The Dozenal Society of America
The Aspirant's Tests

Introduction

For many years, the Dozenal Society of America required prospective members to enroll first as "aspirants," and pass four aspirant's tests prior to admission as members. While the Society no longer requires passing these tests for new members, they are still an excellent way to gain facility in the use of the dozenal system; and while some of these tests are probably beyond what the average citizen will need or be interested in, at least the first two will be extremely useful. Therefore, we offer this remastered version of the Aspirant's Tests; may members and non-members alike find them helpful.

Aspirant's Test Number 1

1. What does "duodecimal" mean?Answer

2. What does "dozenal" mean?Answer

3. What system of counting is in general use?Answer

4. How many symbols are used in decimal counting? What are they?Answer

5. How many are used in the dozenal counting?Answer

6. What is the function of the 0 or zero?Answer

7. What is meant by place-value?Answer

8. Has the zero the same functions in dozenals as in decimals?Answer

9. How is the zero used in fractionals?Answer

ᘔ. What is the general theory of duodecimal counting?Answer

Ɛ. What new symbols are used, and what are they called?Answer

10. What is 10; called, and what does it mean? (The semicolon, ";", indicates duodeci-

mals.)Answer

11. Write in sequence the dozenal numbers from 1 to 100; and under them the corresponding decimal numbers.Answer

12. In decimal counting, what would be the number of this question?Answer

13. What is the quantity commonly known as twelve called, duodecimally, and why?Answer

14. What dozenal quantity corresponds to the decimal quantity 144, and what is it called? Why?Answer

15. 10; articles can be arranged in how many equal piles of how many articles in each pile?Answer

16. 10. articles can be arranged in how many equal piles of how many articles in each pile?Answer

17. There are 5280. feet in a mile. How many inches?Answer

18. There are 3080; feet in a mile. How many inches?Answer

19. Add the following:

$$\begin{array}{rrr} 14; & 689; & 6\text{Ɛ}95; \\ \underline{+\ 36;} & \underline{+\ 4\text{Ɛ}3;} & \underline{+\ 5238;} \end{array}$$

Answer

1Ƨ. Subtract the following:

$$\begin{array}{rr} 689; & 6\text{Ɛ}95; \\ \underline{-\ 4\text{Ɛ}3;} & \underline{-\ 5238;} \end{array}$$

Answer

1Ɛ. Convert the following to dozenals: 14., 30., 84., 100.Answer

20. Convert the following to decimals: 14; 30; 84; 100;.Answer

I hope you got some fun out of these questions. Next time you will get more problems in addition and subtraction, and some questions about the pattern of the duodecimal number system.

Be sure to question anything that is not clear. Try to learn to think in dozens. Avoid using decimal terms for duodecimal numbers.

Aspirant's Test Number 2

1. Add the following:

$$\begin{array}{rr} & 938 \\ 847 & 6425 \\ +\ 372 & +\ 8927 \end{array}$$

Answer

2. Add:

$$\begin{array}{rrrrr} 25062 & 123456 & 4648 & 332832 & 8798765 \\ 89073 & 234567 & 4363 & 23 1514 & 432 1082 \\ 38241 & 345678 & 3894 & 23 2956 & 987 6543 \\ +\ 78205 & 456789 & 5117 & 23 2437 & 210 8298 \\ & 567892 & 7479 & +\ 332694 & 765 4321 \\ & +\ 678932 & 1532 & & 23 9876 \\ & & 2721 & & +\ 543 2103 \\ & & +\ 6282 & & \end{array}$$

Answer

3. Subtract:

$$\begin{array}{rrrrr} 3874 & 43821 & 232229 & 123456 & 1111110 \\ -\ 2732 & -\ 9742 & -\ 33332 & -\ 78932 & -\ 987654 \end{array}$$

Answer

4. When the motorman started his one-man trolley-car, there were 5 passengers aboard, 3 men and 2 women. One of the women was carrying her 8-month-old son. At the first stop, 18 men and 7 women got on. At the second, 1 woman and 2 men got off, while 10 women and 2 men got on, and in addition their 2 sons, one 14 and the other 4 years old. At the next stop, 15 men and 11 women got off, including the woman with the baby. Next stop was the end of the line.

