recurring fractions in the *binal*, in the *quaternal*, in the *sextal* and in the *octaval* systems; *nine* in the *ternal* and *nonal* systems; *ten* in the *quinqual*, *septimal* and *undecimal* systems. To propose any one of them as a practical system of numeration, would therefore be simply an insult to humanity.

But two remain, and to show their respective merits, we will give a tabular statement.

The first column of the following table is a list of ordinary vulgar fractions; in the second column we have the corresponding *decimal* fractions; in the third the corresponding *duodecimal* fractions.

| | | Decimals. | | Duodecimals. |
|---|---|---|---|---|
| $\frac{1}{2}$ | = | 0.5 | = | 0.6 |
| $\frac{1}{3}$ | = | 0.333‘3’. . . | = | 0.4 |
| $\frac{1}{4}$ | = | 0.25 | = | 0.3 |

| | | Decimals. | | Duodecimals. |
|---|---|---|---|---|
| $\frac{1}{5}$ | = | 0.2 | = | 0.‘2497’... |
| $\frac{1}{6}$ | = | 0.16‘6’... | = | 0.2 |
| $\frac{1}{7}$ | = | 0.‘142857’... | = | 0.‘186t35’... |
| $\frac{1}{8}$ | = | 0.125 | = | 0.16 |
| $\frac{1}{9}$ | = | 0.111‘1’... | = | 0.14 |
| $\frac{1}{10}$ | = | 0.1 | = | 0.1‘2497’... |
| $\frac{1}{11}$ | = | 0.09‘09’... | = | 0.111‘1’... |
| $\frac{1}{12}$ | = | 0.083‘3’... | = | 0.1 |

In this table, when a fraction is followed by dots, the meaning is that the figure or group of figures included between inverted commas (‘ ’) is to be repeated to infinity to get the accurate value. Of these there are *six* in the *decimals*, and only *four* in the *duodecimals*, giving a gain of 50 per cent. for these first eleven fractions, and a still greater gain if we were to continue the list further, in favor of the *duodecimal.* Nor is this all, for, among the five decimal fractions which come out without a remainder, only *three* in the decimals (0.5, 0.2, 0.1) are written with one digit each, while in the duodecimals, *five* (0.6, 0.4, 0.3, 0.2, 0.1) enjoy that advantage. Again, in the *decimals*, *one* of them requires *three* digits (0.125), while among the *duodecimals* none has more than two; all of which, summed up, gives in the duodecimal system, another saving of time and labor of about 20 per cent. This advantage is also kept up when we go on to still lower fractions.

Furthermore, the fractions we need oftenest, and use most ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{12}$) are the very ones which are not *repeaters* in the *duodecimal* system; while of the *seven* named above, the *decimal* system shuts off *four* (viz., $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$), and gives us instead, the $\frac{1}{5}$ and the $\frac{1}{10}$, neither of which is of any practical use.

These advantages all hold good for all multiples of these fractions, as, for $\frac{2}{8}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{11}{12}$ and others, all of which, in the *duodecimal* system steer clear of the rock of infinite tails.

Another advantage of the duodecimal system is in the higher degree of accuracy obtained by the use of an equal number of digits, in the case of approximations.

Thus, the greatest error committed by dropping all *decimals* beyond the second place, is less than $\frac{1}{100}$, but in *duodecimals* it is less than $\frac{1}{144}$; dropping all beyond the third place gives a maximum error of less than $\frac{1}{1000}$ in the one and less than $\frac{1}{1728}$ in the other; and dropping all beyond the fourth place less than $\frac{1}{10000}$ and less than $\frac{1}{20736}$. Here, the *duodecimal* error is less than half the *decimal* error. Once more, the maximum error committed by stopping at the seventh *decimal*, in say a table of logarithms is less than $\frac{1}{10000000}$, but in *duodecimals* the error would be reduced to less than $\frac{1}{35831808}$; a degree of accuracy nearly four-fold greater.

These, and other advantages which want of space forbids us to dwell on, prove that the *duodecimal* is theoretically, at least, superior to the *decimal* system.

Let us now take a look at the more practical side of the question, which is of course founded on the theoretical.

It is clear that for an everyday, working system no prime number, as 2, 3, 5, 7, 11, would be at all suitable as a base, and *that* precisely on account of its indivisibility, while 4, 6, 8, and 9 must be rejected for reasons already given; it is also clear that a base greater than 12 would be inconvenient on account of the number of symbols required. The claims, therefore, of the decimal and duodecimal systems are the only ones that need be considered.

The practical value of the duodecimal system depends mainly on two things, the first of which is its slightly greater conciseness, the second the superior divisibility of its base. We have already sufficiently called attention to the question of conciseness, and will add here merely that we do not consider it, if taken alone, of very great importance. Indeed, if that had been the only grounds of a plea for the duodecimal, the present article, would never have been written. The second, however, is of altogether greater weight.

The number *ten* can be factored into $2 \times 5$, and no further. Consequently the only decimal fractions that will not be repeaters are those which come from the vulgar fractions $\frac{1}{2}$ and $\frac{1}{5}$, and from the powers and products of these, as $\frac{1}{2} = 0.5$, $\frac{1}{5} = 0.2$; $\frac{1}{4} = 0.25$, $\frac{1}{8} = 0.125$; $\frac{1}{16} = 0.0625$; $\frac{1}{25} = 0.04$; $\frac{1}{125} = 0.008$; $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10} = 0.1$; $\frac{1}{8} \times \frac{1}{125} = 0.001$, etc. . . .

On the other hand *twelve* can be factored into three different groups: $2 \times 6$, or $2 \times 2 \times 3$, or $3 \times 4$, so that we have 2, 3, 4, and 6 as separate factors. Hence the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, will give even duodecimal fractions; they are 0.6, 0.4, 0.3 and 0.2 respectively. All their powers will be just as obliging, as $\frac{1}{8} = 0.16$, $\frac{1}{9} = 0.14$ $(\frac{1}{16}) = 0.09$ $(\frac{1}{27}) = 0.054$ $(\frac{1}{81}) = 0.0194$ $(\frac{1}{64}) = 0.023$ $(\frac{1}{256}) = 0.0069$, etc.; and all the products of any of these will go and do likewise, as $\frac{1}{2} \times \frac{1}{9} = (\frac{1}{18}) = 0.08$, $\frac{1}{3} \times \frac{1}{4} = (\frac{1}{12}) = 0.1$, $\frac{1}{8} \times \frac{1}{9} = (\frac{1}{72}) = 0.02$ $(\frac{1}{16} \times \frac{1}{27} = \frac{1}{482}) = 0.004$. The fractions just mentioned, $\frac{1}{16}$, $\frac{1}{27}$, etc., *which are enclosed in parentheses* are in decimal notation.

This divisibility of the base in the duodecimal system is therefore more than twice as prolific in useful results as that of the decimal system, and gives it a vast advantage over the latter; an advantage which renders it admirably fitted not only to be *the* system of numeration for arithmetical work in general (because, as we have seen, it is relatively free from repeating fractions), but also fits it to be the foundation of a complete structure of multiples and subdivisions in the everyday matter of weights and measures of all kinds. Well, then, let us reform.

The first step towards a reform in any matter, consists in acknowledging our sinfulness. In the matter of weights and measures our ancestors were terribly sinful, not morally perhaps, but intellectually, arithmetically, scientifically, and they have transmitted their sins, not to the third and fourth generations but to the thirty-third and thirty-fourth. To show this we will take up that one book which of all others we loved most dearly in the days that are long gone by, our Arithmetic. We turn the fond pages till we come to certain Tables of Weights:

| 1. Troy Weight. | | |
|---|---|---|
| 24 grains (gr.) | = | 1 pennyweight (dwt.). |
| 20 pennyweights | = | 1 ounce (oz.). |
| 12 ounces | = | 1 pound (lb.). |

| 2. Apothecaries' Weight | | |
|---|---|---|
| 20 grains | = | 1 scruple (℈) |
| 3 scruples | = | 1 drachm (ʒ) |
| 8 drachms | = | 1 ounce (℥) |
| 12 ounces | = | 1 pound (℔) |

| 2. Avoirdupois Weight | | |
|---|---|---|
| 16 drachms | = | 1 ounce (oz). |
| 16 ounces | = | 1 pound (lb.). |
| 28 or 25 pounds | = | 1 quarter (qr.). |
| 4 quarters | = | 1 hundredweight (cwt.). |
| 20 hundredweights | = | 1 ton (T.). |

The arithmetic then goes on to give some explanations. "Troy Weight," it says, "is used in weighing gold, silver, jewels, liquors, etc., and for ingredients used in philosophical experiments." How "*jewels*" and "*liquors*" got together, what the "*etc.*" may mean (mayhap red-herrings' eyes), what that word "*ingredients*" may signify, are questions outside the range of our finite knowledge. "Apothecaries' weight is used by apothecaries and chemists in mixing medicines, but drugs are bought and sold by avoirdupois weight." So! But the book forgets to tell us how apothecaries and chemists are bought, sold, and mixed. "By avoirdupois weight, coarse and bulky goods are weighed, and *all* the common necessaries of life," such things presumably as cabbage, chewing-gum, base-balls, cigarettes, tooth-picks, and the like (liquors and jewels of course excepted).

Here, then, are the three systems of weights in actual use among beings supposed to be endowed with reason. The first and second agree in two points only, viz., in the number of ounces (12) in the pound, and in the number of grains (480) in the ounce, but they reach the latter result by different roads. From these data luckily the number of grains in the pound turns out, in both cases, to be the same, *i.e.*, 5760.

The third, the avoirdupois, agrees with nothing except itself, and even with that but poorly. Some one, peace to his mud, has said that boys are liars by constitution. We don't believe it, for we remember having been boys ourselves; but when the boy is told by his infallible school-marm, backed up by the infallible arithmetic, that 28 pounds make *one-quarter* of a *hundredweight*, is he to be blamed if his eye for truth takes on a permanent cast? The lie will be four times as big when you tell him that one-hundred-and-twelve pounds make one-hundredweight. Is this the way to instil a spirit of truthfulness into the minds of the rising generation? True, our American arithmetics have corrected this, but it is taught yet to millions of youth in other lands.

We are not done with this avoirdupois weight yet, for we would like to know, not from mere curiosity, but for real practical purposes, how many grains it has to the pound. The "table" is silent, but a supplementary note will tell us that the number is 7000, so with slate and pencil, we figure out that the avoirdupois pound is equal to $1\frac{31}{144}$ lbs. troy, while the troy ounce, to turn the tables again, is equal to $1\frac{17}{175}$ ounce avoirdupois. After that, show us in all this broad land, the boy or girl, or even the woman or man who can tell us, off-hand, how many grains there are in an ounce, and how many there are in a drachm, avoirdupois. We know, because we have just made the calculation; but before you look at the answers, just shut your eyes, and hold your breath, and see if you are ready to answer; but don't all speak at once, please. Even if you should happen to know the former, you will almost surely miss the latter. The answers are: Four-hundred and thirty-seven and a half ($437\frac{1}{2}$) grains in the ounce and twenty-seven and eleven-thirty-seconds ($27\frac{11}{32}$) grains in the drachm. What a mess!