(a) How many persons were then on the car?
(b) How many males, how many females?
(c) How many persons rode altogether?
(d) How many paid fares?

(In this and the following problems, please give the process by which you reach your result.) Answer

5. In the prairie states the roads run on section lines, an even mile apart. Mr. Tweet, a farm adviser, travels here and there, telling the farmers why they are not making any money. Starting from his headquarters at the county seat, he drove 47 miles, then east 32, south 18, west 35, north 16, east 1Ɛ, south 46, west Ɛ, and quit for the night.

(a) How many miles did he travel?
(b) How far was he from his starting point when he quit?

Answer

6. Mr. Dybwad died, leaving a wife and 4 sons, Alfred, Benjamin, Charles, and David. Under his will $\frac{1}{2}$ of his estate went to his wife, excepting that out of her share she was to pay minor beneficiaries to the extent of $\frac{1}{10}$ of the total estate. Al got $\frac{1}{2}$ of his mother's entire half; Ben got half as much as Al; Charlie have as much as Ben; and Dave, poor guy, half as much as Charlie. The attorney got what was left. Dave's share was \$976.

(a) How much did each heir get?
(b) How much did the minor beneficiaries get?
(c) How much did the attorney get?
(d) What was the value of the estate?

Answer

7. Papa wanted to give Willie a lesson in systematic saving, so, on the first of April he said, "Now, Willie, I am dropping a penny into this box. Tomorrow I shall drop 2, the next day 4, and so on, doubling the number each day through the month. By the end of the month there will be a nice little pile of pennies." Assuming that Papa kept his promise,

(a) How many pennies would he drop on the last day of the month?
(b) How many altogether would be in the box?
(c) If it had been March?
(d) February?

Answer

8. Take any number of 3 digits. Double it. Double again. Add 398. Divide by 4. Subtract the original number plus Ɛ5. How much is left?Answer

Aspirant's Test Number 3

1. Multiply in duodecimals:

$$\begin{array}{rrr} 68\text{Ɛ}4 & 2\,0869 & 94\,10\text{Ƨ}8 \\ \times\ 37\text{Ƨ} & \times\ \text{Ɛ}85\text{Ƨ} & \times\ \text{Ɛ}\,\text{Ƨ}987 \end{array}$$

Answer

2. Divide in duodecimals:

$1034 \div 8$ $\qquad$ $1\,2074 \div \text{Ƨ}$ $\qquad$ $9\,7\text{Ɛ}16 \div 79$ $\qquad$ $63\,9560 \div 4\text{Ɛ}39$

Answer

3. In duodecimals there are 260° in a circumference. How many degrees apart are the points of a star with 3 points; 4; 5; 6; 8; 9; Ƨ; 10; 13; 16; 20; 26; 30; 34; 39; 50; 60; 76; Ƨ0; 130 points? This multiplicity of factors, I think, shows why the Babylonians so divided the circle in the first place.Answer

4. Complete Multiplication Table, Base Twelve

	2	3	4	5	6	7	8	9	↊	↋	10
2	4	6	8	↊	10	12	14	16	17	1↊	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	↊	13	18	21	26	2↋	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2↋	36	41	48	53	5↊	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
↊	18	26	34	42	50	5↊	68	76	84	92	↋0
↋	1↊	29	38	47	56	65	74	83	92	↋1	↋0
10	20	30	40	50	60	70	80	90	↋0	↋0	100

Using the multiplication tables to 10; by 10;, which correspond to the decimal tables of 12. by 12., write the tables for 11; and 12;, which will look very familiar.