We turn now to our measures of length. This ought to be a straight-forward business, and we look up, if perchance a ray of light may dawn upon us here. A glance at the "table" soon dispels the illusion, for,

| | | | |
|---|---|---|---|
| 3 | barleycorns | | |
| or 11 | lines | = | 1 inch. |
| or 12 | lines | | |
| 12 | inches | = | 1 foot. |
| 3 | feet | = | 1 yard. |
| $5\frac{1}{2}$ | yards | = | 1 rod, perch, or pole. |
| 40 | rods | = | 1 furlong. |
| 8 | furlongs | = | 1 mile. |
| 3 | miles | = | 1 league. |
| 60 | geographical miles | = | 1 degree on the equator, or on a meridian. |
| $69\frac{1}{6}$ | statute miles (about) | = | 1 degree. |

Ah, that poor little inch that don't know whether it is made of barleycorns or lines, or, if so, of how many. Then the idea of making a mixed number ($5\frac{1}{2}$) of one denomination equal to the unit of the next is unworthy of the intelligence of a Hottentot or Bushman.

Belonging under the head of Linear Measure, we have another table made expressly, it seems, for tailors and milliners, which starts out unblushingly with a mixed number, thus:

| | | | |
|---|---|---|---|
| $2\frac{1}{4}$ | inches | = | 1 nail (kind not stated). |
| 4 | nails | = | 1 quarter. |
| 4 | quarters | = | 1 yard. |

This is nice, but read on:

| | | | |
|---|---|---|---|
| 3 | quarters | = | 1 ell Flemish. |
| 5 | quarters | = | 1 ell English. |
| 6 | quarters | = | 1 ell French. |

And so the poor tailor or milliner must find out whether his customer is a Fleming, or an Englishman, or a Frenchman before he can know how much material he must give him for an ell. How about it if the customer had already been naturalized? What absurd methods and measures!

When we come to Square Measure, we expect to find the absurdities increase in proportion to the square, and we will not be disappointed, for we get:

| | | | |
|---|---|---|---|
| $30\frac{1}{4}$ | square yards | = | 1 square rod. |
| or $272\frac{1}{4}$ | square feet | = | 1 square rod. |

To begin with square inches, square feet, square yards, and square rods is natural under the circumstances and innocent enough, but as soon as that is over we lose our way and strike off into what has no existence in Linear Measure at all, viz., roods and acres, for:

| | | | |
|---|---|---|---|
| 40 | square rods | = | 1 rood. |
| 4 | roods | = | 1 acre. |
| So that 160 | square rods | = | 1 acre. |

Therefore the side of a square whose area is one acre must be $\sqrt{160}$ rods, which gives us 12.6491 . . . . rods. The dots indicate that we have not *finished* the operation; neither do we intend to do so, for it would take too long. What we have struck here is not merely a case of a repeating decimal, as 0.333 . . . . which, though endless as a decimal, can nevertheless be expressed with perfect exactness by the vulgar fraction $\frac{1}{3}$, or by the duodecimal 0.4. No, the disease here is deeper-seated yet, for the exact square root of 160 cannot be extracted at all. Of the several ways of showing this, one will be sufficient.

Reducing to prime factors we have $(2 \times 2)(2 \times 2)(2 \times 5) = 160$. Now, unless all the prime factors of a number can be grouped by twos, so that each group shall be a perfect square, then the number itself will not be a perfect square. But, of the three groups above, the first and second are perfect squares; the third ($2 \times 5 = 10$) is not. For the final digit of a perfect square must be 1, or 4, or 5, or 6, or 9, or one of these followed by an even number of zeros. Now 10 does not fulfil either of these conditions; therefore, it is not a perfect square, and therefore 160 is not a perfect square. Hence, no created intelligence will ever be able to give us in rods, feet, or inches the exact length of the side of a square acre. And yet that acre is the *unit* of land measure for those who measure by rods, feet, and inches, and who expect one day to possess *all* the land.

In Cubic Measure we look for things solid. Well, the cubic inch, foot, and yard are derived honestly from the linear inch, foot, and yard, and are neither better nor worse than their origin; but when we have reached that point the table immediately branches off wildly into:

| | | | |
|---|---|---|---|
| 40 | cubic feet of round timber | = | 1 ton or load. |
| or 50 | cubic feet of hewn timber | | |
| 128 | cubic feet | = | 1 cord of wood. |

How would it be if the timber were hewn *round*, as in masts and spars? But enough. The Anglo-Saxon tongue, rich as it is in strong terms, fails to supply words to characterize such a . . . . (no use, it won't come).

In Liquid Measure things ought to run smoothly, and, at first sight, they do seem a little better than elsewhere. Thus:

| | | | |
|---|---|---|---|
| 4 | fluid ounces | = | 1 gill. |
| 4 | gills | = | 1 pint. |
| 2 | pints | = | 1 quart. |
| 4 | quarts | = | 1 gallon. |
| $31\frac{1}{2}$ | gallons | = | 1 barrel. |
| 63 | gallons (2 barrels) | = | 1 hogshead. |
| 2 | hogsheads | = | 1 pipe. |
| 2 | pipes | = | 1 tun. |

The gill, pint, quart, and gallon we can manage to swallow, but the barrel, hogshead, pipe, and tun are myths and frauds and arithmetical falsehoods.

Moreover, as Liquid Measure is a measure of volume, we naturally want to know what relation there is between it and Cubic Measure, which is also a measure of volume. Now:

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | gallon | = | 231 cubic inches | = | 8.33888 . . . . | lbs. distilled water |
| or 1 | gallon | = | $277\frac{1}{4}$ cubic inches | = | 10. | lbs. distilled water |
| or 1 | gallon | = | 282 cubic inches | = | 10.171325 . . . . | lbs. distilled water |
| or 1 | gallon | = | various other measures. . . . . | | | |

As if this were not bad enough, we have a Dry Measure in which 32 quarts = 1 bushel, and as 4 quarts = 1 gallon, we infer that 8 gallons would be equal in volume to 1 bushel. Therefore the bushel should be:

| | | | | |
|---|---|---|---|---|
| 8 | × | 231 | = | 1848 cubic inches |
| or 8 | × | $277\frac{1}{4}$ | = | 2220 cubic inches |
| or 8 | × | 282 | = | 2256 cubic inches |

Yet it is not, for the bushel is defined, *by law*, as 2150.4 cubic inches; and then, by working back again, we would find our quarts and pints shaky, and we get bewildered entirely. There is then no simple relation between our Weights and Measures, although a futile attempt was made to establish something of the kind by the introduction of that insane *fluid ounce*.

We have not by any means exhausted the potential confusion of this matter; there are yet other tables in the arithmetic, and pages of others in the "Dispensatory," just crammed with similar incongruities, traps, and pitfalls; so much so that there is probably no sane man living who could give from memory all the curls and twists of our systems of Weights and Measures.

Knowing how they originated, it is not hard to account for their vagaries. They were not invented systematically, or built up on any prearranged plan, but just grew up, bit by bit, according to needs and whims of different tribes. Later on, every once in awhile, some king would try to disentangle the snarl by enacting laws to *regulate* the existing standards of Weights and Measures. Their intentions were honest enough, and they accomplished some little good, but it was only palliative, not curative. They did not go to the bottom of the evil, and half-way remedies are usually worthless. The evil here is in the jumble of inconsistencies in the denominations of the systems, taken either separately or in relation to one another, and in the absurdity of having more than one system anyhow. These are the arithmetical sins against good sense which have been handed down to us. Are we going to hand them down to posterity, or are we going to wipe them off the escutcheon of our race, and hand it along, clean and untarnished, to our successors?

A desperate attempt to remedy this state of confusion, in the matter of weights and measures, has indeed been already made by the French, in their metric system, and it becomes necessary for the better understanding of what follows, to say a word about it here. For standard of length they took what they supposed to be the *ten-millionth* part of a quadrant of a meridian, and called it the "*metre*." We have Americanized the word to "*meter*." Its value is 39.37043 . . . inches, about. Then, each denomination ascending is *ten* times as great as the preceding one, and a Greek prefix is used to indicate this. Each denomination descending is *one-tenth* as large as the preceding, and a Latin prefix does duty here. Thus:

| Ascending. | | | Descending. | | |
|---|---|---|---|---|---|
| 10 meters | = | 1 *deca*meter. | 0.1 meter | = | 1 *deci*meter. |
| 10 decameters | = | 1 *hecto*meter. | 0.1 decimeter | = | 1 *centi*meter. |
| 10 hectometers | = | 1 *kilo*meter. | 0.1 centimeter | = | 1 *milli*meter. |
| 10 kilometers | = | 1 *myria*meter. | | | |

And then their vocabulary seems to have given out.

Square Measure and Cubic Measure are derived directly from the Linear Measure, by the simple process of squaring and cubing, and no fag-end irregularities are tolerated.

For unit of weight they took a cubic centimeter of water, and called it a "gramme." We have shortened it to "gram." The prefixes for higher and lower denominations are the same as above, thus :

| | | | |
|---|---|---|---|
| | 10 grams | = | 1 decagram. |
| | 10 decagrams | = | 1 hectogram, etc., etc. |
| And, | 0.1 gram | = | 1 decigram. |
| | 0.1 decigram | = | 1 centigram, etc., etc. |

For Liquid and Dry Measures, the French use their Cubic Measure, but since for ordinary work, the cubic centimeter is too small, and the cubic meter too large, they adopted as a convenient unit the cubic decimeter, which is a little more than our quart. This they called the "litre" (written "liter" by us, and pronounced lee-ter). From this they have decaliter, hectoliter, kiloliter, deciliter, centiliter, and any others that may be desired.

This system is in itself excellent because it establishes a clear, simple, obvious relation between weights and measures, so that we can pass from one to the other without labor, and at the same time be sure of perfectly exact results. This is the reason why it has been adopted by several countries, and by men engaged in scientific pursuits, the world over. Yet, although it has been legalized in the United States and in Great Britain and her colonies generally, more than twenty-five years, English-speaking people *will not* adopt it. The fact is that it has one serious, fatal drawback, that of having been founded on the decimal system of numeration. Had the duodecimal system been brought into use, and a metric system been founded on *it*, the result would have been as near perfection as the nature of numbers will admit of, and such a system, once started, would have gone on, conquering and to conquer, till not a rag of a decimal would be left floating over any spot on the face of the earth. It is time now. Come on comrades, fall in, and keep step in the ranks of a new and true intellectual progress.

Before entering into further details, it becomes necessary to make a digression in order to settle upon a nomenclature for our duodecimal system.

Clearly, it will not do to use the same word-combinations as in the effete decimal system, because the *meanings* would not correspond to the words. Thus, when we write 25 and pronounce it "twenty-five," we mean two tens + five, but in the duodecimal system 25 would mean two twelves + five, and we must have a method of naming which will indicate this fact. Now, we look upon it as a first principle that the names of a simple number (*i.e.*, the name of a single digit) should be a simple (not compound) word, and that the name of a combinational number (formed of two or more digits), should be derived from the names of the components. Most, if not all, systems of nomenclature have violated this principle, more or less.