	2	3	4	5	6	7	8	9	↊	↋	10	11	12
2	4	6	8	↊	10	12	14	16	17	1↊	20		
3	6	9	10	13	16	19	20	23	26	29	30		
4	8	10	14	18	20	24	28	30	34	38	40		
5	↊	13	18	21	26	2↋	34	39	42	47	50		
6	10	16	20	26	30	36	40	46	50	56	60		
7	12	19	24	2↋	36	41	48	53	5↊	65	70		
8	14	20	28	34	40	48	54	60	68	74	80		
9	16	23	30	39	46	53	60	69	76	83	90		
↊	18	26	34	42	50	5↊	68	76	84	92	↋0		
↋	1↊	29	38	47	56	65	74	83	92	↋1	↋0		
10	20	30	40	50	60	70	80	90	↋0	↋0	100		
11													
12													

Perform the following (all in duodecimals), using a single operation in multiplication, and short-division in dividing. Then remember that you are freely using the base dek 13 and 14 tables, which you would never think of learning for decimals:

$49↊ \times 11 =$
$376 \times 11 =$
$253 \times 12 =$
$6↊8 \times 12 =$
$5\,321↊ \div 11 =$
$5\,6478 \div 12 =$

Answer

5. The sun is distant $2718\,↋540$ miles from the earth. The duodecimal π is 3;1848. What

is the length of the earth's orbit? (Leave out the fancy astronomy and assume that the orbit is circular. This is just a matter of multiplication.Answer

6. Assuming circular orbits, if the length of the orbit of Mars is 2 098Ɛ 9880 miles, how near can it approach the earth?Answer

7. How many fractional parts of one hundred come out even? (Avoid duplications.) How many for the gross? How would you state their relative factorability?Answer

8. If the mile were 1000; yards instead of 1,760., what would be the reduction, in percentage? In per "grossage"?Answer

Aspirant's Test Number 4

This is the final test, the subject being the conversion of numbers from the decimal to the duodecimal radix, and the reverse. First study the conversion rules given on the accompanying sheet. (This is sheet is available from the Dozenal Society of America.) Then perform the following exercises, giving in full the methods by which the answers were reached.

1. Convert the following to dozenals (a) by Rule 1, and (b) by Rule 2:
1,920 18,671 324,810
Answer

2. Convert the results of Exercise 1 to decimals (a) by Rule 3, and (b) by Rule 4.Answer

3. Convert the following to common fractions of the dozen system:
$^{13}/_{4}$ $^{189}/_{670}$ $^{99}/_{100}$
Answer

4. Put them back as they were.Answer

5. Use Rule 5 for the following:
0.37 0.144 0.6336 0.916667
Answer

6. Use Rule 6 for the following:
0;Ƨ972 4Ƨ 0;3724 97 0;0555 55
Answer

7. Convert 18.62 by Rule 7, and recovert it by Rule 8.Answer

8. Convert the following by Rule 7, and back by Rule 8:
393.7 39.37 3.937 0.3937

Answer

9. Add the following, giving the answer in both systems. (Note the alternation of decimal and duodecimal numbers.)

$$\begin{array}{r} 23{,}540. \\ 1\mathcal{Z}\,\mathcal{E}987; \\ 422{,}604. \\ 32\,9\mathcal{Z}81; \\ 67{,}148. \\ +\ \mathcal{E}2\,4\mathcal{Z}15; \\ \hline \end{array}$$

Answer

Answers for Test Number 1

1. Relating to twelve or twelfths; counting by twelves.

2. Relating to dozens; counting by dozens; counting by dozens.

3. Base ten or decimal.

4. Ten: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

5. Twelve.

6. To mark the places.

7. Digits take on different values depending [on] where they are placed — 12 differs from 21 and the two 1s in 11 have different values.

8. Yes.

9. To mark places; thus 0;001 differs from 0;1 as do 0.001 and 0.1.

ᘔ. That a good number base should be reasonably small and have lots of factors.

Ɛ. ᘔ (for ten) called "dek" and Ɛ (for eleven) called "el."

10. "Do" (as in "do re mi") means 1 twelve + 0.

11.

1;	2;	3;	4;	5;	6;	7;	8;	9;	ᘔ;	Ɛ;	10;
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
11;	12;	13;	14;	15;	16;	17;	18;	19;	1ᘔ;	1Ɛ;	20;
13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.