In English the compound sign 11, which means *ten-one* (one-ten) is called *eleven* in which word there is no very *obvious* trace of either the word *ten* or the word *one*. The same is true with regard to *twelve*. We do not mean to say that these words did not mean originally one-ten, two-ten; we believe they did, but the relationship has been very much obscured. When we get to *thirteen*, the relation to *three-ten* is clear, and from that on to the end of numbers everything is lovely. The French, not content with these irregularities have allowed yet others to creep in. Thus they have names which, if translated literally, would read: sixty-nine (69), sixty-ten (70), sixty-eleven (71), etc., . . . up to sixty-nineteen (79), and then, four-twenty (80), four-twenty-one (81), etc., . . . up to four-twenty-nineteen (99), and even, though rarely used, six-twenty for 120.

In the scheme proposed below we intend to sweep away all irregularities of whatever kind, and to make the whole nomenclature perfectly regular and consistent. The *names* proposed may not be final; better ones may perhaps be found, and if any one has better to offer, let him stand forth and do so; but the *things* must stand. The table, we think, almost explains itself, yet, to forestall any possible hard feelings, we put, right here, what few remarks we have to make.

The figures in parentheses are of the decimal system, all the others are of the duodecimal. We use the ten digits (0 to 9 included) of the defunct decimal system, with their ancient names, except in the case of "seven" which is ccntracted to "sen." As remarked before, we use provisionally t for ten and l for eleven, which is shortened to "len." The next number being a combinational one, we use two symbols (10), and for a name we contract our present "twelve" to "tel." Then, to be systematically exact, we hitch on the zero, and hence to 10 = tel-zero (formerly twelve), but in practice the word zero will be omitted, and this has been indicated by enclosing it in parentheses.

Having adopted "tel," we coin a new adjective, "*telimal*," and make a vow never to use that worn-out, unnecessary, cumbersome word, duodecimal, again. Also "telth" will take the place of twelfth, "senth" of seventh and "lenth" of eleventh. Another thing to be noticed is that in *true combinational names*, when the larger number precedes, addition is meant, as $16 = 10 + 6 =$ tel-six (the obsolete eighteen); and, on the contrary, when the smaller number precedes, multiplication is intended, as sixtel = six $\times$ tel = $6 \times 10 = 60$ (the obsolete seventy-two).

Telimal (Duodecimal) Nomenclature

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (0) | 0 | Zero | (49) | 41 | fortel-one | (98) | 82 | eightel-two |
| (1) | 1 | one | (50) | 42 | fortel-two | (99) | 83 | eightel-three |
| (2) | 2 | two | (51) | 43 | fortel-three | (100) | 84 | eightel-four |
| (3) | 3 | three | (52) | 44 | fortel-four | (101) | 85 | eightel-five |
| (4) | 4 | four | (53) | 45 | fortel-five | (102) | 86 | eightel-six |
| (5) | 5 | five | (54) | 46 | fortel-six | (103) | 87 | eightel-sen |
| (6) | 6 | six | (55) | 47 | fortel-sen | (104) | 88 | eightel-eight |
| (7) | 7 | sen | (56) | 48 | fortel-eight | (105) | 89 | eightel-nine |
| (8) | 8 | eight | (57) | 49 | fortel-nine | (106) | 8t | eightel-ten |

Telimal (Duodecimal) Nomenclature (cont'd)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (9) | 9 | nine | (58) | 4t | fortel-ten | (107) | 8l | eightel-len |
| (10) | t | ten | (59) | 4l | fortel-len | (108) | 90 | ninetel-(zero) |
| (11) | l | len | (60) | 50 | fivetel-(zero) | (109) | 91 | ninetel-one |
| (12) | 10 | tel | (61) | 51 | fivetel-one | (110) | 92 | ninetel-two |
| (13) | 11 | tel-one | (62) | 52 | fivetel-two | (111) | 93 | ninetel-three |
| (14) | 12 | tel-two | (63) | 53 | fivetel-three | (112) | 94 | ninetel-four |
| (15) | 13 | tel-three | (64) | 54 | fivetel-four | (113) | 95 | ninetel-five |
| (16) | 14 | tel-four | (65) | 55 | fivetel-five | (114) | 96 | ninetel-six |
| (17) | 15 | tel-five | (66) | 56 | fivetel-six | (115) | 97 | ninetel-sen |
| (18) | 16 | tel-six | (67) | 57 | fivetel-sen | (116) | 98 | ninetel-eight |
| (19) | 17 | tel-sen | (68) | 58 | fivetel-eight | (117) | 99 | ninetel-nine |
| (20) | 18 | tel-eight | (69) | 59 | fivetel-nine | (118) | 9t | ninetel-ten |
| (21) | 19 | tel-nine | (70) | 5t | fivetel-ten | (119) | 9l | ninetel-len |
| (22) | 1t | tel-ten | (71) | 5l | fivetel-len | (120) | t0 | tentel-(zero) |
| (23) | 1l | tel-len | (72) | 60 | sixtel-(zero) | (121) | t1 | tentel-one |
| (24) | 20 | twitel-(zero) | (73) | 61 | sixtel-one | (122) | t2 | tentel-two |
| (25) | 21 | twitel-one | (74) | 62 | sixtel-two | (123) | t3 | tentel-three |
| (26) | 22 | twitel-two | (75) | 63 | sixtel-three | (124) | t4 | tentel-four |
| (27) | 23 | twitel-three | (76) | 64 | sixtel-four | (125) | t5 | tentel-five |
| (28) | 24 | twitel-four | (77) | 65 | sixtel-five | (126) | t6 | tentel-six |
| (29) | 25 | twitel-five | (78) | 66 | sixtel-six | (127) | t7 | tentel-sen |
| (30) | 26 | twitel-six | (79) | 67 | sixtel-sen | (128) | t8 | tentel-eight |
| (31) | 27 | twitel-sen | (80) | 68 | sixtel-eight | (129) | t9 | tentel-nine |
| (32) | 28 | twitel-eight | (81) | 69 | sixtel-nine | (130) | tt | tentel-ten |
| (33) | 29 | twitel-nine | (82) | 6t | sixtel-ten | (131) | tl | tentel-len |
| (34) | 2t | twitel-ten | (83) | 6l | sixtel-len | (132) | l0 | lentel-(zero) |
| (35) | 2l | twitel-len | (84) | 70 | sentel-(zero) | (133) | l1 | lentel-one |
| (36) | 30 | thirtel-(zero) | (85) | 71 | sentel-one | (134) | l2 | lentel-two |
| (37) | 31 | thirtel-one | (86) | 72 | sentel-two | (135) | l3 | lentel-three |
| (38) | 32 | thirtel-two | (87) | 73 | sentel-three | (136) | l4 | lentel-four |
| (39) | 33 | thirtel-three | (88) | 74 | sentel-four | (137) | l5 | lentel-five |
| (40) | 34 | thirtel-four | (89) | 75 | sentel-five | (138) | l6 | lentel-six |
| (41) | 35 | thirtel-five | (90) | 76 | sentel-six | (139) | l7 | lentel-sen |
| (42) | 36 | thirtel-six | (91) | 77 | sentel-sen | (140) | l8 | lentel-eight |
| (43) | 37 | thirtel-sen | (92) | 78 | sentel-eight | (141) | l9 | lentel-nine |
| (44) | 38 | thirtel-eight | (93) | 79 | sentel-nine | (142) | lt | lentel-ten |
| (45) | 39 | thirtel-nine | (94) | 7t | sentel-ten | (143) | ll | lentel-len |
| (46) | 3t | thirtel-ten | (95) | 7l | sentel-len | | | |
| (47) | 3l | thirtel-len | (96) | 80 | eightel-(zero) | | | |
| (48) | 40 | fourtel-(zero) | (97) | 81 | eightel-one | | | |

Of course the t's and the l's look strange. We are waiting for some artist to invent better forms.

To pass to numbers of a higher order than are found in the table we need a few new names. Prof. Smith suggests, and the suggestion seems an excellent one, to use "po" (power or position) with the Greek prefixes, di, tri, etc., for coefficients. Hence "dipo" would mean the second power of tel, *i.e.*, $10 \times 10 = 100$ (144 formerly). Next use Latin prefixes for telimal fractions, and you get as follows:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| tel | = | 10. | | | telth | = | $10^{-1}$ | = | 0.1. |
| dipo | = | $10^2$ | = | 100. | semipo | = | $10^{-2}$ | = | 0.01. |
| tripo | = | $10^3$ | = | 1,000. | tertipo | = | $10^{-3}$ | = | 0.001. |
| tetrapo | = | $10^4$ | = | 10,000. | quartipo | = | $10^{-4}$ | = | 0.0001. |
| pentapo | = | $10^5$ | = | 100,000. | quintipo | = | $10^{-5}$ | = | 0.00001. |
| hexapo | = | $10^6$ | = | 1,000,000. | sexipo | = | $10^{-6}$ | = | 0.000001. |
| heptapo | = | $10^7$ | = | 10,000,000. | septipo | = | $10^{-7}$ | = | 0.0000001. |
| octapo | = | $10^8$ | = | 100,000,000. | octipo | = | $10^{-8}$ | = | 0.00000001. |
| ennapo | = | $10^9$ | = | 1,000,000,000. | nonipo | = | $10^{-9}$ | = | 0.000000001. |
| dekapo | = | $10^t$ | = | 10,000,000,000. | decipo | = | $10^{-t}$ | = | 0.0000000001. |
| endekapo | = | $10^l$ | = | 100,000,000,000. | undecipo | = | $10^{-l}$ | = | 0.00000000001. |
| dodekapo | = | $10^{10}$ | = | 1,000,000,000,000. | dodecipo | = | $10^{-10}$ | = | 0.000000000001. |

This will probably be enough, upwards and downwards, for all practical purposes. If, however, any one should have need to count higher or lower, we would advise him to hire a clerk, who can be allowed to waste himself away in writing out Greek and Latin prefixes, to no end.

A word now about the *reading* of numbers in this system. Several ways will suggest themselves; we give one which we think short, clear and simple. For example, 3,8t6,2l1,794,052. Divide it into periods of three figures each, and then read:

*Three dodeka, eight-ten-six enna, two-len-one hexa, sen-nine-four tripo, fiftel=two*; requiring 22 syllables made up of 64 letters. Now, translating that number into its equivalent in the decimal system, we have 33,343,759,669,310 which is read: *Thirty-three trillions, three-hundred and forty-three billions, seven-hundred and fifty-nine millions, six-hundred and sixty-nine thousand, three-hundred and ten*; requiring 38 syllables made up of 134 letters. In this example, taken entirely at random, there is a clear gain of more than 42 per cent. in the reading and of more than 52 per cent. in the writing, by the use of the telimal system. In reading, the "po" is omitted whenever its omission will cause no ambiguity. End of the digression.

We return now to our weights and measures in order to show how they may be brought under the rule of the *Telimal.*

The first thing we need is a standard of length, but there is no use whatever in searching the earth and the skies for a so-called *natural* unit. Any handy length will do, and fortunately we have just such a one already, the yard, which is preserved with such infinite care by our own government as well as by that of Great Britain.