21;	22;	23;	24;	25;	26;	27;	28;	29;	2Ƨ;	2Ɛ;	30;
25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
31;	32;	33;	34;	35;	36;	37;	38;	39;	3Ƨ;	3Ɛ;	40;
37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	48.
41;	42;	43;	44;	45;	46;	47;	48;	49;	4Ƨ;	4Ɛ;	50;
49.	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
51;	52;	53;	54;	55;	56;	57;	58;	59;	5Ƨ;	5Ɛ;	60;
61.	62.	63.	64.	65.	66.	67.	68.	69.	70.	71.	72.
61;	62;	63;	64;	65;	66;	67;	68;	69;	6Ƨ;	6Ɛ;	70;
73.	74.	75.	76.	77.	78.	79.	80.	81.	82.	83.	84.
71;	72;	73;	74;	75;	76;	77;	78;	79;	7Ƨ;	7Ɛ;	80;
85.	86.	87.	88.	89.	90.	91.	92.	93.	94.	95.	96.
81;	82;	83;	84;	85;	86;	87;	88;	89;	8Ƨ;	8Ɛ;	90;
97.	98.	99.	100.	101.	102.	103.	104.	105.	106.	107.	108.
91;	92;	93;	94;	95;	96;	97;	98;	99;	9Ƨ;	9Ɛ;	Ƨ0;
109.	110.	111.	112.	113.	114.	115.	116.	117.	118.	119.	120.
Ƨ1;	Ƨ2;	Ƨ3;	Ƨ4;	Ƨ5;	Ƨ6;	Ƨ7;	Ƨ8;	Ƨ9;	ƧƧ;	ƧƐ;	Ɛ0;
121.	122.	123.	124.	125.	126.	127.	128.	129.	130.	131.	132.
Ɛ1;	Ɛ2;	Ɛ3;	Ɛ4;	Ɛ5;	Ɛ6;	Ɛ7;	Ɛ8;	Ɛ9;	ƐƧ;	ƐƐ;	100;
133.	134.	135.	136.	137.	138.	139.	140.	141.	142.	143.	144.

12. 14.

13. (a) One "do." (b) Because it is one dozen.

14. (a) 100; (b) One gro [as in "grow"]; (c) because it is one gross.

15. 1×10 — 2×6 — 3×4 — 4×3 — 6×2 — 10; $\times 1$.

16. $1 \times 10.$ — 2×5 — 5×2 — $10. \times 1$.

17. 63,360.

18. 3 0800;.

19. 4Ƨ; — Ɛ80; — 1 0211;.

1Ƨ. 196; — 1959;.

1Ɛ. 12; — 26; — 70; — 84;.

20. 16.; — 36.; — 100.; — 144.

Answers for Test Number 2

1. 1000 — 2 3608.

2. 1Ƨ ƧƐ15Ɛ — 1Ɛ6 0713 — 2 Ɛ378 — 4Ɛ7 79ƐƐ — 35Ƨ3 5168.

3. 1142 — 3 615Ɛ — 2222 22 — 6 6667 — 34 5678.

4. (a) 1 + 6 + 8 + 1 = 14.
(b) 1 + 6 + 1 and 8 6 8 and 8.
(c) 14 + 17 + 10 + 1 = 40.
(d) 40 - 3 = 39.

5.

(a)

N	E	S	W
47	32	18	35
16	13	46	3
61	51	62	44

= 198 miles

(b)

N–S	E–W
62	51
− 61	− 44
9	1

$(9^2 + 1^2) = 62$; $\sqrt{62} \sim 9;1$.