Taking that as a unit, and applying the telimal system, we have forthwith:

| | | | | |
|---|---|---|---|---|
| 10 yards | = | 1 tel yard | = | (12 yards old style). |
| 10 tel yards | = | 1 dipo yard | = | (144 yards old style). |
| 10 dipo yards | = | 1 tripo yard | = | (1728 yards old style). |
| 10 tripo yards | = | 1 tetrapo yard, etc. to any extent whatever. | | |

The tripo yard would serve admirably as a unit for long distances, being nearly equal to our present mile.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 yard | = | 10 telth yards | | | = | 1 yard. |
| 1 telth yard | = | 10 semipo yards | = | 0.1 yard | = | (3 inches, old style). |
| 1 semipo yard | = | 10 tertipo yards | = | 0.01 yard | = | ($\frac{1}{4}$ inch, old style). |
| 1 tertipo yard | = | 10 quartipo yards | = | 0.001 yard | = | ($\frac{1}{48}$ inch, old style), |

etc., down to molecular and atomic dimensions.

The telth yard is just a handy length for ordinary small measures, while the semipo yard ($\frac{1}{4}$ inch) is employed constantly in every workshop in the land.

For square and cubic measures the same unit (the yard) squared or cubed, with its multiples and subdivisions, will be used.

Of these, the dipo yard square would be something over four of our acres. This should hardly be thought too large a unit for a country like this; or, at any rate, if you prefer, you can buy a quarter dipo yard instead of a whole one.

The telth yard cube would give a convenient unit for liquid and dry measure, being equal to 27 of our cubic inches, or just a little less than our pint. All higher and lower denomination would of course be telimal multiples and subdivisions of these.

For weights, the standard unit would be the weight of the unit of volume (the telth yard cube) of distilled water, at its maximum density; this would very nearly correspond to our present pound. Multiples and subdivisions as usual.

For a "*set of weights*" to be used with a balance, the 1, 2, 3, 6, would probably be the most convenient; for, taken either separately or by addition, they would serve for weighing anything from *one* up to the unit of the next higher rank inclusively. Thus,

| | | |
|---|---|---|
| With weight 1 we can weigh one | = | 1. |
| With weight 2 we can weigh two | = | 2. |
| With weight 3 we can weigh three | = | 3. |
| With weights 1 + 3 we can weigh four | = | 4. |
| With weights 2 + 3 we can weigh five | = | 5. |
| With weight 6 we can weigh six | = | 6. |
| With weights 1 + 6 we can weigh sen | = | 7. |
| With weights 2 + 6 we can weigh eight | = | 8. |
| With weights 3 + 6 we can weigh nine | = | 9. |
| With weights 1 + 3 + 6 we can weigh ten | = | t. |
| With weights 2 + 3 + 6 we can weigh len | = | l. |
| With weights 1 + 2 + 3 + 6 we can weigh tel | = | 10 (the unit of the rank above). |

For larger numbers take 10, 20, 30, 60, and the same advantages of the greatest return from the least outlay, remains.

An important factor in the running of the machinery of this world is money. We are all naturally somewhat interested in the question of mono-metallism, bi-metallism, silver bills and the like, but these phases of the money question do not enter here. We are concerned just now with the arithmetic of money. All things considered, we think our telth pound of coin silver (len-telths fine), which is very near our present dollar, would be the best unit. Then divide and multiply that to your heart's content, but always telimally.

The coins or bills to be issued should be according to the scheme given above for weights, viz., the 1's, 2's, 3's, 6's of each rank. This would greatly facilitate the counting and reduce the handling of money; besides, silver coins could then be used for weights on a small scale.

The French, in their metric system, tried to get a grip on circular and time measure, but failed completely. We hope to be more successful.

In circular measure we have now,

| | | |
|---|---|---|
| Sixty seconds | = | one minute. |
| Sixty minutes | = | one degree. |
| Sixty degrees | = | one sextant. |
| Six sextants | = | one circle. |

This is probably a remnant of an attempted *sexagesimal* system. In time measure we have:

| | | |
|---|---|---|
| Sixty seconds | = | one minute. |
| Sixty minutes | = | one hour, and then we go wild again, for, |
| Twenty-four hours | = | one day. |

Now, a day is measured by one turn in a circle of a point on the earth, and an every-day problem is to convert time into longitude; and yet the *words* which correspond to minute and second, do not mean the same *things*; for a minute of time equals fifteen minutes of longitude. Comment is unnecessary.

In our new system circular and time measures are the same. After mature consideration we have come to the conclusion that it will perhaps be better to take the semi-circumference as a unit instead of the entire circle. This granted, divide and subdivide that unit according to the telimal system, and you have:

Tel fourths ($^{\text{iv}}$) = $1'''$ ( = about $\frac{1}{6}$ second, old style).

Tel thirds = $1''$ (= about 2 seconds, old style).

Tel seconds = $1'$ ( = 25 seconds, old style).

Tel primes = $1^\circ\begin{cases}\text{grade (of time) (= 5 minutes, old style).}\\ \text{degree (of cycle) } (= 1\frac{1}{4}^\circ\text{, old style).}\end{cases}$

Tel$\begin{Bmatrix}\text{grades}\\ \text{degrees}\end{Bmatrix}$= 1 hour (= 1 hour = 15°, old style).

Tel hours = $\begin{cases}\text{1 day = 1 night (of time) (= 12 hours, old style).}\\ \text{1 hemicycle (of circle) (= 180°, old style).}\end{cases}$

If it were preferred to use the whole circumference as a unit, then each of the above values would be doubled. This would make the *hour* very long.