6. Let V = Value of the total estate.
W gets $\frac{1}{2}V - 4(\frac{1}{10})V = (0;6 - 0;4)V = 0;2V$
A gets $\frac{1}{4}V + \frac{1}{10}V = (0;3 + 0;1)V = 0;4V$
B gets $\frac{1}{8}V + \frac{1}{10}V = (0;16 + 0;1)V = 0;26V$
C gets $\frac{1}{14}V + \frac{1}{10}V = (0;09 + 0;1)V = 0;19V$
D gets $\frac{1}{28}V + \frac{1}{10}V = (0;046 + 0;1)V = 0;146V = \$926$
L gets $(1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{14} - \frac{1}{28})V = 0;046V$
From D we know that $\frac{1}{28}V + \frac{1}{10}V = \$926$.
The LCM [lowest common denominator] of 28 and 10 is 80. Multiplying by 80 we obtain:
$3V + 8V = 926(80)$
$\text{Ɛ}V = 6\,7000$
$V = 6\,7000 / \text{Ɛ} = \$7222;22$
Thus:
W gets $0;2(\$7222;22) = \$1244;44$
A gets $0;4(\$7222;22) = \$2488;89$
B gets $0;26(\$7222;22) = \$15\text{Ɛ}5;55$
C gets $0;19(\$7222;22) = \$1069;92$
D gets $0;146(\$7222;22) = \$926;00$
L gets $0;046(\$7222;22) = \$283;92$
$\$7222;22$

7. (a) $2^{24} = 75\text{Ƨ}9\,4714$.
(b) $2^{25} - 1 = 1\,2\text{Ɛ}96\,9227$
(c) $2^{25} = 1\,2\text{Ɛ}96\,9228$ and $2^{26} - 1 = 2\,5\text{Ɛ}71\,6453$.
(d) $2^{23} = 36\text{Ɛ}4\,8368$ and $2^{24} - 1 = 75\text{Ƨ}9\,4713$.

8. Nothing, zero, 0.

Answers for Test Number 3

1. 207 8094 — 2 013Ƨ ƐƐ46 — 931 Ɛ55Ɛ Ƨ768.

2. 165 — 14Ƨ4 — 12Ɛ6 — 134.

3.

Points	Degrees	Points	Degrees
3	Ƨ0°	18	16°
4	76°	20	13°
5	60°	26	10°
6	50°	30	Ƨ°
8	39°	34	9°
9	34°	39	8°
Ƨ	30°	50	6°
10	26°	60	5°
13	20°	76	4°
16	18°	Ƨ0	3°
		130	2°

4.

	2	3	4	5	6	7	8	9	Ƨ	Ɛ	10	11	12
2	4	6	8	Ƨ	10	12	14	16	17	1Ƨ	20	22	24
3	6	9	10	13	16	19	20	23	26	29	30	33	36
4	8	10	14	18	20	24	28	30	34	38	40	44	48
5	Ƨ	13	18	21	26	2Ɛ	34	39	42	47	50	55	5Ƨ
6	10	16	20	26	30	36	40	46	50	56	60	66	70
7	12	19	24	2Ɛ	36	41	48	53	5Ƨ	65	70	77	82
8	14	20	28	34	40	48	54	60	68	74	80	88	94
9	16	23	30	39	46	53	60	69	76	83	90	99	9Ƨ
Ƨ	18	26	34	42	50	5Ƨ	68	76	84	92	Ƨ0	ƨƨ	Ɛ8
Ɛ	1Ƨ	29	38	47	56	65	74	83	92	Ƨ1	Ɛ0	ƐƐ	10Ƨ
10	20	30	40	50	60	70	80	90	Ƨ0	Ɛ0	100	110	120
11	22	33	44	55	66	77	88	99	ƨƨ	ƐƐ	110	121	132
12	24	36	48	5Ƨ	70	82	94	9Ƨ	Ɛ8	10Ƨ	120	132	144

5. C = $2\pi r$ = 2(3;1848)(2718 Ɛ540) = 1 4383 9254;194. 1 4384 0000 miles correct to 5 significant digits.

6. 2 098Ɛ 9880 = $2\pi r$ = 2(3;1848)r.
r = 2 098Ɛ 9880 ÷ 6;3494 = 3Ɛ47 Ɛ000 miles.
d = 1 4384 0000 − 3Ɛ47 Ɛ000 = 1 0438 0000.

7. Eight:

$$
\begin{aligned}
{}^{1}/_{2}(100.) &= 50.\\
{}^{1}/_{4}(100.) &= 25.\\
{}^{1}/_{5}(100.) &= 20.\\
{}^{1}/_{10.}(100.) &= 10.\\
{}^{1}/_{20.}(100.) &= 5\\
{}^{1}/_{25.}(100.) &= 4\\
{}^{1}/_{50.}(100.) &= 2\\
{}^{1}/_{100.}(100.) &= 1
\end{aligned}
$$

One dozen two:

$$
\begin{aligned}
{}^{1}/_{2}(100;) &= 60;\\
{}^{1}/_{3}(100;) &= 40;\\
{}^{1}/_{4}(100;) &= 30;\\
{}^{1}/_{6}(100;) &= 20;\\
{}^{1}/_{8}(100;) &= 16;\\
{}^{1}/_{9}(100;) &= 14;\\
{}^{1}/_{10;}(100;) &= 10;\\
{}^{1}/_{14;}(100;) &= 9\\
{}^{1}/_{16;}(100;) &= 8\\
{}^{1}/_{20;}(100;) &= 6\\
{}^{1}/_{30;}(100;) &= 4\\
{}^{1}/_{40;}(100;) &= 3\\
{}^{1}/_{60;}(100;) &= 2\\
{}^{1}/_{100;}(100;) &= 1
\end{aligned}
$$

12; (14.) is $1\,{}^{3}/_{4}$ or 1;9 (1.75) times as great as 8.

8. 32 / 1760. = 1 / 55. = 0.0181818...1.8%.
28 / 1028; = 1 / 47; = 0;027502750275...2;7 ${}^{\text{p}}/_{\text{g}}$.

Answers to Test Number 4

1. 1,920. = 1140;
18,671. = ƻ97Ɛ;
324,810. = 13 7Ɛ76;

2. See Exercise 1.

3. ${}^{13}/_{44}$. = ${}^{11}/_{38}$;
${}^{189}/_{670}$. = ${}^{139}/_{47ƻ}$;
${}^{99}/_{100}$. = ${}^{83}/_{84}$;

4. See Exercise 3.

5. $0.37 = 0;453 = 0;45$
$0.144 = 0;1889 = 0;189$
$0.6336 = 0;772\mathcal{Z}\,3 = 0;772\mathcal{Z}$
$0.916\,667 = 0;\mathcal{E}000\,00\mathcal{E} = 0;\mathcal{E}000\,01$

6. Use Rule 6 for the following:
$0;\mathcal{Z}972\,4\mathcal{Z} = 0.900\,000\,1 = 0.900\,000$
$0;3724\,97 = 0.299\,999\,9 = 0.300\,000$
$0;0555\,55 = 0.037\,878\,6 = 0.037\,879$

7. $18.62 = 16;753 = 16;75 = 18.618 = 18.62.$

8. Convert the following by Rule 7, and back by Rule 8: $3937. = 2341; = 3937\dot{}$
$393.7 = 289;84 = 289;8 = 393.666\ldots = 393.7$
$39.37 = 33;453 = 33;45 = 39.368 = 39.37$
$3.937 = 3;\mathcal{E}2\mathcal{E}1 = 3;\mathcal{E}2\mathcal{E} = 3.9369 = 9.937$
$0.393\,7 = 0;4883\,9 = 0;4884 = 0.393\,71 = 0.393\,7$

9. $513{,}292. + 144\,2661; =$
$20\,9064; + 144\,2661 = 164\,\mathcal{E}705$

Errata: Q3:5: replaced a "p" with π, which was clearly the intent, and added a verb into the question. Q4:9: added a "+." A2:6: Subtracted one from both "\$1070;9$\mathcal{Z}$" and "\$284;9$\mathcal{Z}$," to reflect the correct answers. A3:6: Added a "=." A3:8: corrected incorrect rounding of 2;75 to 2;8. A4:9: corrected "144 2691" to "144 2661" and "164 $\mathcal{E}$735" to "164 $\mathcal{E}$705," to reflect the correct answers. Finally, in preamble to Test 4, changed the link and the language somewhat concerning the link to the conversion rules sheet.

Throughout: typeset dozenal number grouped by fours separated by small spaces, decimal numbers by threes separated by commas, or by small spaces in the fractional parts; also throughout, used Pitman symbols "$\mathcal{Z}$" and "$\mathcal{E}$" rather than the crossed "X" and "#" of the DSA during the time the digital copies were created.

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