Holding close relationship with the foregoing is the division of the year. The day (as used now) is a *natural* division of time which man can neither lengthen nor shorten; the week of sen (old seven) days has been settled by higher legislation than ours, and it behooves us to leave it alone. The year is another natural unit of time, and its division into tel months falls in with our telimal system. The only regulating that should be attempted in this matter is to make the months of equal length by giving to each thirty days. There will then remain five days over in common years and six in leap-years. Let these be made legal holidays, belonging to no month, and bearing no interest, and let them be spaced at even distances throughout the year, as follows. We leave the numbers in decimal notation:

| | | | |
|---|---|---|---|
| | 1 | day | New Year's Day |
| January, | 30 | days. | |
| February, | 30 | days. | |
| | 1 | day | Leap-Year Day every fourth year. |
| March, | 30 | days. | |
| April, | 30 | days. | |
| | 1 | day | Spring (Vernal?) Day. |
| May, | 30 | days. | |
| June, | 30 | days. | |
| | 1 | day | Independence Day (put the "Glorious *old* Fourth" here). |
| July, | 30 | days. | |
| August, | 30 | days. | |
| | 1 | day | Autumn Day |
| September, | 30 | days. | |
| October, | 30 | days. | |
| | 1 | day | Election Day every fourth year, but celebrate it every year. |
| November, | 30 | days. | |
| December, | 30 | days. | |

This suggestion about the year is considered only as an ornament, and whether it be adopted or not, the telimal system will not suffer any loss.

In looking back over our "tables," we are struck with a curious fact, which seems to indicate that there always existed a natural longing for that number *twelve*.

The *old* multiplication tables always went to twelve times twelve. Then we have:

| | | |
|---|---|---|
| Twelve pence | = | 1 shilling. |
| Twelve lines | = | 1 inch. |
| Twelve inches | = | 1 foot. |
| Twelve ounces | = | 1 pound (in two systems). |
| Twelve units | = | 1 dozen. |
| Twelve dozen | = | 1 gross. |
| Twelve gross | = | 1 great-gross. |
| Twelve hours | = | 1 day (often used thus). |
| Twelve signs | = | 1 zodiac. |
| Twelve months | = | 1 year. |

Is it not by the *dozen* that you buy shirts, collars, cuffs, socks, eggs, cups and saucers, knives and forks, spoons, hardware, chemical-ware, handkerchiefs, buttons, candles, and dozens and dozens of the smaller articles of every-day use? No other number occurs anything like so often in the tables, except that most useful factor of twelve, viz., four; but ten and five are conspicuously absent. This shows that common sense, in spite of the heathen decimal, still succeeded, to some extent, in making itself heard. Who ever wanted to buy $\frac{1}{5}$ of a pound of tea, or $\frac{1}{10}$ of a gallon of molasses? The five and the ten, *i.e.*, the decimal system, is doomed, and the sooner it is knocked down and carried out the better.

To sum up, the telimal system has, among others, the following advantages over the decimal:

It is more concise; it has fewer repeating fractions and, therefore, less need of the crazy vulgar fractions; it is very much more exact in the matter of approximations; the divisibility of its base stamps it as the *natural* system of numeration; arithmetical operations in it are easier, especially multiplication; it has no perplexing tables to be learned and forgotten, or left, as is so often the case. unlearned altogether; it requires no multiplications or divisions to reduce from one denomination to another; it is just as easy, in it, to handle compound numbers of *any kind* as to work with abstract numbers; much of what to us is now hard and tedious work will be done with ease, even mentally, in the telimal system.

To the astronomer, the surveyor, the physicist, the chemist; the mechanical, the civil, the electrical engineer, the architect; to every one who uses an instrument of precision; to the machinist, the carpenter, to the business man, the teacher, the scholar; to every one who deals largely in figures, it would save more than one year in every tel years if the change were made.

And, after all, the change is not so very great. Two new symbols, three or four new names, a few Greek and Latin prefixes, and all is ready. A grown man would *understand* it at sight, be able to use it in a week or two, be perfect in it in a couple of months, *think* in it at the end of a year, and would bless God for the rest of his days for having fallen in with so superior a method of managing numbers. The school-boy would grow fat on it, and know more arithmetic at tel years of age than his father did at twitel.

The only real difficulty in the way is the unlearning of the decimal system; and "the beast with ten horns" would make a hard fight, but that applies only to the unfortunates of this generation. "*Whatever man has done man may do*." Six or seven centuries back all western Europe abandoned its old systems of numeration and adopted the decimal. That system has been weighed in the balance and found wanting. Where should be the insurmountable difficulty in throwing it over now and in putting in its place the most perfect system which the nature of numbers will admit of, and which will never need replacing so long as the world endures—nor afterwards?

A century back France woke up and brushed away all her foolish old systems of measures and weights, introduced her metric system, and induced several other countries to do the like. In a short time everybody was talking "kilogrammes," "centimetres," "hectolitres," etc., or words to that effect, and yet no bones were broken over the matter, the literature of the country was not overthrown, nor was the science of mathematics destroyed. Yet the change proposed here is no more violent than was then brought about.

About the same time the United States gave up *£*, *s.*, *d.*, and introduced $, dimes, cents, without creating a revolution or stirring up any bad blood. The change proposed now is dipo, tripo, tetrapo times more useful, and the cost bears no proportion to the gain.

"Let us then be up and doing;"
  Rouse ye now, ye valiant men,
The light of science still pursuing,
  Thunder o'er the ranks of TEN.
Onward to the conflict press ye,
  Bearing high the flag of TEL;
And may coming ages bless ye,
  Proclaiming that ye have done well.

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